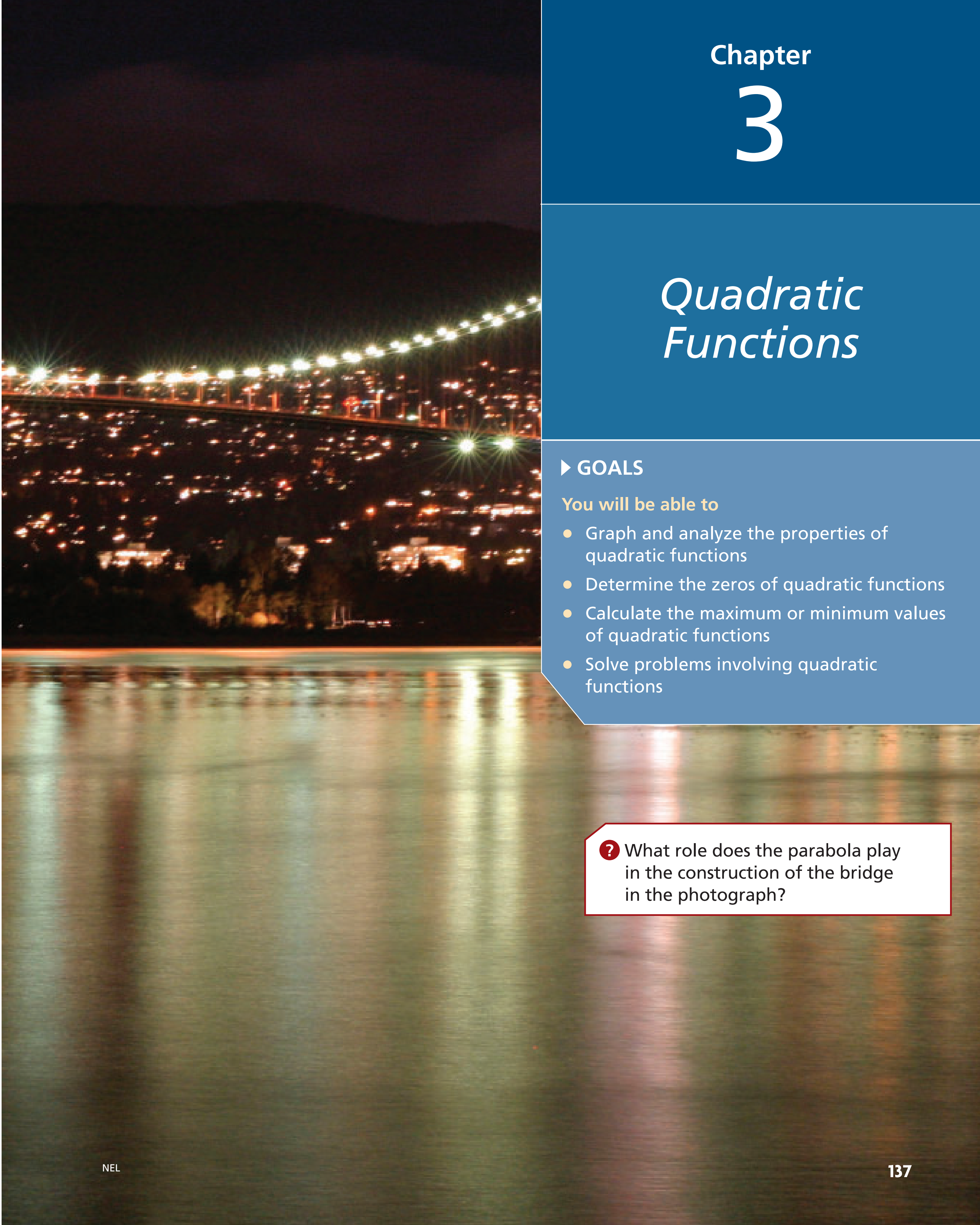


Chapter 3

Quadratic Functions

GOALS

You will be able to

- Graph and analyze the properties of quadratic functions
- Determine the zeros of quadratic functions
- Calculate the maximum or minimum values of quadratic functions
- Solve problems involving quadratic functions

? What role does the parabola play in the construction of the bridge in the photograph?

3 Getting Started

SKILLS AND CONCEPTS You Need

Study | *Aid*

- For help, see Essential Skills Appendix.

Question	Appendix
1	A-7
2	A-8
3, 4, 7	A-12
5, 6	A-9, A-10

1. Given $f(x) = -3x^2 + 4x - 1$, evaluate the following.

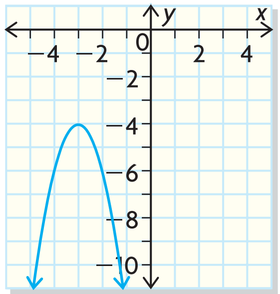
a) $f(1)$
b) $f(-2)$
c) $f\left(\frac{1}{3}\right)$
d) $f(0)$
e) $f(k)$
f) $f(-k)$

2. Express each function in standard form, $f(x) = ax^2 + bx + c$.

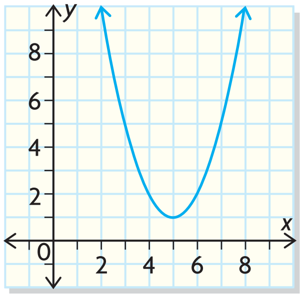
a) $f(x) = (x - 3)(x + 5)$
b) $f(x) = 2x(x + 6)$
c) $f(x) = -3(x + 2)^2 + 3$
d) $f(x) = (x - 1)^2$

3. State the **vertex**, **axis of symmetry**, **domain**, and **range** of each parabola.

a)



b)



4. State the vertex, axis of symmetry, and **direction of opening** for each parabola.

a) $y = x^2 + 4$
b) $y = 3(x - 4)^2 + 1$
c) $y = -0.5(x + 7)^2 - 3$
d) $y = -3(x + 2)(x - 5)$

5. Solve each equation. Answer to two decimal places if necessary.

a) $x^2 - 11x + 24 = 0$
b) $x^2 - 6x + 3 = 0$
c) $3x^2 - 2x - 5 = 0$
d) $3x^2 + 2x = x^2 + 9x - 3$

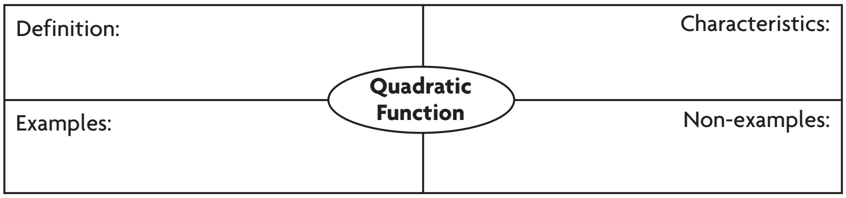
6. Determine the *x*-intercepts of each function. Answer to two decimal places if necessary.

a) $f(x) = x^2 - 9$
b) $f(x) = x^2 - 8x - 18$
c) $f(x) = -3x^2 + 10x - 8$
d) $f(x) = 6x - 2x^2$

7. Sketch the graph of each function.

a) $f(x) = x^2 + 3$
b) $f(x) = 2x^2 - 4$
c) $f(x) = -(x - 2)(x + 8)$
d) $f(x) = -(x + 2)^2 + 3$

8. Complete the chart by writing what you know about quadratic functions.



APPLYING What You Know

Building Rectangles

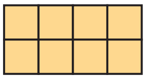
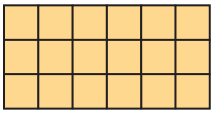
A set of rectangles can be formed out of 1 cm squares on centimetre grid paper. Martina draws the first rectangle with dimensions 1 cm × 2 cm, the second 2 cm × 4 cm, and the third 3 cm × 6 cm.

YOU WILL NEED

- centimetre grid paper
- graph paper

If this pattern continues, what function can be used to model the relationship between width and area?

A. Use centimetre grid paper to draw the next four rectangles in the pattern. Use your diagrams to extend and complete the table.

Shape	Width (cm)	Length (cm)	Perimeter (cm)	Area (cm)^2
	1	2	6	2
	2	4	12	8
	3	6	18	

B. Calculate the first differences for the Perimeter and Area columns. Is the relation between width and perimeter linear or nonlinear? How do you know?

C. Is the relation between width and area linear or nonlinear? How do you know?

D. Determine the second differences for the Area column. What do they tell you about the relationship between width and area?

E. Create a scatter plot of area versus width. Draw a curve of good fit. Does the shape of your graph support your answer to part D? Explain.

F. What is the relationship between length and width for each rectangle?

G. Write the function that models the relationship between width and area.

3.1 Properties of Quadratic Functions

GOAL

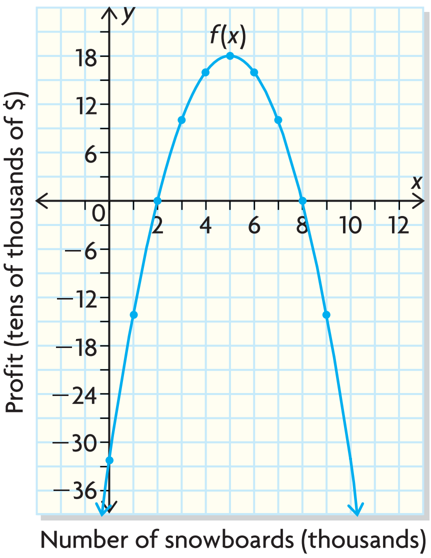
Represent and interpret quadratic functions in a number of different forms.

LEARN ABOUT the Math

Francisco owns a business that sells snowboards. His accountants have presented him with data on the business' profit in a table and a graph.

Snowboards Sold, *x* (1000s)	0	1	2	3	4	5	6	7	8	9
Profit, *f*(*x*) ($10 000s)	−32	−14	0	10	16	18	16	10	0	−14

Profit from Snowboard Sales



? What function models Francisco's profit?

EXAMPLE 1 Selecting a strategy to describe the algebraic model

Develop an algebraic expression for the function that models Francisco's profit from selling snowboards.

Kelly's Solution: Using the Vertex Form of the Quadratic Function

Snowboards Sold (1000s)	Profit ($10 000s)	First Differences	Second Differences
0	−32		
		18	
1	−14		−4
		14	
2	0		−4
		10	
3	10		−4
		6	
4	16		−4
		2	
5	18		−4
		−2	
6	16		−4
		−6	
7	10		−4
		−10	
8	0		−4
		−14	
9	−14		

This function looks quadratic, since its graph appears to be a parabola. To make sure, I checked the first and second differences. Since the first differences are not constant, the function is nonlinear. The second differences are all equal and negative. So the function is quadratic. This confirms that the graph is a parabola that opens downward.

From the graph, the vertex is (5, 18).
The parabola also passes through (2, 0).

I could determine the quadratic function model if I knew the vertex and at least one other point on the graph.

$$f(x) = a(x - h)^2 + k$$

$$= a(x - 5)^2 + 18$$

I used the **vertex form** of the quadratic function and substituted the coordinates of the vertex from the graph.

$$0 = a(2 - 5)^2 + 18$$

$$0 = 9a + 18$$

$$-18 = 9a$$

$$-2 = a$$

$f(2) = 0$, so I substituted (2, 0) into the function. Once I did that, I solved for a.

The function $f(x) = -2(x - 5)^2 + 18$ models Francisco's profit.

Jack's Solution: Using the Factored Form of the Quadratic Function

The graph is a parabola, opening down with axis of symmetry $x = 5$.

The graph looks like a parabola, so it has to be a quadratic function. The graph is symmetric about the line $x = 5$.

The x-intercepts are the points $(2, 0)$ and $(8, 0)$.

I could find the **factored form** of the quadratic function if I knew the x-intercepts, or zeros. I read these from the graph and used the table of values to check.

$$f(x) = a(x - r)(x - s)$$

I took the factored form of a quadratic function and substituted the values of the x-intercepts for r and s.

$$f(x) = a(x - 2)(x - 8)$$

$$10 = a(3 - 2)(3 - 8)$$

I then chose the point (3, 10) from the table of values and substituted its coordinates into $f(x)$ to help me find the value of a.

$$10 = a(1)(-5)$$

$$10 = -5a$$

$$-2 = a$$

The function $f(x) = -2(x - 2)(x - 8)$ models Francisco's profit.

Reflecting

A. What information do you need to write the vertex form of the quadratic function?

B. What information do you need to write the factored form of the quadratic function?

C. Use the graph to state the domain and range of the function that models Francisco's profit. Explain.

D. Will both of the models for Francisco's profit lead to the same function when expressed in **standard form**?

APPLY the Math

EXAMPLE 2 Determining the properties of a quadratic function

A construction worker repairing a window tosses a tool to his partner across the street. The height of the tool above the ground is modelled by the quadratic function $h(t) = -5t^2 + 20t + 25$, where $h(t)$ is height in metres and t is the time in seconds after the toss.

a) How high above the ground is the window?
b) If his partner misses the tool, when will it hit the ground?
c) If the path of the tool's height were graphed, where would the axis of symmetry be?
d) Determine the domain and range of the function in this situation.

André's Solution

a)
$$\begin{aligned} h(0) &= -5(0)^2 + 20(0) + 25 \\ &= 25 \end{aligned}$$

The height of the window must be the value of the function at $t = 0$ s.

The window is 25 m above the ground.

b)
$$\begin{aligned} 0 &= -5t^2 + 20t + 25 \\ 0 &= -5(t^2 - 4t - 5) \\ 0 &= -5(t + 1)(t - 5) \end{aligned}$$

If the partner missed the tool, it would hit the ground. The height at the ground is zero. I set $h(t)$ equal to zero. Then I factored the quadratic.

$$t = -1 \quad \text{or} \quad t = 5$$

I found two values for t, but the negative answer is not possible, since time must be positive.

The tool will hit the ground after 5 s.

c)
$$t = \frac{-1 + 5}{2}$$
$$t = \frac{4}{2}$$
$$t = 2$$

The axis of symmetry passes through the midpoint of the two zeros of the function. I added the zeros together and divided by two to find that t-value. The axis of symmetry is a vertical line, so its equation is $t = 2$.

The axis of symmetry is $t = 2$.

d) Domain $= \{t \in \mathbf{R} \mid 0 \le t \le 5\}$

In this situation, t must be 0 or greater, and the tool will stop when it hits the ground.

$$\begin{aligned} h(2) &= -5(2)^2 + 20(2) + 25 \\ &= -20 + 40 + 25 \\ &= 45 \end{aligned}$$

The least possible value of $h(t)$ is zero. I found the value of h at $t = 2$, since the greatest value is the y-coordinate of the vertex, which is always on the axis of symmetry.

Range $= \{h \in \mathbf{R} \mid 0 \le h \le 45\}$

EXAMPLE 3 Graphing a quadratic function from the vertex form

Given $f(x) = 2(x - 1)^2 - 5$, state the vertex, axis of symmetry, direction of opening, y-intercept, domain, and range. Graph the function.

Sacha's Solution

Vertex: $(1, -5)$

The x-coordinate of the vertex is 1 and the y-coordinate is -5.

Axis of symmetry: $x = 1$

The axis of symmetry is a vertical line through the vertex at $(1, -5)$.

Direction of opening: up

Since a is positive, the parabola opens up.

$$\begin{aligned} f(0) &= 2(0 - 1)^2 - 5 \\ &= 2(-1)^2 - 5 \\ &= 2 - 5 \\ &= -3 \end{aligned}$$

I substituted $x = 0$ to calculate the y-intercept and solved the equation.

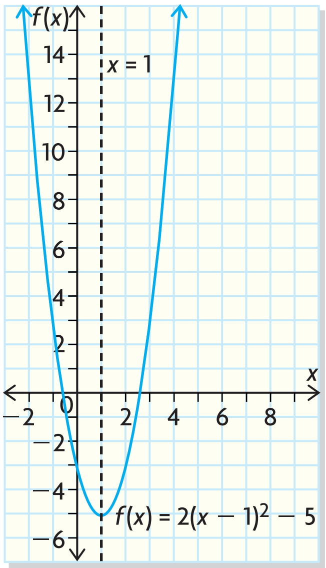
y-intercept: -3

Domain: $\{x \in \mathbf{R}\}$

There are no restrictions on the values for x.

Range: $\{y \in \mathbf{R} \mid y \geq -5\}$

Because the vertex has a y-value of -5 and the parabola opens up, the y-values have to be greater than or equal to -5.



To graph the function, I plotted the vertex and the axis of symmetry.

I found the values of $f(x)$ when $x = 2$, 3, and 4.

$f(2) = -3$

$f(3) = 3$

$f(4) = 13$

The values had to be the same for 0, -1, and -2 because the graph is symmetric about the line $x = 1$. I plotted the points and joined them with a smooth curve.

In Summary

Key Ideas

- Graphs of quadratic functions with no domain restrictions are parabolas.
- Quadratic functions have constant nonzero second differences. If the second differences are positive, the parabola opens up and the coefficient of x^2 is positive. If the second differences are negative, the parabola opens down and the coefficient of x^2 is negative.

Need to Know

- Quadratic functions can be represented by equations in function notation, by tables of values, or by graphs.
- Quadratic functions have a degree of 2.
- Quadratic functions can be expressed in different algebraic forms:
  - standard form: $f(x) = ax^2 + bx + c, a \neq 0$
  - factored form: $f(x) = a(x - r)(x - s), a \neq 0$
  - vertex form: $f(x) = a(x - h)^2 + k, a \neq 0$

CHECK *Your understanding*

1. Determine whether each function is linear or quadratic. Give a reason for your answer.

a)

x	y
−2	15
−1	11
0	7
1	3
2	−1

b)

x	y
−2	1
−1	3
0	6
1	10
2	15

c)

x	y
−2	4
−1	8
0	12
1	16
2	20

d)

x	y
−2	7
−1	4
0	3
1	4
2	7

2. State whether each parabola opens up or down.

a) $f(x) = 3x^2$

b) $f(x) = -2(x - 3)(x + 1)$

c) $f(x) = -(x + 5)^2 - 1$

d) $f(x) = \frac{2}{3}x^2 - 2x - 1$

3. Given $f(x) = -3(x - 2)(x + 6)$, state

a) the zeros

b) the direction of opening

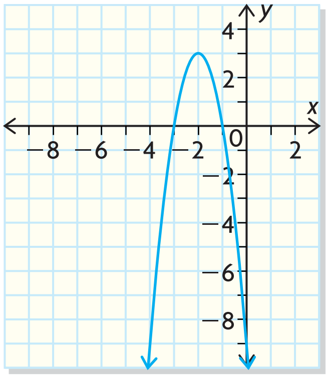
c) the equation of the axis of symmetry

4. Given the parabola at the right, state

a) the vertex

b) the equation of the axis of symmetry

c) the domain and range



PRACTISING

5. Graph each function. State the direction of opening, the vertex, and the equation of the axis of symmetry.

a) $f(x) = x^2 - 3$

b) $f(x) = -(x + 3)^2 - 4$

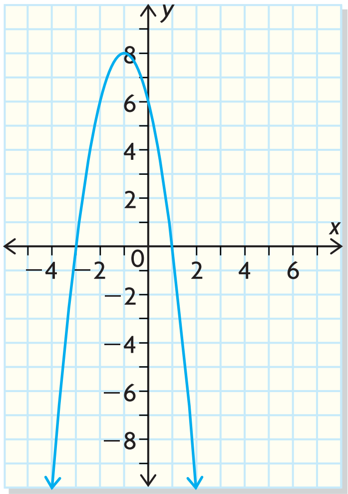
c) $f(x) = 2(x - 4)(x + 2)$

d) $f(x) = -\frac{1}{2}x^2 + 4$

6. Express each quadratic function in standard form. State the y-intercept of each.

a) $f(x) = -3(x - 1)^2 + 6$

b) $f(x) = 4(x - 3)(x + 7)$



7. Examine the parabola at the left.

K **a)** State the direction of opening.

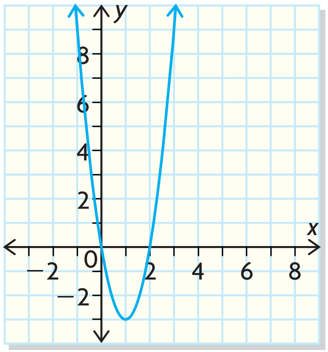
b) Name the coordinates of the vertex.

c) List the values of the x-intercepts.

d) State the domain and range of the function.

e) If you calculated the second differences, what would their sign be? How do you know?

f) Determine the algebraic model for this quadratic function.



8. Examine the parabola at the left.

a) State the direction of opening.

b) Find the coordinates of the vertex.

c) What is the equation of the axis of symmetry?

d) State the domain and range of the function.

e) If you calculated the second differences, what would their sign be? Explain.

9. Each pair of points (x, y) are the same distance from the vertex of their parabola. Determine the equation of the axis of symmetry of each parabola.

a) $(-2, 2), (2, 2)$

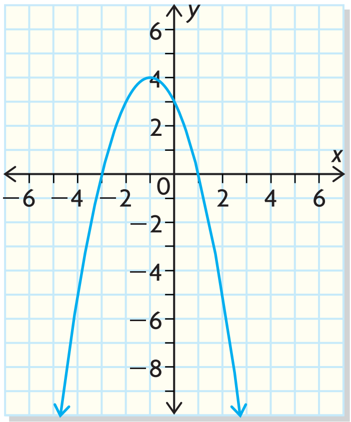
b) $(-9, 1), (-5, 1)$

c) $(6, 3), (18, 3)$

d) $(-5, 7), (1, 7)$

e) $(-6, -1), (3, -1)$

f) $\left(-\frac{11}{8}, 0\right), \left(\frac{3}{4}, 0\right)$



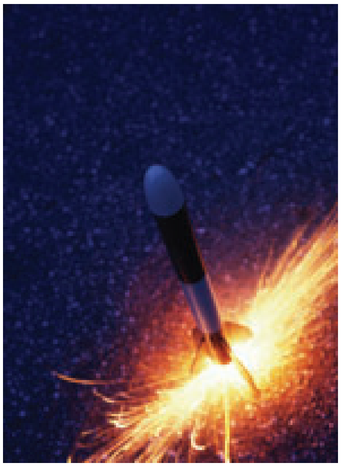
10. Examine the parabola shown at the left.

a) Copy and complete this table.

x	−2	−1	0	1	2
***f*(*x*)**					

b) Calculate the second differences of the function. How could you have predicted their signs?

c) Determine the equation of the function.



11. The height of a rocket above the ground is modelled by the quadratic
A function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

- **a)** Graph the quadratic function.
- **b)** How long will the rocket be in the air? How do you know?
- **c)** How high will the rocket be after 3 s?
- **d)** What is the maximum height that the rocket will reach?

12. A quadratic function has these characteristics:
T
- $x = -1$ is the equation of the axis of symmetry.
- $x = 3$ is the x-intercept.
- $y = 32$ is the maximum value.

Determine the y-intercept of this parabola.

13. Describe two ways in which the functions $f(x) = 2x^2 - 4x$ and
C $g(x) = -(x - 1)^2 + 2$ are alike, and two ways in which they are different.

Extending

14. The first differences and second differences of a quadratic function with domain ranging from $x = -2$ to $x = 3$ are given. If $f(-2) = 19$, copy the table and complete the second row by determining the missing values of the function.

x	−2	−1	0	1	2	3
***f*(*x*)**	19					
First Differences		−10	−6	−2	2	6
Second Differences		4	4	4	4	

15. A company's profit, in thousands of dollars, on sales of computers is modelled by the function $P(x) = -2(x - 3)^2 + 50$, where x is in thousands of computers sold. The company's profit, in thousands of dollars, on sales of stereo systems is modelled by the function $P(x) = -(x - 2)(x - 7)$, where x is in thousands of stereo systems sold. Calculate the maximum profit the business can earn.

16. Jim has a difficult golf shot to make. His ball is 100 m from the hole. He wants the ball to land 5 m in front of the hole, so it can roll to the hole. A 20 m tree is between his ball and the hole, 40 m from the hole and 60 m from Jim's ball. With the base of the tree as the origin, write an algebraic expression to model the height of the ball if it just clears the top of the tree.

3.2 Determining Maximum and Minimum Values of a Quadratic Function

GOAL

Use a variety of strategies to determine the maximum or minimum value of a quadratic function.

LEARN ABOUT the Math

A golfer attempts to hit a golf ball over a gorge from a platform above the ground. The function that models the height of the ball is $h(t) = -5t^2 + 40t + 100$, where $h(t)$ is the height in metres at time t seconds after contact. There are power lines 185 m above the ground.

? Will the golf ball hit the power lines?

EXAMPLE 1 Selecting a strategy to find the vertex

Using the function for the golf ball's height, determine whether the ball will hit the power line.

Jonah's Solution: Completing the Square

$h(t) = -5t^2 + 40t + 100$

I needed to find the maximum height of the ball to compare it to the height of the power lines.

a is negative. The graph of *h*(*t*) is a parabola that opens down. Its maximum value occurs at the vertex.

$h(t) = -5(t^2 - 8t) + 100$

I put the function into vertex form by **completing the square**.

I factored -5 from the t^2 and t terms.

$= -5(t^2 - 8t + 16 - 16) + 100$

I divided the coefficient of t in half, then squared it to create a perfect-square trinomial.

By adding 16, I changed the value of the expression. To make up for this, I subtracted 16.

$$= -5(t^2 - 8t + 16) + 80 + 100$$

I grouped the first 3 terms that formed the perfect square and moved the subtracted value of 16 outside the brackets by multiplying by -5.

$$= -5(t - 4)^2 + 180$$

I factored the perfect square and collected like terms.

The vertex is (4, 180). The maximum height will be 180 m after 4 s.

Since the power lines are 185 m above the ground, the ball will not hit them.

Since the vertex is at the maximum height, the ball goes up only 180 m.

Sophia's Solution: Factoring to Determine the Zeros

$$h(t) = -5t^2 + 40t + 100$$

The maximum height of the golf ball is at the vertex of the parabola.

The vertex is located on the axis of symmetry, which is always in the middle of the two zeros of the function. To find the zeros, I factored the quadratic.

$$h(t) = -5(t^2 - 8t - 20)$$

$$h(t) = -5(t - 10)(t + 2)$$

$$0 = -5(t - 10)(t + 2)$$

I divided -5 out as a common factor. Inside the brackets was a simple trinomial I could factor.

$$t = 10 \quad \text{or} \quad t = -2$$

The zeros are the values that make $h(t) = 0$. I found them by setting each factor equal to 0 and solving the resulting equations.

For the axis of symmetry,

$$t = \frac{10 + (-2)}{2}$$

$$t = \frac{8}{2}$$

$$t = 4$$

I added the zeros and divided the result by 2 to locate the axis of symmetry. This was also the *x*-coordinate, or in this case, the *t*-coordinate of the vertex.

The *t*-coordinate of the vertex is 4.

$$h(4) = -5(4 - 10)(4 + 2)$$

$$= -5(-6)(6)$$

$$= 180$$

To find the *y*-value, or height *h*, I substituted $t = 4$ into the factored form of the equation. Alternatively, I could have substituted into the function in standard form.

The vertex is (4, 180). The maximum height will be 180 m, after 4 s. Since the power lines are 185 m above the ground, the ball will not hit them.

Reflecting

A. How can you tell from the algebraic form of a quadratic function whether the function has a maximum or a minimum value?

B. Compare the two methods for determining the vertex of a quadratic function. How are they the same? How are they different?

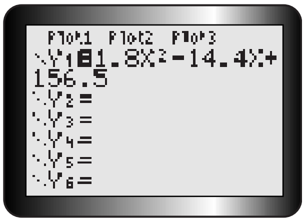
C. Not all quadratic functions have zeros. Which method allows you to find the vertex without finding the zeros? Explain.

APPLY the Math

EXAMPLE 2 Using the graphing calculator as a strategy to determine the minimum value

The cost, $c(x)$, in dollars per hour of running a certain steamboat is modelled by the quadratic function $c(x) = 1.8x^2 - 14.4x + 156.5$, where x is the speed in kilometres per hour. At what speed should the boat travel to achieve the minimum cost?

Rita's Solution



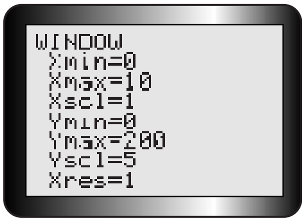
```
Plot1 Plot2 Plot3
\Y1=1.8X²-14.4X+
156.5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

This parabola opens up. Therefore, the minimum value will be at the vertex.

I used a graphing calculator because the numbers in the question were decimals and it would not have been easy to complete the square or factor.

Tech Support

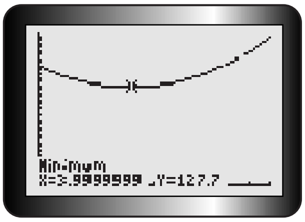
For help using the graphing calculator to determine the minimum value of a function, see Technical Appendix, B-9.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=200
 Yscl=5
 Xres=1
```

I chose a **WINDOW** that would display the graph. I chose both the *x*- and *y*-values to have a minimum of 0, since neither cost nor speed could be negative.

I picked a maximum *x* of 10 and an *x* scale of 1. I estimated the corresponding maximum *y* from the function.



Minimum
X=3.9999999 Y=127.7

I used the minimum operation to locate the vertex.

The minimum cost to operate the steamboat is \$127.70/h, when the boat is travelling at about 4 km/h.

The vertex is (4,127.70).

EXAMPLE 3 Solving a problem to determine when the maximum value occurs

The demand function for a new magazine is $p(x) = -6x + 40$, where $p(x)$ represents the selling price, in thousands of dollars, of the magazine and x is the number sold, in thousands. The cost function is $C(x) = 4x + 48$. Calculate the maximum profit and the number of magazines sold that will produce the maximum profit.

Communication ***Tip***

The demand function $p(x)$ is the relation between the price of an item and the number of items sold, x. The cost function $C(x)$ is the total cost of making x items. Revenue is the money brought in by selling x items. Revenue is the product of the demand function and the number sold. Profit is the difference between revenue and cost.

Levi's Solution

$$\begin{aligned}\text{Revenue} &= \text{Demand} \times \text{Number sold}\\ &= [p(x)](x)\end{aligned}$$

I found the revenue function by multiplying the demand function by the number of magazines sold.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

To find the profit function, I subtracted the cost function from the revenue function and simplified.

$$\begin{aligned}P(x) &= [p(x)](x) - C(x)\\ &= (-6x + 40)(x) - (4x + 48)\\ &= -6x^2 + 40x - 4x - 48\\ &= -6x^2 + 36x - 48\end{aligned}$$

The coefficient of x^2 is negative, so the parabola opens down with its maximum value at the vertex. Instead of completing the square, I determined two points symmetrically opposite each other.

$$P(x) = -6x(x - 6) - 48$$

I started by factoring the common factor $-6x$ from $-6x^2$ and $36x$.

$$-6x(x-6)=0$$

$$x=0 \quad \text{or} \quad x=6$$

Points on the graph of the profit function are $(0, -48)$ and $(6, -48)$.

I knew that the x-intercepts of the graph of $y = -6x(x - 6)$ would help me find the two points I needed on the graph of the profit function, since both functions have the same axis of symmetry.

The axis of symmetry is $x = \frac{0+6}{2}$ or $x = 3$. So the x-coordinate of the vertex is 3.

I found the axis of symmetry, which gave me the x-coordinate of the vertex.

$$\begin{aligned} P(3) &= -6(3)^2 + 36(3) - 48 \\ &= -54 + 108 - 48 \\ &= 6 \end{aligned}$$

The maximum profit is \$6000, when 3000 magazines are sold.

I substituted $x = 3$ into the function to determine the profit. I remembered that x is in *thousands* of magazines sold, and $P(x)$ is in *thousands* of dollars.

In Summary

Key Idea

- The maximum or minimum value of a quadratic function is the y-coordinate of the vertex.

Need to Know

- If $a > 0$ in standard form, factored form, or vertex form, then the parabola opens up. The quadratic has a minimum value.
- If $a < 0$ in standard form, factored form, or vertex form, then the parabola opens down. The quadratic has a maximum value.
- The vertex can be found from the standard form $f(x) = ax^2 + bx + c$ algebraically in several ways:
    - by completing the square to put the quadratic in vertex form $f(x) = a(x - h)^2 + k$
    - by expressing the quadratic in factored form $f(x) = a(x - r)(x - s)$, if possible, and averaging the zeros at r and s to locate the axis of symmetry. This will give the x-coordinate of the vertex
    - by factoring out the common factor from $ax^2 + bx$ to determine two points on the parabola that are symmetrically opposite each other, and averaging the x-coordinates of these points to determine the x-coordinate of the vertex
    - by using a graphing calculator

CHECK Your Understanding

1. Which of the following quadratic functions will have a maximum value? Explain how you know.

a) $y = -x^2 + 7x$

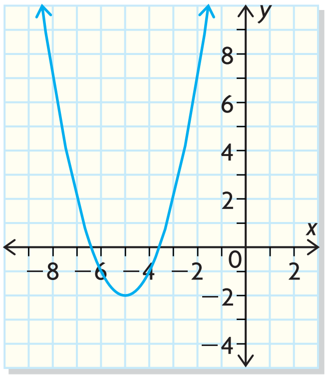
b) $f(x) = 3(x - 1)^2 - 4$

c) $f(x) = -4(x + 2)(x - 3)$

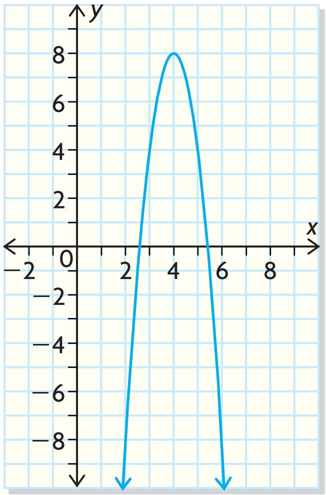
d) $g(x) = 4x^2 + 3x - 5$

2. State the vertex of each parabola and indicate the maximum or minimum value of the function.

a)



b)



3. Determine the maximum or minimum value for each.

a) $y = -4(x + 1)^2 + 6$

b) $f(x) = (x - 5)^2$

c) $f(x) = -2x(x - 4)$

d) $g(x) = 2x^2 - 7$

PRACTISING

4. Determine the maximum or minimum value. Use at least two different methods.
K

a) $y = x^2 - 4x - 1$

b) $f(x) = x^2 - 8x + 12$

c) $y = 2x^2 + 12x$

d) $y = -3x^2 - 12x + 15$

e) $y = 3x(x - 2) + 5$

f) $g(x) = -2(x + 1)^2 - 5$

5. Each function is the demand function of some item, where x is the number of items sold, in thousands. Determine

i) the revenue function

ii) the maximum revenue in thousands of dollars

a) $p(x) = -x + 5$

b) $p(x) = -4x + 12$

c) $p(x) = -0.6x + 15$

d) $p(x) = -1.2x + 4.8$

6. Use a graphing calculator to determine the maximum or minimum value. Round to two decimal places where necessary.

a) $f(x) = 2x^2 - 6.5x + 3.2$

b) $f(x) = -3.6x^2 + 4.8x$

7. For each pair of revenue and cost functions, determine

i) the profit function

ii) the value of x that maximizes profit

a) $R(x) = -x^2 + 24x,\ C(x) = 12x + 28$

b) $R(x) = -2x^2 + 32x,\ C(x) = 14x + 45$

c) $R(x) = -3x^2 + 26x,\ C(x) = 8x + 18$

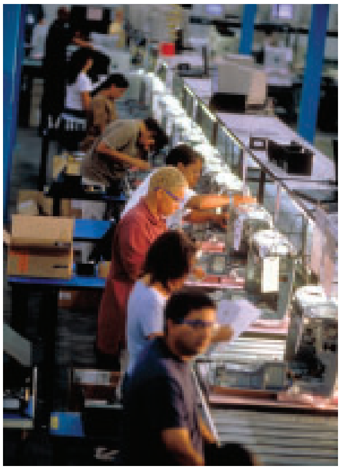
d) $R(x) = -2x^2 + 25x,\ C(x) = 3x + 17$

8. The height of a ball thrown vertically upward from a rooftop is modelled by $h(t) = -5t^2 + 20t + 50$, where $h(t)$ is the ball's height above the ground, in metres, at time t seconds after the throw.

a) Determine the maximum height of the ball.

b) How long does it take for the ball to reach its maximum height?

c) How high is the rooftop?



9. The cost function in a computer manufacturing plant is $C(x) = 0.28x^2 - 0.7x + 1$, where $C(x)$ is the cost per hour in millions of dollars and x is the number of items produced per hour in thousands. Determine the minimum production cost.

10. Show that the value of $3x^2 - 6x + 5$ cannot be less than 1.

11. **A** The profit $P(x)$ of a cosmetics company, in thousands of dollars, is given by $P(x) = -5x^2 + 400x - 2550$, where x is the amount spent on advertising, in thousands of dollars.

a) Determine the maximum profit the company can make.

b) Determine the amount spent on advertising that will result in the maximum profit.

c) What amount must be spent on advertising to obtain a profit of at least $4 000 000?

12. **T** A high school is planning to build a new playing field surrounded by a running track. The track coach wants two laps around the track to be 1000 m. The football coach wants the rectangular infield area to be as large as possible. Can both coaches be satisfied? Explain your answer.

13. **C** Compare the methods for finding the minimum value of the quadratic function $f(x) = 3x^2 - 7x + 2$. Which method would you choose for this particular function? Give a reason for your answer.

Extending

14. A rock is thrown straight up in the air from an initial height h_0, in metres, with initial velocity v_0, in metres per second. The height in metres above the ground after t seconds is given by $h(t) = -4.9t^2 + v_0t + h_0$. Find an expression for the time it takes the rock to reach its maximum height.

15. A ticket to a school dance is $8. Usually, 300 students attend. The dance committee knows that for every $1 increase in the price of a ticket, 30 fewer students attend the dance. What ticket price will maximize the revenue?

3.3

The Inverse of a Quadratic Function

GOAL

Determine the inverse of a quadratic function, given different representations.

YOU WILL NEED

- graph paper
- ruler
- graphing calculator

INVESTIGATE *the Math*

Suzanne needs to make a box in the shape of a cube. She has 864 cm^2 of cardboard to use. She wants to use all of the material provided.

? How long will each side of Suzanne's box be?

A. Copy and complete this table.

Cube Side Length (cm)	1	2	3	4	5	6	7	8	9	10
Area of Each Face (cm^2)	1	4								
Surface Area (cm^2)	6	24								

B. Draw a graph of surface area versus side length. What type of function is this? Explain how you know.

C. Determine the equation that represents the cube's surface area as a function of its side length. Use function notation and state the domain and range.

D. How would you calculate the inverse of this function to describe the side length of the cube if you know its surface area?

E. Make a table of values for the inverse of the surface area function.

F. Draw a graph of the inverse. Compare the graph of the inverse with the original graph. Is the inverse a function? Explain.

G. State the domain and range of the inverse.

H. Write the equation that represents the cube's side length for a given surface area.

I. Use your equation from part H to determine the largest cube Suzanne will be able to construct.

Reflecting

J. How are the surface area function and its inverse related

a) in the table of values?

b) in their graphs?

c) in their domains and ranges?

K. How is any quadratic function related to its inverse
  a) in their domains and ranges?
  b) in their equations?

APPLY the Math

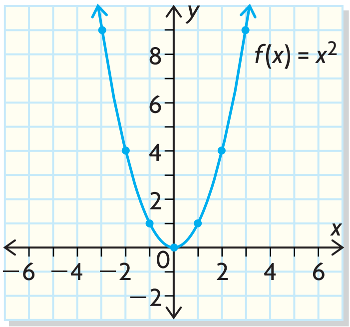
EXAMPLE 1 Determining the domain and range of the inverse of a quadratic function

Given the graph of $f(x) = x^2$,

a) graph the inverse relation

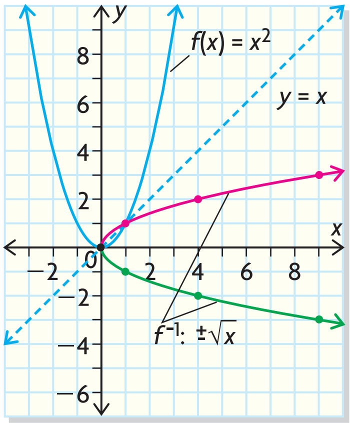
b) state the domain and range of $f(x) = x^2$ and the inverse relation

c) determine whether the inverse of $f(x) = x^2$ is also a function. Give a reason for your answer.



Paul's Solution

a)



To graph the inverse of $f(x) = x^2$, I took the coordinates of each point on the original graph and switched the x- and y-coordinates. For example, (2, 4) became (4, 2). I had to do this because the input value becomes the output value in the inverse, and vice versa.

The graph of the inverse is a reflection of the original function about the line $y = x$.

b) The domain of $f(x) = x^2$ is $\{x \in \mathbf{R}\}$. The range of $f(x) = x^2$ is $\{y \in \mathbf{R} \mid y \geq 0\}$. Therefore, the domain of f^{-1} is $\{x \in \mathbf{R} \mid x \geq 0\}$, and the range is $\{y \in \mathbf{R}\}$.

The domain of the inverse is the range of the original function. The range of the inverse is the domain of the original function.

c) The inverse of $f(x) = x^2$ is not a function.

I knew this because the inverse fails the vertical-line test: For each number in the domain except 0, there are two values in the range.

EXAMPLE 2 Determining the equation of the inverse of a quadratic function

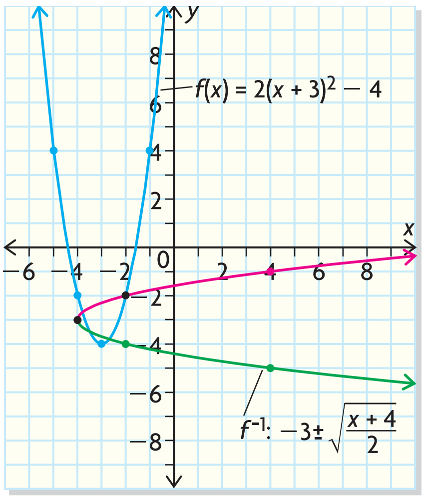
Given the quadratic function $f(x) = 2(x + 3)^2 - 4$,

a) graph $f(x)$ and its inverse

b) determine the equation of the inverse

Prashant's Solution

a)



I graphed $f(x)$ by plotting the vertex, $(-3, -4)$. The parabola opens up because the value of a is positive. I found $f(-2) = -2$ and $f(-1) = 4$, which are also the same values of $f(-4)$ and $f(-5)$, respectively.

To graph the inverse, I interchanged the x- and y-coordinates of the points on the original function.

b)

$$f(x) = 2(x + 3)^2 - 4$$
$$y = 2(x + 3)^2 - 4$$
$$x = 2(y + 3)^2 - 4$$

First I wrote the equation with y replacing $f(x)$, because y represents the output value in the function. To find the equation of the inverse, I interchanged x and y in the original function.

$$x + 4 = 2(y + 3)^2$$
$$\frac{x + 4}{2} = (y + 3)^2$$
$$\pm\sqrt{\frac{x + 4}{2}} = y + 3$$
$$-3 \pm \sqrt{\frac{x + 4}{2}} = y$$

I then rearranged the equation and solved for y by using the inverse of the operations given in the original function. I could tell from the graph of the inverse that there were two parts to the inverse function, an upper branch and a lower branch. The upper branch came from taking the positive square root of both sides, the lower from taking the negative square root.

For $f(x)$ restricted to $x \geq -3$,

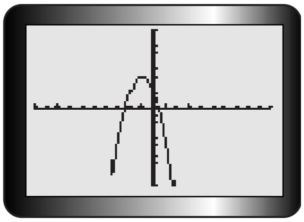
$$f^{-1}(x) = -3 + \sqrt{\frac{x + 4}{2}}$$

I couldn't write $f^{-1}(x)$ for y, since the inverse is not a function. But if I restricted the original domain to $x \geq -3$, then I would use only one branch of the inverse, and I could write it in function notation.

EXAMPLE 3 Using a graphing calculator as a strategy to graph a quadratic function and its inverse

Using a graphing calculator, graph $f(x) = -2(x + 1)^2 + 4$ and its inverse.

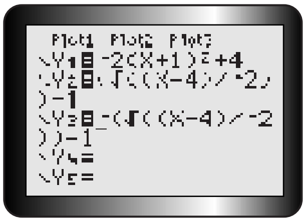
Bonnie's Solution



I entered the function into the equation editor at **Y1**. I used **WINDOW** settings $-10 \leq x \leq 10$, $-10 \leq y \leq 10$, because I knew the vertex was located at $(-1, 4)$ and the parabola opened down.

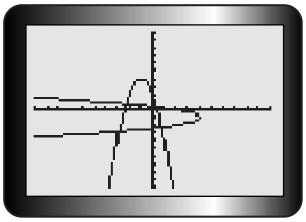
$$f(x) = -2(x + 1)^2 + 4$$
$$y = -2(x + 1)^2 + 4$$
$$x = -2(y + 1)^2 + 4$$
$$x - 4 = -2(y + 1)^2$$
$$\frac{x - 4}{-2} = (y + 1)^2$$
$$\pm\sqrt{\frac{x - 4}{-2}} = y + 1$$
$$\pm\sqrt{\frac{x - 4}{-2}} - 1 = y$$

To graph the inverse, I needed to find the equation of the inverse. I switched x and y and used inverse operations with the original function.



```
Plot1 Plot2 Plot3
\Y1=-2(X+1)²+4
\Y2=(√((X-4)/-2)
)-1
\Y3=-(√((X-4)/-2
))-1
\Y4=
\Y5=
```

I entered both parts of the inverse separately. I entered $y = \sqrt{\frac{x - 4}{-2}} - 1$ into Y2 and $y = -\sqrt{\frac{x - 4}{-2}} - 1$ into Y3.



The calculator displayed the function and its inverse.

EXAMPLE 4 Solving a problem by using the inverse of a quadratic function

The rate of change in the surface area of a cell culture can be modelled by the function $S(t) = -0.005(t - 6)^2 + 0.18$, where $S(t)$ is the rate of change in the surface area in square millimetres per hour at time t in hours, and $0 \leq t \leq 12$.

a) State the domain and range of $S(t)$.

b) Determine the model that describes time in terms of the surface area.

c) Determine the domain and range of the new model.

Thomas' Solution

a) Domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 12\}$

The domain is given in the problem as part of the model.

Range $= \{S \in \mathbf{R} \mid 0 \leq S \leq 0.18\}$

This function is a parabola that opens down. The vertex is (6, 0.18), so the maximum value is 0.18. The surface area also cannot be negative, so 0 is the minimum value.

$$S = -0.005(t - 6)^2 + 0.18$$

To find the inverse of the original function, I solved the given equation for t by using the inverse operations.

$$S - 0.18 = -0.005(t - 6)^2$$

$$\frac{S - 0.18}{-0.005} = (t - 6)^2$$

$$\pm\sqrt{\frac{S - 0.18}{-0.005}} = t - 6$$

I did not interchange S and t in this case because S always means surface area and t always means time.

$$t = 6 \pm \sqrt{\frac{S - 0.18}{-0.005}}$$

$$t = 6 \pm \sqrt{\frac{-S + 0.18}{0.005}}$$

The domain and range of the new model:
Domain $= \{S \in \mathbf{R} \mid 0 \leq S \leq 0.18\}$

The value under the square root sign has to be positive, so the greatest value S can have is 0.18. For values greater than 0.18, the numerator would be positive, so the value under the square root would be negative.

Surface area cannot be less than zero, so S must be at least 0.

Range $= \{t \in \mathbf{R} \mid 0 \leq t \leq 12\}$

If $S = 0.18$, then the value of t is 6. If $S = 0$, then $t = 6 \pm 6$, so $t = 0$ or $t = 12$.

Therefore, the range values are between 0 and 12.

In Summary

Key Ideas

- The inverse of the original function undoes what the original function has done. It can be used to determine which values of the original dependent variable produce given values for the original independent variable.
- The inverse of a quadratic function defined over all the real numbers is not a function. It is a parabolic relation that opens either to the left or to the right. If the original quadratic opens up $(a > 0)$, the inverse opens to the right. If the original quadratic opens down $(a < 0)$, the inverse opens to the left.

Need to Know

- The equation of the inverse of a quadratic can be found by interchanging x and y in vertex form and solving for y.
- In the equation of the inverse of a quadratic function, the positive square root function represents the upper branch of the parabola, while the negative root represents the lower branch.
- The inverse of a quadratic function can be a function if the domain of the original function is restricted.

CHECK Your Understanding

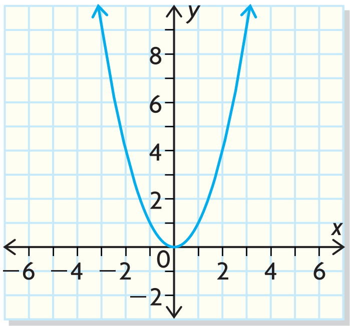
1. Each set of ordered pairs defines a parabola. Graph the relation and its inverse.

a) $\{(0, 0), (1, 3), (2, 12), (3, 27)\}$

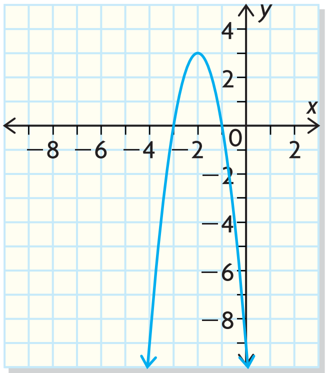
b) $\{(-3, -4), (-2, 1), (-1, 4), (0, 5), (1, 4), (2, 1), (3, -4)\}$

2. Given the graph of $f(x)$, graph the inverse relation.

a)



b)



3. Given $f(x) = 2x^2 - 1$, determine the equation of the inverse.

PRACTISING

4. Given $f(x) = 7 - 2(x - 1)^2, x \geq 1$, determine
a) $f(3)$ **b)** $f^{-1}(x)$ **c)** $f^{-1}(5)$ **d)** $f^{-1}(2a + 7)$

5. **a)** Sketch the graph of $f(x) = 3(x - 2)^2 - 2$.
b) Sketch the graph of its inverse on the same axes.

6. **a)** Graph $g(x) = -\sqrt{x}$ for $x \geq 0$.
b) Graph its inverse on the same axes.
c) State the domain and range of $g^{-1}(x)$.
d) Determine the equation for $g^{-1}(x)$.

7. Given $f(x) = -(x + 1)^2 - 3$ for $x \geq -1$, determine the equation for
K $f^{-1}(x)$. Graph the function and its inverse on the same axes.

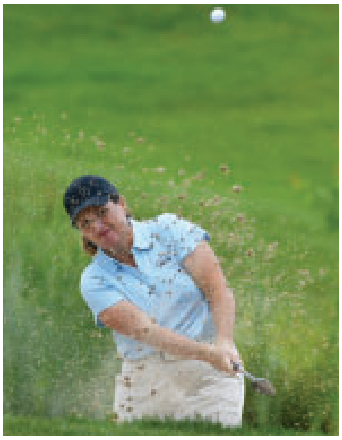
8. Given $f(x) = \frac{1}{2}(x - 5)^2 + 3$, find the equation for $f^{-1}(x)$ for the part of the function where $x \leq 5$. Use a graphing calculator to graph $f^{-1}(x)$.

9. For $-2 < x < 3$ and $f(x) = 3x^2 - 6x$, determine
a) the domain and range of $f(x)$
b) the equation of $f^{-1}(x)$ if $f(x)$ is further restricted to $1 < x < 3$

10. The height of a ball thrown from a balcony can be modelled by the function
A $h(t) = -5t^2 + 10t + 35$, where $h(t)$ is the height above the ground, in metres, at time t seconds after it is thrown.
a) Write $h(t)$ in vertex form.
b) Determine the domain and range of $h(t)$.
c) Determine the model that describes time in terms of the height.
d) What are the domain and range of the new model?

11. The height of a golf ball after Lori Kane hits it is shown in the table.

Time (s)	0	0.5	1	1.5	2	2.5
Height (m)	0	12.375	22.5	30.375	36.0	39.375

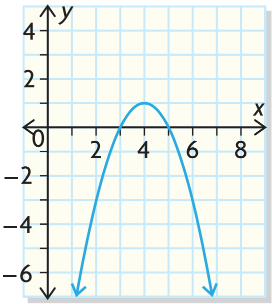


a) Use first and second differences to extend the table.
b) Graph the data and a curve of good fit for the relationship.
c) Graph the inverse relation and its curve of good fit.
d) Is the inverse a function? Explain.

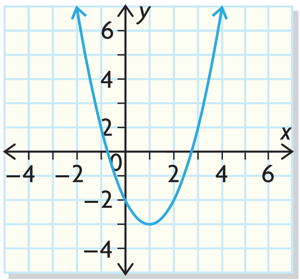
12. Consider $f(x) = -2x^2 + 3x - 1$.
T **a)** Determine the vertex of the parabola.
b) Graph $f(x)$.
c) Graph $f^{-1}(x)$ for $y \geq 0.75$.
d) Determine the domain and range of $f^{-1}(x)$ for $y \geq 0.75$.
e) Why were the values of x restricted in parts (c) and (d)?

13. Each graph shows a function f that is a parabola or a branch of a parabola.

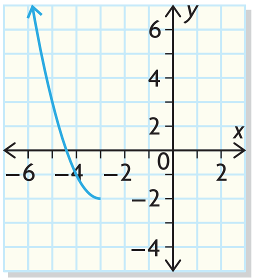
a)



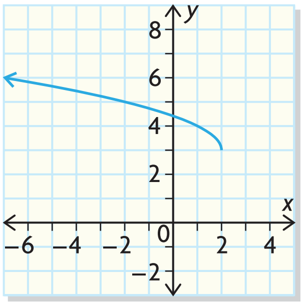
b)



c)



d)



i) Determine $f(x)$.

ii) Graph f^{-1}.

iii) State restrictions on the domain or range of f to make its inverse a function.

iv) Determine the equation(s) for f^{-1}.

14. What must happen for the inverse of a quadratic function defined over all the real numbers also to be a function?

15. C **a)** If you are given a quadratic function in standard form, explain how you could determine the equation of its inverse.

b) If the domain of the quadratic function is $\{x \in \mathbf{R}\}$, will its inverse be a function? Explain.

Extending

16. A meat department manager discovers that she can sell $m(x)$ kilograms of ground beef in a week, where $m(x) = 14\,700 - 3040x$, if she sells it at x dollars per kilogram. She pays her supplier \$3.21/kg for the beef.

a) Determine an algebraic expression for $P(x)$, where $P(x)$ represents the total profit in dollars for 1 week.

b) Find the equation for the inverse relation. Interpret its meaning.

c) Write an expression in function notation to represent the price that will earn \$1900 in profit. Evaluate and explain.

d) Determine the price that will maximize profit.

e) The supply cost drops to \$3.10/kg. What price should the manager set? How much profit will be earned at this price?

17. You are given the relation $x = 4 - 4y + y^2$.

a) Graph the relation.

b) Determine the domain and range of the relation.

c) Determine the equation of the inverse.

d) Is the inverse a function? Explain.

3.4 Operations with Radicals

GOAL

Simplify and perform operations on mixed and entire radicals.

INVESTIGATE the Math

The distance, $s(t)$, in millimetres of a particle from a certain point at any time, t, is given by $s(t) = 10\sqrt{4 + t}$. Don needs to find the exact distance between the point and the particle after 20 s. His answer must be in simplest form and he is not permitted to use a decimal.

? What is the exact value of $s(20)$, the distance between the particle and the given point at 20 s?

A. Copy and complete these products of radicals:

$$\sqrt{25} \times \sqrt{4} = 5 \times 2 = 10 \qquad \sqrt{25 \times 4} = \sqrt{100} = 10$$

$$\sqrt{16} \times \sqrt{9} = \qquad \sqrt{16 \times 9} = \sqrt{} =$$

$$\sqrt{4} \times \sqrt{36} = \qquad \sqrt{4 \times 36} = \sqrt{} =$$

$$\sqrt{100} \times \sqrt{9} = \qquad \sqrt{100 \times 9} = \sqrt{} =$$

radical

a square, cube, or higher root, such as $\sqrt{4} = 2$ or $\sqrt[3]{27} = 3$; $\sqrt{}$ is called the radical symbol

B. Compare the results in each pair of products.

C. Consider $\sqrt{4} \times \sqrt{6}$. From the preceding results, express this product as

a) an entire radical **b)** a mixed radical

Use your calculator to verify that your products are equivalent.

entire radical

a radical with coefficient 1; for example, $\sqrt{12}$

mixed radical

a radical with coefficient other than 1; for example, $2\sqrt{3}$

D. Determine $s(20)$. Use what you observed in parts A to C to simplify the expression so that your answer uses the smallest possible radical.

Reflecting

E. Determine $s(20)$ as a decimal. Why would the decimal answer for $\sqrt{24}$ not be considered exact?

F. To express $\sqrt{24}$ as a mixed radical, explain why using the factors $\sqrt{4} \times \sqrt{6}$ is a better choice than using $\sqrt{3} \times \sqrt{8}$.

G. If a and b are positive whole numbers, describe how $\sqrt{ab}$ is related to $\sqrt{a} \times \sqrt{b}$.

H. If $a > 0$, why is $b\sqrt{a}$ a simpler form of $\sqrt{ab^2}$?

APPLY the Math

EXAMPLE 1 Simplifying radicals by using a strategy involving perfect-square factors

Express each of the following as a mixed radical in lowest terms.

a) $\sqrt{72}$ **b)** $5\sqrt{27}$

Jasmine's Solution

a) $\sqrt{72} = \sqrt{36} \times \sqrt{2}$

$= 6\sqrt{2}$

I needed to find a perfect square number that divides evenly into 72. I could have chosen 4 or 9, but to put the mixed radical in lowest terms, I had to choose the greatest perfect square, which was 36. Once I expressed $\sqrt{72}$ as $\sqrt{36} \times \sqrt{2}$, I evaluated $\sqrt{36}$.

b) $5\sqrt{27} = 5 \times \sqrt{9} \times \sqrt{3}$

$= 5 \times 3 \times \sqrt{3}$

$= 15\sqrt{3}$

I found the largest perfect square that would divide evenly into 27; it was 9. I evaluated the square root of 9 and multiplied it by the coefficient 5.

EXAMPLE 2 Changing mixed radicals to entire radicals

Express each of the following as entire radicals.

a) $4\sqrt{5}$ **b)** $-6\sqrt{3}$

Sami's Solution

a) $4\sqrt{5} = 4 \times \sqrt{5}$

$= \sqrt{16} \times \sqrt{5}$

$= \sqrt{80}$

To create an entire radical, I had to change 4 into a square root. I expressed 4 as the square root of 16. Then I was able to multiply the numbers under the radical signs.

b) $-6\sqrt{3} = (-6) \times \sqrt{3}$

$= (-1) \times 6 \times \sqrt{3}$

$= (-1) \times \sqrt{36} \times \sqrt{3}$

$= -\sqrt{108}$

I knew that the negative sign would not go under the radical, since squares of real numbers are always positive. So I wrote -6 as the product of -1 and 6. I expressed 6 as $\sqrt{36}$ so that I could multiply the radical parts together to make an entire radical.

EXAMPLE 3 Multiplying radicals

Simplify.

a) $\sqrt{5} \times \sqrt{11}$ **b)** $-4\sqrt{6} \times 2\sqrt{6}$

Caleb's Solution

a) $\sqrt{5} \times \sqrt{11} = \sqrt{55}$

I multiplied the numbers under the radical signs together. 55 was not divisible by a perfect square, so my answer was in lowest terms.

b)
$$\begin{aligned} -4\sqrt{6} \times 2\sqrt{6} &= (-4) \times 2 \times \sqrt{6} \times \sqrt{6} \\ &= (-8)\sqrt{36} \end{aligned}$$

A mixed radical is the product of the integer and the radical, so I grouped together the integer products and the radical products.

$$\begin{aligned} &= (-8) \times 6 \\ &= -48 \end{aligned}$$

Since 36 is a perfect square, I was able to simplify.

EXAMPLE 4 Adding radicals

In Don's research, he may also have to add expressions that contain radicals. Can he add radicals that are like radicals? What about other radicals?

like radicals

radicals that have the same number under the radical symbol, such as $3\sqrt{6}$ and $-2\sqrt{6}$

Marta's Solution

$$\begin{aligned} \sqrt{3} &\doteq 1.732 \\ \sqrt{5} &\doteq 2.236 \\ \sqrt{3} + \sqrt{5} &= 1.732 + 2.236 \\ &= 3.968 \\ \sqrt{8} &\doteq 2.828 \end{aligned}$$

So $\sqrt{3} + \sqrt{5} \neq \sqrt{8}.$

I used my calculator to evaluate two radicals that were not like each other: $\sqrt{3}$ and $\sqrt{5}$. I rounded each value to 3 decimal places and then performed the addition. When I calculated $\sqrt{8}$, I found that it was not equal to $\sqrt{3} + \sqrt{5}$.

It looks like I cannot add radicals together if the numbers under the radical signs are different.

$$\begin{aligned} 3\sqrt{2} &= \sqrt{2} + \sqrt{2} + \sqrt{2} \\ 3\sqrt{2} + \sqrt{2} &= \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

Also,

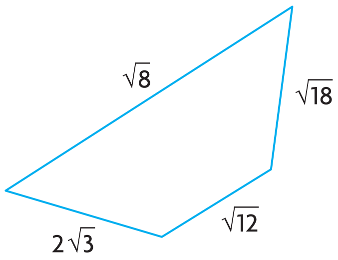
$$\begin{aligned} 3\sqrt{2} + \sqrt{2} &= \sqrt{2}(3 + 1) \\ &= \sqrt{2} \times 4 \\ &= 4\sqrt{2} \end{aligned}$$

Then I tried two like radicals. I used $3\sqrt{2}$ and $\sqrt{2}$. I expressed $3\sqrt{2}$ as the sum of three $\sqrt{2}$s. When I added $\sqrt{2}$ to this sum, I had 4 of them altogether, or $4\sqrt{2}$.

It makes sense that I can add like radicals by adding the integers in front of the radicals together.

EXAMPLE 5 Solving a problem involving radicals

Calculate the perimeter. Leave your answer in simplest radical form.



Robert's Solution

$$\begin{aligned}P &= \sqrt{8} + 2\sqrt{3} + \sqrt{12} + \sqrt{18}\\ &= \sqrt{4} \times \sqrt{2} + 2\sqrt{3} + \sqrt{4} \times \sqrt{3} + \sqrt{9} \times \sqrt{2}\\ &= 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{3} + 3\sqrt{2}\\ &= 2\sqrt{2} + 3\sqrt{2} + 2\sqrt{3} + 2\sqrt{3}\\ &= 5\sqrt{2} + 4\sqrt{3}\end{aligned}$$

To find the perimeter I needed to add up the sides. I can add only like radicals. I factored the numbers I could by using perfect squares to see if any of these are like radicals.

I grouped, and then added the like radicals together.

EXAMPLE 6 Multiplying binomial radical expressions

Simplify $\left(3 - \sqrt{6}\right)\left(2 + \sqrt{24}\right)$.

Barak's Solution

$$\begin{aligned}&(3 - \sqrt{6})(2 + \sqrt{24})\\ &= 6 + 3\sqrt{24} - 2\sqrt{6} - \sqrt{144}\\ &= 6 + 3(\sqrt{4} \times \sqrt{6}) - 2\sqrt{6} - 12\\ &= 6 + 3(2\sqrt{6}) - 2\sqrt{6} - 12\\ &= 6 - 12 + 6\sqrt{6} - 2\sqrt{6}\\ &= -6 + 4\sqrt{6}\end{aligned}$$

I simplified this expression by first expanding the quantities in brackets.

After I multiplied the terms, I noticed that some of them could be simplified further. I factored $\sqrt{24}$ by using a perfect square. I then evaluated $\sqrt{144}$ and simplified.

I collected and combined like radicals.

In Summary

Key Idea

- Entire radicals can sometimes be simplified by expressing them as the product of two radicals, one of which contains a perfect square. This results in a mixed radical.
  - $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ for $a \geq 0, b \geq 0$
  - $c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$ for $a \geq 0, b \geq 0$

Need to Know

- The only radicals that can be added or subtracted into a single term are like radicals.
- An answer containing a radical is an exact answer. An answer containing a decimal is an approximate answer.
- A mixed radical is in simplest form when the smallest possible number is written under the radical sign.

CHECK YOUR Understanding

1. Express each of these as mixed radicals in simplest form.

a) $\sqrt{27}$

b) $\sqrt{50}$

c) $\sqrt{98}$

d) $\sqrt{32}$

2. Simplify.

a) $\sqrt{5} \times \sqrt{7}$

b) $\sqrt{11} \times \sqrt{6}$

c) $2\sqrt{3} \times 5\sqrt{2}$

d) $-4\sqrt{3} \times 8\sqrt{13}$

3. Simplify.

a) $4\sqrt{5} + 3\sqrt{5}$

b) $9\sqrt{7} - 4\sqrt{7}$

c) $3\sqrt{3} + 8\sqrt{2} - 4\sqrt{3} + 11\sqrt{2}$

d) $\sqrt{8} - \sqrt{18}$

PRACTISING

4. Express as a mixed radical in simplest form.

a) $3\sqrt{12}$

b) $-5\sqrt{125}$

c) $10\sqrt{40}$

d) $-\frac{1}{2}\sqrt{60}$

e) $\frac{2}{3}\sqrt{45}$

f) $-\frac{9}{10}\sqrt{1200}$

5. Simplify.

a) $\sqrt{3}(2 - \sqrt{5})$

b) $2\sqrt{2}(\sqrt{7} + 3\sqrt{3})$

c) $(4\sqrt{2})^2$

d) $(-2\sqrt{3})^3$

e) $4\sqrt{3} \times 3\sqrt{6}$

f) $-7\sqrt{2} \times 5\sqrt{8}$

6. Simplify.

- **a)** $\sqrt{8} - \sqrt{32}$
- **b)** $\sqrt{12} + \sqrt{18} - \sqrt{27} + \sqrt{50}$
- **c)** $3\sqrt{98} - 5\sqrt{72}$
- **d)** $-4\sqrt{200} + 5\sqrt{242}$
- **e)** $-5\sqrt{45} + \sqrt{52} + 3\sqrt{125}$
- **f)** $7\sqrt{12} - 3\sqrt{28} + \frac{1}{2}\sqrt{48} + \frac{2}{3}\sqrt{63}$

7. Simplify.

- K **a)** $(6 - \sqrt{5})(3 + 2\sqrt{10})$
- **b)** $(2 + 3\sqrt{3})^2$
- **c)** $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})$
- **d)** $(3\sqrt{3} + 4\sqrt{2})(\sqrt{3} - 2\sqrt{2})$
- **e)** $(2\sqrt{5} - 3\sqrt{7})^2$
- **f)** $(1 - \sqrt{3})(2 + \sqrt{6})(5 + \sqrt{2})$

For questions 8 to 12, calculate the exact values and express your answers is simplest radical form.

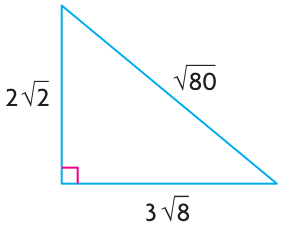
8. Calculate the length of the diagonal of a square with side length 4 cm.

9. A square has an area of 450 cm^2. Calculate the side length.

10. Determine the length of the diagonal of a rectangle with dimensions 3 cm × 9 cm.

11. A Determine the length of the line segment from $A(-2, 7)$ to $B(4, 1)$.

12. Calculate the perimeter and area of this triangle.



13. T If $a > 0$ and $b > 0$, which is greatest, $\left(\sqrt{a} + \sqrt{b}\right)^2$ or $\sqrt{a^2} + \sqrt{b^2}$?

14. C Give three mixed radicals that are equivalent to $\sqrt{200}$. Which answer is in simplest radical form? Explain how you know.

Extending

15. Express each radical in simplest radical form.

- **a)** $\sqrt{a^3}$
- **b)** $\sqrt{x^5y^6}$
- **c)** $5\sqrt{n^7} - 2n\sqrt{n^5}$
- **d)** $(\sqrt{p} + 2\sqrt{q})(\sqrt{q} - \sqrt{p})$

16. Simplify $\sqrt{\sqrt{\sqrt{4096}}}$.

17. Solve $(\sqrt{2})^x = 256$ for x.

3 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How can you tell if a function is quadratic from its table of values? its graph? its equation?

A:
- If the second differences are constant, then the function is quadratic.
- If the graph is a parabola opening either up or down, then the function is quadratic.
- If the equation is of degree 2, the function is quadratic.

Study *Aid*

- See Lesson 3.1, Example 2.
- Try Mid-Chapter Review Questions 1 and 2.

Q: How can you determine the maximum or minimum value of a quadratic function?

A1: If the equation is in standard form $f(x) = ax^2 + bx + c$, complete the square to find the vertex, which is the maximum point if the parabola opens down $(a < 0)$ and the minimum point if the parabola opens up $(a > 0)$.

A2: If the equation is in factored form $f(x) = a(x - r)(x - s)$, find the x-intercepts of the function. The maximum or minimum value occurs at the x-value that is the average of the two x-intercepts.

A3: If two points are known that are the same distance from the vertex and opposite each other on the graph, then the maximum or minimum value occurs at the x-value that is the average of the x-coordinates of the two points.

A4: If the equation is in standard form and the values of the coefficients are decimals, graph the function on a graphing calculator and determine the maximum or minimum value.

Study *Aid*

- See Lesson 3.2, Examples 1 to 3.
- Try Mid-Chapter Review Questions 5 and 8.

Q: How can you determine the equation of the inverse of a quadratic function?

A: Interchange the values of x and y in the equation that defines the function, and then solve the new equation for y. Remember that when you take the square root of an expression, there are two possible solutions, one positive and one negative.

Study *Aid*

- See Lesson 3.3, Examples 2 and 3.
- Try Mid-Chapter Review Questions 9, 10, and 12.

Q: How do you know if a radical can be simplified?

A: A radical can be simplified if the number under the radical sign is divisible by a perfect-square number other than 1.

Study *Aid*

- See Lesson 3.4, Example 1.
- Try Mid-Chapter Review Question 13.

PRACTICE Questions

Lesson 3.1

1. For each table, calculate the second differences. Determine whether the function is quadratic.

a)

x	y
−2	−8
−1	−2
0	0
1	−2
2	−8

b)

x	y
−2	0
−1	1
0	4
1	9
2	16

2. Graph each function.

a) $f(x) = -3(x - 2)^2 + 5$

b) $f(x) = 2(x + 4)(x - 6)$

3. For each function in question 2, state the vertex, the equation of the axis of symmetry, and the domain and range.

4. Express each function in question 2 in standard form.

Lesson 3.2

5. Determine the maximum or minimum value of each quadratic function.

a) $f(x) = x^2 - 6x + 2$

b) $f(x) = 2(x - 4)(x + 6)$

c) $f(x) = -2x^2 + 10x$

d) $f(x) = 3.2x^2 + 15x - 7$

6. The profit function for a business is given by the equation $P(x) = -4x^2 + 16x - 7$, where x is the number of items sold, in thousands, and $P(x)$ is dollars in thousands. Calculate the maximum profit and how many items must be sold to achieve it.

7. The cost per hour of running an assembly line in a manufacturing plant is a function of the number of items produced per hour. The cost function is $C(x) = 0.3x^2 - 1.2x + 2$, where $C(x)$ is the cost per hour in thousands of dollars, and x is the number of items produced per hour, in thousands. Determine the most economical production level.

8. The sum of two numbers is 16. What is the largest possible product between these numbers?

Lesson 3.3

9. a) Determine the equation of the inverse of the quadratic function $f(x) = x^2 - 4x + 3$.

b) List the domain and range of $f(x)$ and its inverse.

c) Sketch the graphs of $f(x)$ and its inverse.

10. The revenue for a business is modelled by the function $R(x) = -2.8(x - 10)^2 + 15$, where x is the number of items sold, in thousands, and $R(x)$ is the revenue in thousands of dollars. Express the number sold in terms of the revenue.

11. Almost all linear functions have an inverse that is a function, but quadratic functions do not. Explain why?

12. Graph $f(x) = -\sqrt{x + 3}$ and determine

a) the domain and range of $f(x)$

b) the equation of f^{-1}

Lesson 3.4

13. Express each radical in simplest form.

a) $\sqrt{48}$

b) $\sqrt{68}$

c) $\sqrt{180}$

d) $-3\sqrt{75}$

e) $5\sqrt{98}$

f) $-8\sqrt{12}$

14. Simplify.

a) $\sqrt{7} \times \sqrt{14}$

b) $3\sqrt{5} \times 2\sqrt{15}$

c) $\sqrt{12} + 2\sqrt{48} - 5\sqrt{27}$

d) $3\sqrt{28} - 2\sqrt{50} + \sqrt{63} - 3\sqrt{18}$

e) $(4 - \sqrt{3})(5 + 2\sqrt{3})$

f) $(3\sqrt{5} + 2\sqrt{10})(-2\sqrt{5} + 5\sqrt{10})$

Curious **Math**

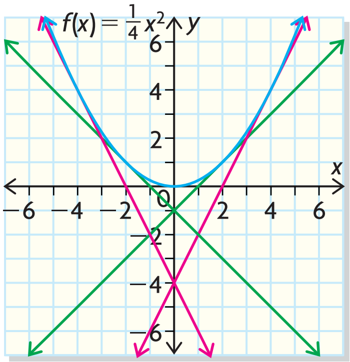
Investigating a Special Property of Parabolas

In a parabolic satellite reception dish, all of the signal rays are directed to a feed horn, which is positioned at a fixed point called the focus. You can locate this point by using math. Let the function $f(x) = \frac{1}{4}x^2$ model the shape of a satellite dish.

The graph of $f(x) = \frac{1}{4}x^2$ shows four tangent lines. (Each tangent line just touches the curve at one point.) You can use the tangent lines to determine the focus.

YOU WILL NEED

- ruler
- protractor



tangent lines

1. Mark these points on a copy of the graph: $(-4, 4)$, $(4, 4,)$, $(-2, 1)$, $(2, 1)$.
2. Draw a line parallel to the y-axis through each of the points.
3. Each vertical line represents a satellite signal ray, and the parabola represents the dish. Each signal ray will be reflected. With a protractor, measure the angle made between the ray and the tangent line.
4. The reflected ray will make the same angle with the tangent line. Use the protractor to draw the line representing the reflected ray.
5. Extend the lines representing the reflected ray until they cross the y-axis.
6. All reflected rays should pass through the same point on the y-axis, called the focus of the parabola. What point is the focus of this parabola?

3.5

Quadratic Function Models: Solving Quadratic Equations

YOU WILL NEED

- graphing calculator

Tech Support

For help using the graphing calculator to graph functions and find their *x*-intercepts, see Technical Appendix, B-2 and B-8.

GOAL

Solve problems involving quadratic functions in different ways.

LEARN ABOUT the Math

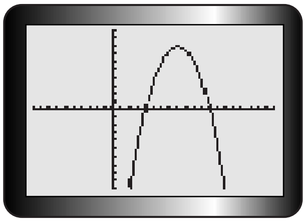
Anthony owns a business that sells parts for electronic game systems. The profit function for his business can be modelled by the equation $P(x) = -0.5x^2 + 8x - 24$, where x is the quantity sold, in thousands, and $P(x)$ is the profit in thousands of dollars.

? How many parts must Anthony sell in order for his business to break even?

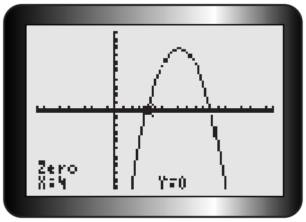
EXAMPLE 1 Selecting a strategy to solve a quadratic equation

Determine the number of parts Anthony must sell to break even.

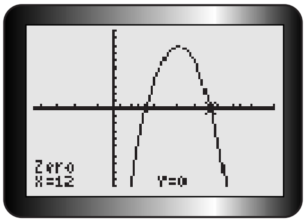
Derek's Solution: Using Graphing Technology



I graphed $P(x)$. Breaking even means that the profit is zero, so I looked for the x-intercepts of my function.



```
Zero
X=4          Y=0
```



```
Zero
X=12         Y=0
```

Anthony must sell 4000 parts or 12 000 parts.

I used the zero operation and found the x-intercepts from my graph. There were two possible values, at $x = 4$ and $x = 12$.

Because x is measured in thousands, I knew that the break-even values were 4000 and 12 000.

Tina's Solution: By Factoring

$$P(x) = 0$$

If Anthony's business breaks even, the profit is zero.

$$-0.5x^2 + 8x - 24 = 0$$

$$-0.5(x^2 - 16x + 48) = 0$$

$$-0.5(x - 4)(x - 12) = 0$$

I divided all the terms by the common factor −0.5. Inside the brackets was a simple trinomial that I could also factor.

$$x - 4 = 0 \quad \text{or} \quad x - 12 = 0$$

$$x = 4 \quad \text{or} \quad x = 12$$

I found the values of *x* that would give me zero in each bracket.

Anthony's business must sell 4000 parts or 12 000 parts to break even.

Since *x* is measured in thousands, my answer was 4000 or 12 000 parts.

Tracey's Solution: Using the Quadratic Formula

$$P(x) = 0$$

$$-0.5x^2 + 8x - 24 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

I needed to find the values of *x* that would make the profit function equal to zero. To solve the equation, I used the **quadratic formula**.

$$= \frac{-8 \pm \sqrt{(8)^2 - 4(-0.5)(-24)}}{2(-0.5)}$$

$$= \frac{-8 \pm \sqrt{64 - 48}}{-1}$$

$$= \frac{-8 \pm \sqrt{16}}{-1}$$

I substituted the values $a = -0.5$, $b = 8$, and $c = -24$ from the equation into the quadratic formula and simplified.

$$x = \frac{-8 + 4}{-1} \quad \text{or} \quad x = \frac{-8 - 4}{-1}$$

$$x = 4 \quad \text{or} \quad x = 12$$

Anthony must sell 4000 parts or 12 000 parts to break even.

My answer would have to be in thousands, since the number sold was in thousands.

Reflecting

A. How are the three methods for calculating the break-even points for Anthony's business the same? How are they different?

B. Will there always be two break-even points for a profit function? Why or why not?

C. If break-even points exist, which method may not work to determine where they are?

APPLY the Math

EXAMPLE 2 Solving a problem involving a quadratic equation

A water balloon is catapulted into the air from the top of a building. The height, $h(t)$, in metres, of the balloon after t seconds is $h(t) = -5t^2 + 30t + 10$.

a) What are the domain and range of this function?

b) When will the balloon reach a height of 30 m?

Brian's Solution

a) $h(t) = -5(t^2 - 6t) + 10$

The graph must be a parabola opening down because the value of *a* is negative.

$$
\begin{aligned}
&= -5(t^2 - 6t + 9 - 9) + 10 \\
&= -5(t^2 - 6t + 9) + 45 + 10 \\
&= -5(t - 3)^2 + 55
\end{aligned}
$$

To get the range, I found the vertex by completing the square. The vertex is (3, 55), so the maximum height is 55 m and the minimum height is 0.

Range $= \{h(t) \in \mathbf{R} \mid 0 \le h(t) \le 55\}$

$$
\begin{aligned}
-5(t - 3)^2 + 55 &= 0 \\
-5(t - 3)^2 &= -55 \\
(t - 3)^2 &= 11 \\
t - 3 &= \pm\sqrt{11}
\end{aligned}
$$

The domain is the interval of time the balloon was in flight. It stops when it hits the ground. The time *t* must be greater than 0. I calculated the values of *t* that would make the height 0.

$t = 3 + \sqrt{11}$ or $t = 3 - \sqrt{11}$

$t = 3 + 3.32$ or $t = 3 - 3.32$

$t = 6.32$ or $t = -0.32$

Domain $= \{t \in \mathbf{R} \mid 0 \le t \le 6.32\}$

One value of *t* is negative, so the domain must start at 0 and go to the positive value of *t* that I found.

b) $30 = -5t^2 + 30t + 10$

$$0 = -5t^2 + 30t - 20$$

To know when the ball reached 30 m, I replaced $h(t)$ with 30 and solved for t.

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4(-5)(-20)}}{2(-5)}$$

I used the quadratic formula to solve for t.

$$= \frac{-30 \pm \sqrt{500}}{-10}$$

$$\doteq \frac{-30 \pm 22.36}{-10}$$

$$t \doteq \frac{-30 + 22.36}{-10} \quad \text{or} \quad t \doteq \frac{-30 - 22.36}{-10}$$

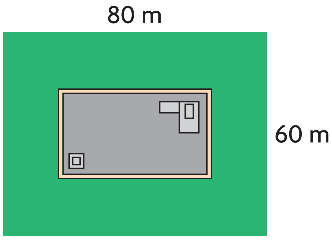
$$t \doteq 0.764 \quad \text{or} \quad t \doteq 5.236$$

The ball will reach a height of 30 m after 0.764 s or 5.236 s.

Both answers are possible in this question. The ball reaches 30 m going up and again coming down.

EXAMPLE 3 Representing and solving a problem by using a quadratic equation

A factory is to be built on a lot that measures 80 m by 60 m. A lawn of uniform width, equal to the area of the factory, must surround it. How wide is the strip of lawn, and what are the dimensions of the factory?



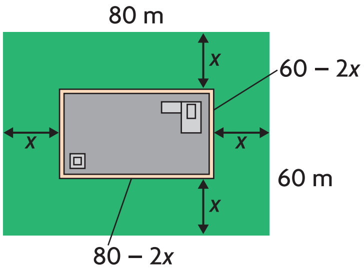
Communication *Tip*

Any solution of an equation that does not work in the context of a problem is said to be an inadmissible solution.

Rachael's Solution

Let the width of the lawn be x metres.

I chose a variable for the width of the lawn.



The dimensions of the factory are $(60 - 2x)$ m and $(80 - 2x)$ m.

Since the lawn was the same width all around, I had to subtract $2x$ from 60 and $2x$ from 80 to get the dimensions of the factory.

Area of factory = length × width

$$= (60 - 2x)(80 - 2x)$$
$$= 4800 - 120x - 160x + 4x^2$$
$$= 4800 - 280x + 4x^2$$

I wrote down and simplified an expression for the area of the factory.

Area of lawn = Area of lot − Area of factory

$$= 4800 - (4800 - 280x + 4x^2)$$
$$= -4x^2 + 280x$$

The area of the lawn is the difference between the area of the lot and the area of the factory.

$$-4x^2 + 280x = 4800 - 280x + 4x^2$$
$$-8x^2 + 560x - 4800 = 0$$
$$-8(x^2 - 70x + 600) = 0$$
$$-8(x - 60)(x - 10) = 0$$
$$x = 60 \quad \text{or} \quad x = 10$$

The area of the lawn is equal to the area of the factory, so I set the two expressions equal to each other. This equation was quadratic, so I rearranged it so that it was equal to zero and solved it by factoring.

But $x = 60$ is inadmissible in this problem, so $x = 10$.

I found two possible values of x. Since a width of 60 for the strip made the dimensions of the factory negative, the width of the lawn had to be 10 m.

$$60 - 2(10) = 40$$
$$80 - 2(10) = 60$$

The lawn is 10 m wide, and the dimensions of the factory are 60 m by 40 m.

I then substituted $x = 10$ into the expressions for the length and width of the factory to find its dimensions.

In Summary

Key Idea

- All quadratic equations can be expressed in the form $ax^2 + bx + c = 0$ by algebraic techniques. The equations can be solved in a number of ways.

Need to Know

- Quadratic equations can be solved by graphing the corresponding functions $f(x) = ax^2 + bx + c$ and locating the x-intercepts, or zeros, either by hand or with technology. These zeros are the solutions or roots of the equation $ax^2 + bx + c = 0$.
- Quadratic equations can also be solved
  - by factoring
  - with the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Depending on the problem and the degree of accuracy required, the solutions of a quadratic equation may be expressed exactly by using radicals or rational numbers, or approximately with decimals.

CHECK Your Understanding

1. Determine the roots of each equation by factoring.

a) $x^2 + 5x + 4 = 0$

b) $x^2 - 11x + 18 = 0$

c) $4x^2 - 9 = 0$

d) $2x^2 - 7x - 4 = 0$

2. Use the quadratic formula to determine each of the roots to two decimal places.

a) $x^2 - 4x - 9 = 0$

b) $3x^2 + 2x - 8 = 0$

c) $-2x^2 + 3x - 6 = 0$

d) $0.5x^2 - 2.2x - 4.7 = 0$

3. Use a graphing calculator to solve each equation.

a) $0 = -4x^2 - 5x - 1$

b) $0 = 2x^2 - 11x + 9$

PRACTISING

4. **K** **i)** For each equation, decide on a strategy to solve it and explain why you chose that strategy.

ii) Use your strategy to solve the equation. When appropriate, leave your answer in simplest radical form.

a) $2x^2 - 3x = x^2 + 7x$

b) $4x^2 + 6x + 1 = 0$

c) $x^2 + 4x - 3 = 0$

d) $(x + 3)^2 = -2x$

e) $3x^2 - 5x = 2x^2 + 4x + 10$

f) $2(x + 3)(x - 4) = 6x + 6$

5. Locate the x-intercepts of the graph of each function.

a) $f(x) = 3x^2 - 7x - 2$

b) $f(x) = -4x^2 + 25x - 21$

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

a) $P(x) = -x^2 + 12x + 28$

b) $P(x) = -2x^2 + 18x - 40$

c) $P(x) = -2x^2 + 22x - 17$

d) $P(x) = -0.5x^2 + 6x - 5$

7. **A** The flight of a ball hit from a tee that is 0.6 m tall can be modelled by the function $h(t) = -4.9t^2 + 6t + 0.6$, where $h(t)$ is the height in metres at time t seconds. How long will it take for the ball to hit the ground?

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995.

a) What was the population in 1995?

b) What will be the population in 2010?

c) In what year will the population be at least 450 000? Explain your answer.

9. A rectangle has an area of 330 m^2. One side is 7 m longer than the other. What are the dimensions of the rectangle?

10. The sum of the squares of two consecutive integers is 685. What could the integers be? List all possibilities.

11. A right triangle has a height 8 cm more than twice the length of the base. If the area of the triangle is 96 cm^2, what are the dimensions of the triangle?

12. **T** Jackie mows a strip of uniform width around her 25 m by 15 m rectangular lawn and leaves a patch of lawn that is 60% of the original area. What is the width of the strip?

13. A small flare is launched off the deck of a ship. The height of the flare above the water is given by the function $h(t) = -4.9t^2 + 92t + 9$, where $h(t)$ is measured in metres and t is time in seconds.

a) When will the flare's height be 150 m?

b) How long will the flare's height be above 150 m?

14. A bus company has 4000 passengers daily, each paying a fare of \$2. For each \$0.15 increase, the company estimates that it will lose 40 passengers per day. If the company needs to take in \$10 450 per day to stay in business, what fare should be charged?

15. **C** Describe three possible ways that you could determine the zeros of the quadratic function $f(x) = -2x^2 + 14x - 24$.

Extending

16. The perimeter of a right triangle is 60 cm. The length of the hypotenuse is 6 cm more than twice the length of one of the other sides. Find the lengths of all three sides.

17. Find the zeros of the function $f(x) = 3x - 1 + \frac{1}{x + 1}$.

3.6 The Zeros of a Quadratic Function

GOAL

Use a variety of strategies to determine the number of zeros of a quadratic function.

LEARN ABOUT the Math

Samantha has been asked to predict the number of zeros for each of three quadratic functions without using a graphing calculator. Samantha knows that quadratics have 0, 1, or 2 zeros. The three functions are:

$$f(x) = -2x^2 + 12x - 18$$
$$g(x) = 2x^2 + 6x - 8$$
$$h(x) = x^2 - 4x + 7$$

? How can Samantha predict the number of zeros each quadratic has without graphing?

EXAMPLE 1 Connecting functions to their graphs

Determine the properties of each function that will help you determine the number of x-intercepts each has.

Tara's Solution: Using Properties of the Quadratic Function

$$\begin{aligned} f(x) &= -2x^2 + 12x - 18 \\ f(x) &= -2(x^2 - 6x + 9) \\ &= -2(x - 3)^2 \end{aligned}$$

I decided to find the vertex of the first function. I factored -2 out as a common factor. The trinomial that was left was a perfect square, so I factored it. This put the function in vertex form. Because the vertex is on the x-axis, there is only one zero.

Vertex is (3, 0) and the parabola opens down. This function has one zero.

$$\begin{aligned} g(x) &= 2x^2 + 6x - 8 \\ g(x) &= 2(x^2 + 3x - 4) \\ &= 2(x + 4)(x - 1) \end{aligned}$$

I factored 2 out as a common factor in the second function, then factored the trinomial inside the brackets. I used the factors to find the zeros, so this function has two.

This function has two zeros, at $x = -4$ and $x = 1$.

$$\begin{aligned} h(x) &= x^2 - 4x + 7 \\ &= (x^2 - 4x + 4 - 4) + 7 \\ &= (x^2 - 4x + 4) - 4 + 7 \\ &= (x - 2)^2 + 3 \end{aligned}$$

This function would not factor, so I found the vertex by completing the square. The vertex is above the *x*-axis, and the parabola opens up because *a* is positive. Therefore, the function has no zeros.

The vertex is (2, 3) and the parabola opens up. This function has no zeros.

Asad's Solution: Using the Quadratic Formula

$$\begin{aligned} f(x) &= -2x^2 + 12x - 18 \\ 0 &= -2x^2 + 12x - 18 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-12 \pm \sqrt{(12)^2 - 4(-2)(-18)}}{2(-2)} \\ &= \frac{-12 \pm \sqrt{144 - 144}}{-4} \\ &= \frac{-12 \pm \sqrt{0}}{-4} \\ &= \frac{-12}{-4} \\ &= 3 \end{aligned}$$

The zeros or *x*-intercepts occur when each function equals 0. I set $f(x) = 0$, then solved the resulting equation using the quadratic formula with $a = -2$, $b = 12$, and $c = -18$.

The first function has only one value for the *x*-intercept, so there is only one zero. This can be seen from the value of the **discriminant** (the quantity under the radical sign), which is zero.

This function has one zero.

$$\begin{aligned} g(x) &= 2x^2 + 6x - 8 \\ 0 &= 2x^2 + 6x - 8 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{(6)^2 - 4(2)(-8)}}{2(2)} \\ &= \frac{-6 \pm \sqrt{36 + 64}}{4} \end{aligned}$$

I used the quadratic formula again with $a = 2$, $b = 6$, and $c = -8$. There were two solutions, since the discriminant was positive. So the function has two zeros.

$$= \frac{-6 \pm \sqrt{100}}{4}$$

$$x = \frac{-6 - 10}{4} \quad \text{or} \quad x = \frac{-6 + 10}{4}$$

$$x = -4 \quad \text{or} \quad x = 1$$

This function has two zeros.

$$h(x) = x^2 - 4x + 7$$

$$0 = x^2 - 4x + 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(7)}}{2}$$

$$= \frac{4 \pm \sqrt{16 - 28}}{2}$$

$$= \frac{4 \pm \sqrt{-12}}{2}$$

This function has no zeros.

I used the quadratic formula with $a = 1$, $b = -4$, and $c = 7$. The discriminant was negative, so there were no real-number solutions. The function has no zeros.

Reflecting

A. Describe the possibilities for the number of zeros of a quadratic function.

B. How can finding the vertex help determine the number of zeros?

C. Why is the factored form useful in determining the number of zeros of a quadratic function?

D. Explain how the quadratic formula can be used to predict the number of zeros of a quadratic function.

APPLY the Math

EXAMPLE 2 Using the discriminant to determine the number of zeros

Find the value of the discriminant to determine the number of zeros of each quadratic function.

a) $f(x) = 2x^2 - 3x - 5$
b) $g(x) = 4x^2 + 4x + 1$
c) $h(x) = -5x^2 + x - 2$

Larry's Solution

a)
$$\begin{aligned} b^2 - 4ac &= (-3)^2 - 4(2)(-5) \\ &= 9 + 40 \\ &= 49 \end{aligned}$$

The discriminant $b^2 - 4ac$ is the value under the square root sign in the quadratic formula.

If $b^2 - 4ac$ is a positive number, the function has two zeros.

Since $49 > 0$, there are two distinct zeros.

b)
$$\begin{aligned} b^2 - 4ac &= (4)^2 - 4(4)(1) \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

This time, the discriminant is equal to zero, so the function has only one zero.

Therefore, there is one zero.

c)
$$\begin{aligned} b^2 - 4ac &= (1)^2 - 4(-5)(-2) \\ &= 1 - 40 \\ &= -39 \end{aligned}$$

In this function, $b^2 - 4ac$ is a negative number, so the function has no zeros.

Since $-39 < 0$, there are no zeros.

EXAMPLE 3 Solving a problem involving a quadratic function with one zero

Determine the value of k so that the quadratic function $f(x) = x^2 - kx + 3$ has only one zero.

Ruth's Solution

$$\begin{aligned} b^2 - 4ac &= 0 \\ (-k)^2 - 4(1)(3) &= 0 \\ k^2 - 12 &= 0 \\ k^2 &= 12 \\ k &= \pm\sqrt{12} \\ k &= \pm 2\sqrt{3} \end{aligned}$$

If there is only one zero, then the discriminant is zero. I put the values for a, b, and c into the equation $b^2 - 4ac = 0$ and solved for k.

Since I had to take the square root of both sides to solve for k, there were two possible values. I expressed the values of k using a mixed radical in simplest form.

EXAMPLE 4 Solving a problem by using the discriminant

A market researcher predicted that the profit function for the first year of a new business would be $P(x) = -0.3x^2 + 3x - 15$, where x is based on the number of items produced. Will it be possible for the business to break even in its first year?

Raj's Solution

$$\begin{aligned} P(x) &= 0 \\ -0.3x^2 + 3x - 15 &= 0 \end{aligned}$$

At a break-even point, the profit is zero.

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(-0.3)(-15) \\ &= 9 - 18 \\ &= -9 \end{aligned}$$

I just wanted to know if there was a break-even point and not what it was, so I only needed to know if the profit function had any zeros. I used the value of the discriminant to decide.

Since $b^2 - 4ac < 0$, there are no zeros for this function. Therefore, it is not possible for the business to break even in its first year.

In Summary

Key Idea

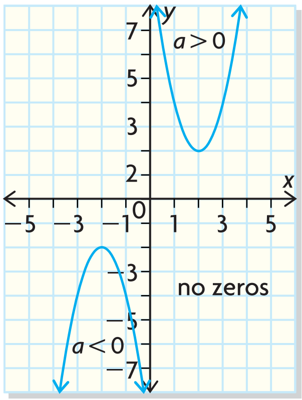
- A quadratic function can have 0, 1, or 2 zeros. You can determine the number of zeros either by graphing or by analyzing the function.

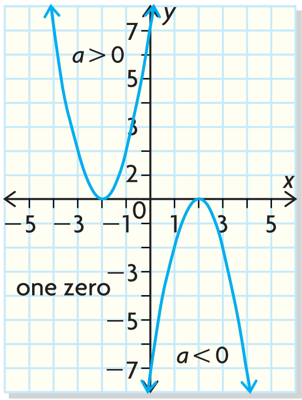
Need to Know

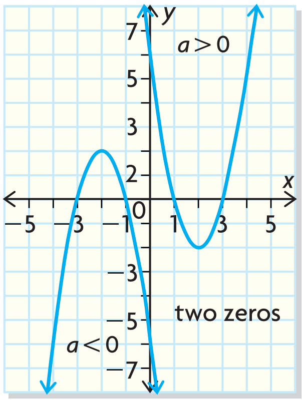
- The number of zeros of a quadratic function can be determined by looking at the graph of the function and finding the number of *x*-intercepts.
- For a quadratic equation $ax^2 + bx + c = 0$ and its corresponding function $f(x) = ax^2 + bx + c$, see the table below:

Value of the Discriminant	**Number of Zeros/Solutions**
$b^2 - 4ac > 0$	2
$b^2 - 4ac = 0$	1
$b^2 - 4ac < 0$	0

- The number of zeros can be determined by the location of the vertex relative to the *x*-axis, and the direction of opening:
  - If $a > 0$, and the vertex is above the *x*-axis, there are no zeros.
  - If $a > 0$, and the vertex is below the *x*-axis, there are two zeros.
  - If $a < 0$, and the vertex is above the *x*-axis, there are two zeros.
  - If $a < 0$, and the vertex is below the *x*-axis, there are no zeros.
  - If the vertex is on the *x*-axis, there is one zero.







CHECK Your Understanding

1. Determine the vertex and the direction of opening for each quadratic function. Then state the number of zeros.

a) $f(x) = 3x^2 - 5$
b) $f(x) = -4x^2 + 7$
c) $f(x) = 5x^2 + 3$
d) $f(x) = 3(x + 2)^2$
e) $f(x) = -4(x + 3)^2 - 5$
f) $f(x) = 0.5(x - 4)^2 - 2$

2. Factor each quadratic function to determine the number of zeros.

a) $f(x) = x^2 - 6x - 16$
b) $f(x) = 2x^2 - 6x$
c) $f(x) = 4x^2 - 1$
d) $f(x) = 9x^2 + 6x + 1$

3. Calculate the value of $b^2 - 4ac$ to determine the number of zeros.

a) $f(x) = 2x^2 - 6x - 7$
b) $f(x) = 3x^2 + 2x + 7$
c) $f(x) = x^2 + 8x + 16$
d) $f(x) = 9x^2 - 14.4x + 5.76$

PRACTISING

4. **K** Determine the number of zeros. Do not use the same method for all four parts.

a) $f(x) = -3(x - 2)^2 + 4$
b) $f(x) = 5(x - 3)(x + 4)$
c) $f(x) = 4x^2 - 2x$
d) $f(x) = 3x^2 - x + 5$

5. **A** For each profit function, determine whether the company can break even. If the company can break even, determine in how many ways it can do so.

a) $P(x) = -2.1x^2 + 9.06x - 5.4$
b) $P(x) = -0.3x^2 + 2x - 7.8$
c) $P(x) = -2x^2 + 6.4x - 5.12$
d) $P(x) = -2.4x^2 + x - 1.2$

6. For what value(s) of k will the function $f(x) = 3x^2 - 4x + k$ have one x-intercept?

7. For what value(s) of k will the function $f(x) = kx^2 - 4x + k$ have no zeros?

8. For what values of k will the function $f(x) = 3x^2 + 4x + k = 0$ have no zeros? one zero? two zeros?

9. The graph of the function $f(x) = x^2 - kx + k + 8$ touches the x-axis at one point. What are the possible values of k?

10. Is it possible for $n^2 + 25$ to equal $-8n$? Explain.

11. Write the equation of a quadratic function that meets each of the given conditions.

a) The parabola opens down and has two zeros.
b) The parabola opens up and has no zeros.
c) The parabola opens down and the vertex is also the zero of the function.

12. The demand function for a new product is $p(x) = -4x + 42.5$, where x is the quantity sold in thousands and p is the price in dollars. The company that manufactures the product is planning to buy a new machine for the plant. There are three different types of machine. The cost function for each machine is shown.

Machine A: $C(x) = 4.1x + 92.16$

Machine B: $C(x) = 17.9x + 19.36$

Machine C: $C(x) = 8.8x + 55.4$

Investigate the break-even quantities for each machine. Which machine would you recommend to the company?

13. Describe how each transformation or sequence of transformations of the function $f(x) = 3x^2$ will affect the number of zeros the function has.

a) a vertical stretch of factor 2

b) a horizontal translation 3 units to the left

c) a horizontal compression of factor 2 and then a reflection in the x-axis

d) a vertical translation 3 units down

e) a horizontal translation 4 units to the right and then a vertical translation 3 units up

f) a reflection in the x-axis, then a horizontal translation 1 unit to the left, and then a vertical translation 5 units up

14. **T** If $f(x) = x^2 - 6x + 14$ and $g(x) = -x^2 - 20x - k$, determine the value of k so that there is exactly one point of intersection between the two parabolas.

15. Determine the number of zeros of the function $f(x) = 4 - (x - 3)(3x + 1)$ without solving the related quadratic equation or graphing. Explain your thinking.

16. **C** Describe how you can determine the number of zeros of a quadratic function if the equation of the function is in

a) vertex form **b)** factored form **c)** standard form

Extending

17. Show that $(x^2 - 1)k = (x - 1)^2$ has one solution for only one value of k.

18. Investigate the number of zeros of the function $f(x) = (k + 1)x^2 + 2kx + k - 1$ for different values of k. For what values of k does the function have no zeros? one zero? two zeros?

3.7

Families of Quadratic Functions

GOAL

Determine the properties of families of quadratic functions.

INVESTIGATE the Math

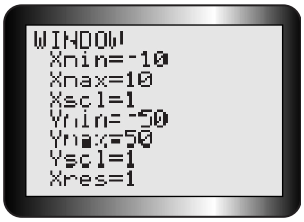
Equations that define quadratic functions can look quite different, yet their graphs can have similar characteristics.

Group 1	Group 2	Group 3
$f(x) = x^2 - 3x - 10$	$m(x) = -2x^2 + 4x + 1$	$r(x) = -3x^2 + 5x - 2$
$g(x) = -2x^2 + 6x + 20$	$n(x) = 0.5x^2 - 1x + 3.5$	$s(x) = 2x^2 + x - 2$
$h(x) = 4x^2 - 12x - 40$	$p(x) = -6x^2 + 12x - 3$	$t(x) = 7x^2 - 2x - 2$
$k(x) = -0.5x^2 + 1.5x + 5$	$q(x) = 10x^2 - 20x + 13$	$u(x) = -4x^2 - 4x - 2$

? What characteristics do the graphs in each of these groups have in common?

A. Graph each of the functions in Group 1 on a graphing calculator. Use the window settings shown. How are the graphs the same? How are they different?

A.



WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-50
Ymax=50
Yscl=1
Xres=1

B. Write each of the functions in Group 1 in factored form. What do you notice?

C. Clear all functions, and then graph each of the functions for Group 2 on a graphing calculator. Use the window settings shown.

C.

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

D. How are the graphs the same? How are they different?

E. Write each of the functions in Group 2 in vertex form. What do you notice?

F. Clear all functions, and then graph each of the functions in Group 3 on a graphing calculator. Use the window settings shown. What do these functions have in common?

F.

WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

G. Summarize your findings for each group.

Reflecting

H. Each of the three groups of functions forms a **family of parabolas.** Describe the common characteristics of each of the groups.

family of parabolas

a group of parabolas that all share a common characteristic

I. In each of Groups 1 and 2, what single value was varied to create the family? What transformation is this parameter associated with?

J. What common characteristic appears in all quadratic functions in the same family if the equation is in factored form? vertex form? standard form?

APPLY the Math

EXAMPLE 1 Looking for quadratics that share a vertex

Given the function $f(x) = -3(x + 2)^2 - 1$, determine another quadratic function with the same vertex.

Ian's Solution

$f(x) = -3(x + 2)^2 - 1$

I identified the vertex.

Vertex is $(-2, -1)$.

Family of parabolas is of the form

$f(x) = a(x + 2)^2 - 1$

So another quadratic in the family is $g(x) = 2(x + 2)^2 - 1$.

To get another quadratic function with the same vertex, I needed to change the value of *a* because parabolas with the same vertex are vertically stretched or compressed, but not horizontally or vertically translated.

EXAMPLE 2 Determining a specific equation of a member of the family

Determine the equation of the quadratic function that passes through $(-3, 20)$ if its zeros are 2 and -1.

Preet's Solution

$$\begin{aligned} f(x) &= a(x - 2)[x - (-1)] \\ &= a(x - 2)(x + 1) \\ &= a[x^2 - 2x + x - 2] \\ &= a(x^2 - x - 2) \end{aligned}$$

I wrote the general function of all parabolas that have zeros at 2 and -1, then simplified by expanding.

$$f(x) = a(x^2 - x - 2)$$
$$20 = a[(-3)^2 - (-3) - 2)]$$
$$20 = 10a$$
$$a = 2$$

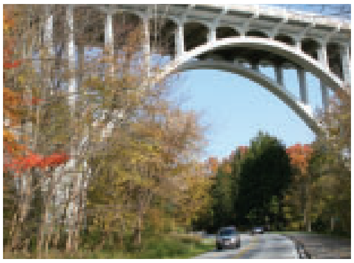
To determine the equation passing through $(-3, 20)$, I had to find the correct value of a.

I substituted the point into the equation and solved for a.

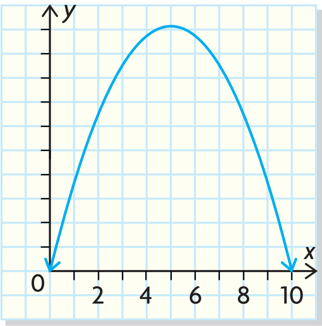
Therefore, $f(x) = 2(x^2 - x - 2)$.

EXAMPLE 3 Solving a problem by applying knowledge of the vertex form of a quadratic

A highway overpass has a shape that can be modelled by the equation of a parabola. If the edge of the highway is the origin and the highway is 10 m wide, what is the equation of the parabola if the height of the overpass 2 m from the edge of the highway is 13 m?



Elizabeth's Solution



I drew a sketch. If the edge of the highway is at the origin, then one of the zeros is 0. If the highway is 10 m wide, then the other zero is at (10, 0).

$$h = ax(x - 10)$$

Since I had the zeros, I wrote the equation in factored form. The equation that would model the overpass would be in the same family, so I needed to find the value of a.

$$13 = a(2)(2 - 10)$$
$$13 = -16a$$
$$a = -\frac{13}{16}$$

(2, 13) is a point on the curve, so I substituted those values into the equation and solved for a. Once I had the value of a, I wrote the equation in factored form.

Therefore, the equation that models the overpass is

$$h = -\frac{13}{16}x(x - 10)$$

EXAMPLE 4 Selecting a strategy to determine the quadratic function from data

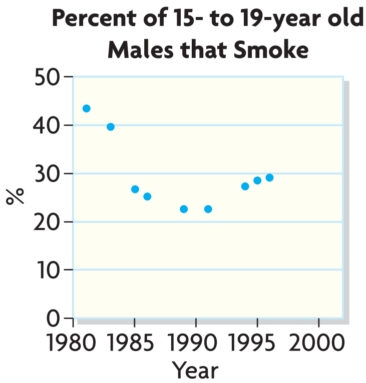
The percent of 15- to 19-year-old males who smoke has been tracked by Health Canada. The data from 1981 to 1996 are given in the table.

Year	1981	1983	1985	1986	1989	1991	1994	1995	1996
Smokers (%)	43.4	39.6	26.7	25.2	22.6	22.6	27.3	28.5	29.1

a) Draw a scatter plot of the data.
b) Draw a curve of good fit.
c) Estimate the location of the vertex.
d) Determine a quadratic function that will model the data.

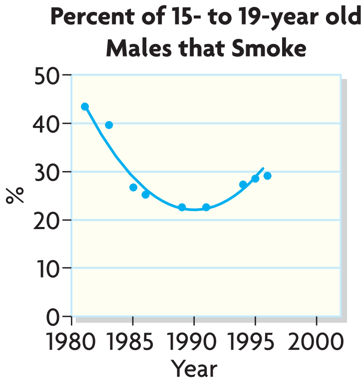
Bryce's Solution

a)



I plotted the points on a graph. They could be represented by a quadratic function.

b)



I drew a curve of good fit by hand so that I could estimate the vertex. Since the values were 22.6 in both 1989 and 1991, I used (1990, 22) as my estimated vertex.

c) The graph models a parabola with the vertex above the x-axis. Estimated vertex: (1990, 22)

I used vertex form with the vertex I knew. I needed to find the value of a to approximate the data. I chose the point (1995, 28.5) as the point on the curve. I substituted the point into the equation and solved for a.

d) $f(x) = a(x - 1990)^2 + 22$

$$28.5 = a(1995 - 1990)^2 + 22$$

$$28.5 = a(-5)^2 + 22$$

$$28.5 = 25a + 22$$

$$6.5 = 25a$$

$$\frac{65}{25} = a$$

$$a \doteq 0.26$$

Therefore, a model for the data is $f(x) = 0.26(x - 1990)^2 + 22$.

In Summary

Key Ideas

- If the value of a is varied in a quadratic function expressed in vertex form, $f(x) = a(x - h)^2 + k$, a family of parabolas with the same vertex and axis of symmetry is created.
- If the value of a is varied in a quadratic function in factored form, $f(x) = a(x - r)(x - s)$, a family of parabolas with the same x-intercepts and axis of symmetry is created.
- If the values of a and b are varied in a quadratic function expressed in standard form, $f(x) = ax^2 + bx + c$, a family of parabolas with the same y-intercept is created.

Need to Know

- The algebraic model of a quadratic function can be determined algebraically.
  - If the zeros are known, write in factored form with a unknown, substitute another known point, and solve for a.
  - If the vertex is known, write in vertex form with a unknown, substitute a known point, and solve for a.

CHECK Your Understanding

1. What characteristics will two parabolas in the family $f(x) = a(x - 3)(x + 4)$ share?

2. How are the parabolas $f(x) = -3(x - 2)^2 - 4$ and $g(x) = 6(x - 2)^2 - 4$ the same? How are they different?

3. What point do the parabolas $f(x) = -2x^2 + 3x - 7$ and $g(x) = 5x^2 + 3x - 7$ have in common?

PRACTISING

4. Determine the equation of the parabola with x-intercepts
- **a)** -4 and 3, and that passes through (2, 7)
- **b)** 0 and 8, and that passes through $(-3, -6)$
- **c)** $\sqrt{7}$ and $-\sqrt{7}$, and that passes through $(-5, 3)$
- **d)** $1 - \sqrt{2}$ and $1 + \sqrt{2}$, and that passes through (2, 4)

5. Determine the equation of the parabola with vertex
- **a)** $(-2, 5)$ and that passes through $(4, -8)$
- **b)** (1, 6) and that passes through $(0, -7)$
- **c)** $(4, -5)$ and that passes through $(-1, -3)$
- **d)** (4, 0) and that passes through (11, 8)

6. Determine the equation of the quadratic function $f(x) = ax^2 - 6x - 7$ if $f(2) = 3$.

7.
- **a)** Sketch the graph of $f(x) = (x - 2)(x + 6)$.
- **b)** Use your graph to sketch the graph of $g(x) = -2(x - 2)(x + 6)$.
- **c)** Sketch the graph of $h(x) = 3(x - 2)(x + 6)$.

8. **K** Determine the equation of the parabola with x-intercepts ±4 and passing through (3, 6).

9. Determine the equation of the quadratic function that passes through $(-4, 5)$ if its zeros are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

10. **T** A tunnel with a parabolic arch is 12 m wide. If the height of the arch 4 m from the left edge is 6 m, can a truck that is 5 m tall and 3.5 m wide pass through the tunnel? Justify your decision.

11. **A** A projectile is launched off the top of a platform. The table gives the height of the projectile at different times during its flight.

Time (s)	0	1	2	3	4	5	6
Height (m)	11	36	51	56	51	36	11

- **a)** Draw a scatter plot of the data.
- **b)** Draw a curve of good fit.
- **c)** Determine the equation that will model this set of data.

12. Jason tossed a ball over a motion detector and it recorded these data.

Time (s)	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Height above Ground (m)	0	2.1875	3.75	4.6875	5	4.6875	3.75	2.1875	0

a) Draw a scatter plot of the data.

b) Draw a curve of good fit.

c) Determine an algebraic expression that models the data. Express the function in standard form.

13. Students at an agricultural school collected data showing the effect of different annual amounts of water (rainfall plus irrigation), x, in hectare-metres $(\text{ha} \cdot \text{m})$, on the yield of broccoli, y, in hundreds of kilograms per hectare (100 kg/ha).

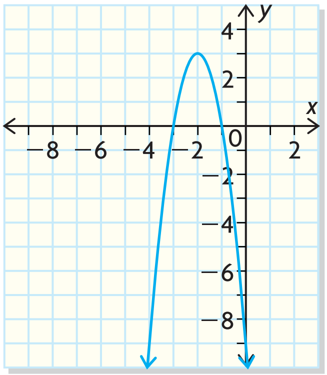
Amount of Water, *x* (ha • m)	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35	1.50
Yield, *y* (100 kg/ha)	35	104	198	287	348	401	427	442	418

a) Draw a scatter plot and a curve of good fit.

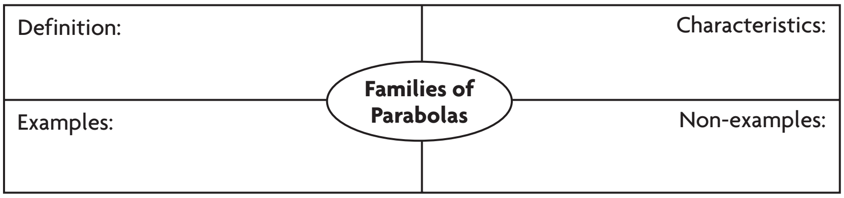
b) Estimate the location of the vertex.

c) Determine an algebraic model for the data.

14. What is the equation of the parabola at the right if the point $(-4, -9)$ is on the graph?



15. **C** Complete the chart shown. Include what you know about families of parabolas in standard, vertex, and factored form.



Extending

16. A parabolic bridge is 40 m wide. Determine the height of the bridge 12 m in from the outside edge, if the height 5 m in from the outside edge is 8 m.

17. A family of cubic equations with zeros -3, 1, and 5 can be represented by the function $f(x) = a(x + 3)(x - 1)(x - 5)$. Which equation describes the cubic in the family that passes through the point (3, 6)?

3.8

Linear–Quadratic Systems

GOAL

Solve problems involving the intersection of a linear and a quadratic function.

LEARN ABOUT the Math

Adam has decided to celebrate his birthday by going skydiving. He loves to freefall, so he will wait for some time before opening his parachute.

- His height after jumping from the airplane before he opens his parachute can be modelled by the quadratic function $h_1(t) = -4.9t^2 + 5500$, where t is time in seconds and $h_1(t)$ is the height above the ground, in metres, t seconds after jumping out.
- After he releases his parachute, he begins falling at a constant rate. His height above the ground can be modelled by the linear function $h_2(t) = -5t + 4500$.

? According to these models, how long after jumping out of the airplane should Adam release his parachute? At what height will this occur?

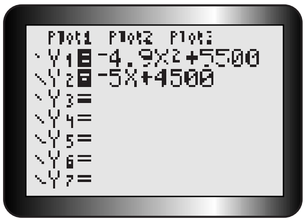
EXAMPLE 1 Selecting a strategy to solve a linear–quadratic system of equations

Determine when the two functions have the same height values.

Tech Support

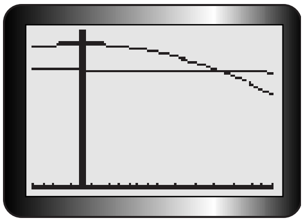
For help using the graphing calculator to find points of intersection, see Technical Appendix, B-12.

Kobi's Solution: Using a Graphical Approach

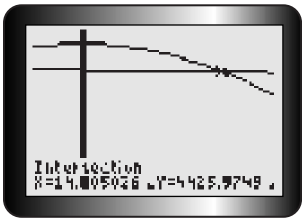


```
Plot1 Plot2 Plot3
\Y1■-4.9X²+5500
\Y2■-5X+4500
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

I graphed both functions on my graphing calculator. I was looking for the time at which both function values were the same. I looked for the point of intersection of the two graphs.



I adjusted the window settings until both graphs and the point of intersection on the right was visible. I was only interested in this point since time must be positive.



Intersection
X=14.805026 Y=4425.9749

I used the intersection operation to locate the point of intersection: (14.8, 4426). That means that Adam should release his parachute 14.8 s after jumping out of the airplane. He will be 4426 m above the ground at that time.

Christina's Solution: Using Algebra

Just before parachute opens:
$h(t) = -4.9t^2 + 5500$

Just after parachute opens:
$h(t) = -5t + 4500$

At the moment the parachute is opened, the heights represented by each equation will be the same, so I set the right-hand sides of the equations equal to each other. This resulted in a quadratic equation that I needed to solve.

$$-4.9t^2 + 5500 = -5t + 4500$$

$$-4.9t^2 + 5t + 1000 = 0$$

I put the equation into standard form and then used the quadratic formula with $a = -4.9$, $b = 5$, and $c = 1000$ to solve for t.

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{5^2 - 4(-4.9)(1000)}}{2(-4.9)} \\ &= \frac{-5 \pm \sqrt{25 + 19\,600}}{-9.8} \\ &= \frac{-5 \pm \sqrt{19\,625}}{-9.8} \end{aligned}$$

$t \doteq 14.8$ or $t \doteq -13.78 \leftarrow$ inadmissible

I got two possible values for t, but the time after jumping cannot be negative, so the only answer was $t = 14.8$ s.

Adam should open his parachute after 14.8 s.

$$h_2 = -5(14.8) + 4500$$
$$= 4426$$

Adam will be about 4426 m above the ground when he opens his parachute.

To find the height above the ground, I substituted my value for *t* into one of the equations. I chose the second one because it was an easier calculation.

Reflecting

A. Explain how you would determine the point of intersection of a linear function and a quadratic function graphically and algebraically.

B. What are an advantage and a disadvantage of each method?

C. Do you think that a linear function always intersects a quadratic function in two places? Why or why not?

APPLY the Math

EXAMPLE 2 Selecting a strategy to predict the number of points of intersection

Determine the number of points of intersection of the quadratic and linear functions $f(x) = 3x^2 + 12x + 14$ and $g(x) = 2x - 8$.

Julianne's Solution

$$3x^2 + 12x + 14 = 2x - 8$$
$$3x^2 + 10x + 22 = 0$$
$$b^2 - 4ac = (10)^2 - 4(3)(22)$$
$$= 100 - 264$$
$$= -164$$

If there is a point of intersection, the values of $f(x)$ and $g(x)$ will be equal at that point, so I set the expressions for $f(x)$ and $g(x)$ equal to each other. I put the resulting quadratic function into standard form. I then calculated the value of the discriminant.

The line and the parabola do not intersect.

Since $-164 < 0$, there are no real roots.

EXAMPLE 3 Solving a problem involving the point of intersection

Justin is skeet shooting. The height of the skeet is modelled by the function $h(t) = -5t^2 + 32t + 2$, where $h(t)$ is the height in metres t seconds after the skeet is released. The path of Justin's bullet is modelled by the function $g(t) = 31.5t + 1$, with the same units. How long will it take for the bullet to hit the skeet? How high off the ground will the skeet be when it is hit?

Stephanie's Solution

$$h(t) = -5t^2 + 32t + 2$$

$$g(t) = 31.5t + 1$$

$$-5t^2 + 32t + 2 = 31.5t + 1$$

$$-5t^2 + 0.5t + 1 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0.5 \pm \sqrt{(0.5)^2 - 4(-5)(1)}}{-10}$$

$$= \frac{-0.5 \pm \sqrt{20.25}}{-10}$$

I needed to find the point of intersection of the quadratic and the linear functions. I set them equal to each other. Then I put the resulting quadratic equation into standard form. I used the quadratic formula to solve for t.

$$t = \frac{-0.5 + 4.5}{-10} \quad \text{or} \quad t = \frac{-0.5 - 4.5}{-10}$$

$$t = -0.4 \quad \text{or} \quad t = 0.5$$

I got two possible values for t, but time cannot be negative, so I couldn't use the solution $t = -0.4$.

The bullet will hit the skeet after 0.5 s.

$$g(0.5) = 31.5(0.5) + 1$$

$$= 16.75$$

I substituted the value of t into $g(t)$ to solve for the height.

The skeet will be 16.75 m off the ground when it is hit.

In Summary

Key Ideas

- A linear function and a quadratic function can intersect at a maximum of two points.
- The point(s) of intersection of a line and a parabola can be found
  - graphically
  - algebraically

Need to Know

- To determine the points of intersection algebraically, use substitution to replace $f(x)$ in the quadratic function with the expression for $g(x)$ from the linear function. This results in a quadratic equation whose solutions correspond to the x-coordinates of the points of intersection.
- In many situations, one of the two solutions will be inadmissible.

CHECK Your Understanding

1. Find the point(s) of intersection by graphing.

a) $f(x) = x^2, g(x) = x + 6$

b) $f(x) = -2x^2 + 3, g(x) = 0.5x + 3$

c) $f(x) = (x - 3)^2 + 1, g(x) = -2x - 2$

2. Determine the point(s) of intersection algebraically.

a) $f(x) = -x^2 + 6x - 5, g(x) = -4x + 19$

b) $f(x) = 2x^2 - 1, g(x) = 3x + 1$

c) $f(x) = 3x^2 - 2x - 1, g(x) = -x - 6$

3. Determine the number of points of intersection of $f(x) = 4x^2 + x - 3$ and $g(x) = 5x - 4$ without solving.

PRACTISING

4. Determine the point(s) of intersection of each pair of functions.

K **a)** $f(x) = -2x^2 - 5x + 20, g(x) = 6x - 1$

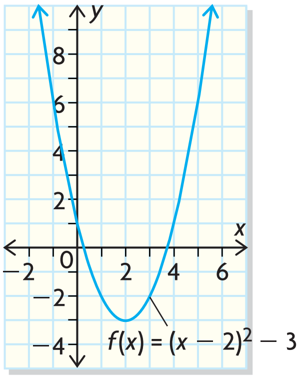
b) $f(x) = 3x^2 - 2, g(x) = x + 7$

c) $f(x) = 5x^2 + x - 2, g(x) = -3x - 6$

d) $f(x) = -4x^2 - 2x + 3, g(x) = 5x + 4$

5. An integer is two more than another integer. Twice the larger integer is one more than the square of the smaller integer. Find the two integers.

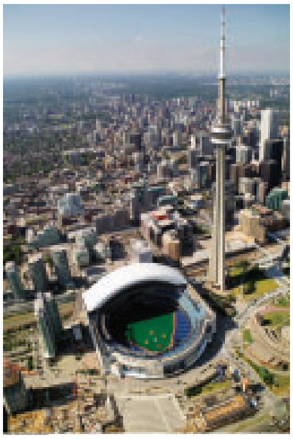
6. The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.



7. **a)** Copy the graph of $f(x) = (x - 2)^2 - 3$. Then draw lines with slope -4 that intersect the parabola at **(i)** one point, **(ii)** two points, and **(iii)** no points.
   b) Write the equations of the lines from part (a).
   c) How are all of the lines with slope -4 that do not intersect the parabola related?

8. Determine the value of k such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.

9. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.



10. **A** A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

    $h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
    $h_2(t) = -4t + 142$ after he released his parachute.

    How long after jumping did the daredevil release his parachute?

11. **T** A quadratic function is defined by $f(x) = 3x^2 + 4x - 2$. A linear function is defined by $g(x) = mx - 5$. What value(s) of the slope of the line would make it a tangent to the parabola?

12. A punter kicks a football. Its height, $h(t)$, in metres, t seconds after the kick is given by the equation $h(t) = -4.9t^2 + 18.24t + 0.8$. The height of an approaching blocker's hands is modelled by the equation $g(t) = -1.43t + 4.26$, using the same t. Can the blocker knock down the punt? If so, at what point will it happen?

13. **C** Given a quadratic function $f(x)$ and a linear function $g(x)$, describe two ways you could determine the number of points of intersection of the two functions without solving for them.

Extending

14. Determine the coordinates of any points of intersection of the functions $x^2 - 2x + 3y + 6 = 0$ and $2x + 3y + 6 = 0$.

15. In how many ways could the graphs of two parabolas intersect? Draw a sketch to illustrate each possibility.

16. Determine the equation of the line that passes through the points of intersection of the graphs of the quadratic functions $f(x) = x^2 - 4$ and $g(x) = -3x^2 + 2x + 8$.

3 Chapter Review

FREQUENTLY ASKED Questions

Study | *Aid*

- See Lesson 3.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 12, 13, and 14.

Q: How can you determine the solutions to a quadratic equation?

A: Arrange the equation into standard form $ax^2 + bx + c = 0$, then:

- try to factor; the numbers that make each factor zero are the solutions to the original equation
- graph the corresponding function $f(x) = ax^2 + bx + c = 0$ and determine its x-intercepts; these points are the solutions to the original equation
- use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Study | *Aid*

- See Lesson 3.6, Examples 2, 3, and 4.
- Try Chapter Review Questions 15 and 16.

Q: How can you use the discriminant to determine the number of solutions of a quadratic equation?

A: The value of the discriminant, $b^2 - 4ac$, determines the number of roots of a quadratic equation. If $b^2 - 4ac > 0$, there are two distinct roots. If $b^2 - 4ac = 0$, there is one root. If $b^2 - 4ac < 0$, there are no roots.

Study | *Aid*

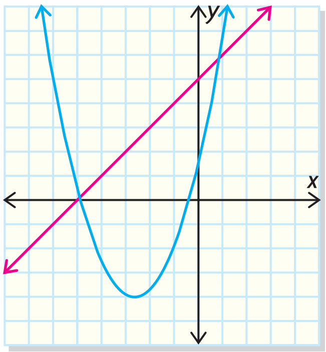
- See Lesson 3.7, Example 2.
- Try Chapter Review Questions 18 and 19.

Q: What characteristics do the members of the family of parabolas $f(x) = a(x - 2)(x + 6)$ have in common? the family of parabolas $g(x) = a(x - 2)^2 - 5$? the family of parabolas $h(x) = ax^2 + bx - 7$?

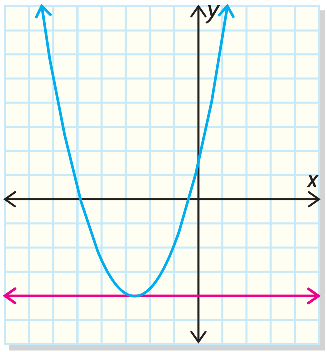
A: The members of the first family of parabolas will all have the same x-intercepts, 2 and -6, and the same axis of symmetry, $x = -2$. The members of the second family will have the same vertex, $(2, -5)$. The members of the third family will have the same y-intercept, $y = -7$.

Q: In how many ways can a line intersect a parabola?

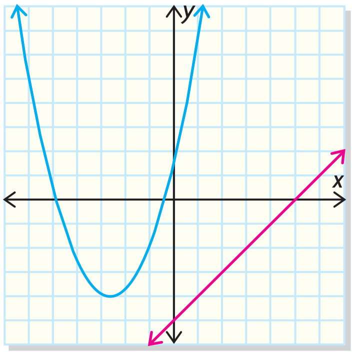
A: A line can intersect a parabola in at most two places.



It may intersect a parabola at only one point.



There may be no points of intersection.



Q: How can you determine the points of intersection between a linear and a quadratic function?

A1: You can graph both functions on the same set of axes and determine the point(s) of intersection from the graphs.

A2: You could equate the two functions. This results in a quadratic equation whose solutions are the *x*-coordinates of the points of intersection.

Study *Aid*

- See Lesson 3.8, Examples 1 and 3.
- Try Chapter Review Questions 21 and 23.

PRACTICE Questions

Lesson 3.1

1. Consider the quadratic function $f(x) = -3(x - 2)^2 + 5$.

a) State the direction of opening, the vertex, and the axis of symmetry.

b) State the domain and range.

c) Graph the function.

2. Consider the quadratic function $f(x) = 4(x - 2)(x + 6)$.

a) State the direction of opening and the zeros of the function.

b) Determine the coordinates of the vertex.

c) State the domain and range.

d) Graph the function.

3. Determine the equation of the axis of symmetry of the parabola with points $(-5, 3)$ and $(3, 3)$ equally distant from the vertex on either side of it.

Lesson 3.2

4. For each quadratic function, state the maximum or minimum value and where it will occur.

a) $f(x) = -3(x - 4)^2 + 7$

b) $f(x) = 4x(x + 6)$

5. The height, $h(t)$, in metres, of the trajectory of a football is given by $h(t) = 2 + 28t - 4.9t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached.

Lesson 3.3

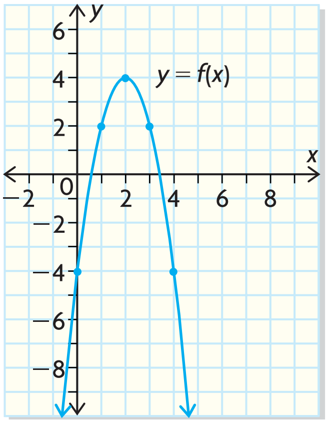
6. Describe the relationship between $f(x) = x^2$, $g(x) = \sqrt{x}$, and $h(x) = -\sqrt{x}$.

7. Is the inverse of a quadratic function also a function? Give a reason for your answer.

8. a) Given the graph of $f(x)$, sketch the graph of the inverse relation.

b) State the domain and range of the inverse relation.

c) Is the inverse relation a function? Why or why not?



Lesson 3.4

9. Express each number as a mixed radical in simplest form.

a) $\sqrt{98}$

b) $-5\sqrt{32}$

c) $4\sqrt{12} - 3\sqrt{48}$

d) $(3 - 2\sqrt{7})^2$

10. The area of a triangle can be calculated from Heron's formula,

$$A = \sqrt{s(s - a)(s - b)(s - c)},$$

where a, b, and c are the side lengths and $s = \frac{a + b + c}{2}$. Calculate the area of a triangle with side lengths 5, 7, and 10. Leave your answer in simplest radical form.

11. What is the perimeter of a right triangle with legs 6 cm and 3 cm? Leave your answer in simplest radical form.

Lesson 3.5

12. Determine the x-intercepts of the quadratic function $f(x) = 2x^2 + x - 15$.

13. The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40\ 000$, where t is the time in years. When $t = 0$, the year is 2007.

a) According to the model, what will the population be in 2020?

b) In what year is the population predicted to be 300 000?

14. A rectangular field with an area of 8000 m^2 is enclosed by 400 m of fencing. Determine the dimensions of the field to the nearest tenth of a metre.

Lesson 3.6

15. The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m? Explain.

16. Determine the values of k for which the function $f(x) = 4x^2 - 3x + 2kx + 1$ has two zeros. Check these values in the original equation.

17. Determine the break-even points of the profit function $P(x) = -2x^2 + 7x + 8$, where x is the number of dirt bikes produced, in thousands.

Lesson 3.7

18. Determine the equation of the parabola with roots $2 + \sqrt{3}$ and $2 - \sqrt{3}$, and passing through the point (2, 5).

19. Describe the characteristics that the members of the family of parabolas $f(x) = a(x + 3)^2 - 4$ have in common. Which member passes through the point $(-2, 6)$?

20. An engineer is designing a parabolic arch. The arch must be 15 m high, and 6 m wide at a height of 8 m.

a) Determine a quadratic function that satisfies these conditions.

b) What is the width of the arch at its base?

Lesson 3.8

21. Calculate the point(s) of intersection of $f(x) = 2x^2 + 4x - 11$ and $g(x) = -3x + 4$.

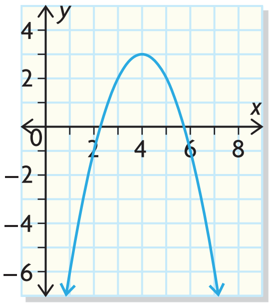
22. The height, $h(t)$, of a baseball, in metres, at time t seconds after it is tossed out of a window is modelled by the function $h(t) = -5t^2 + 20t + 15$. A boy shoots at the baseball with a paintball gun. The trajectory of the paintball is given by the function $g(t) = 3t + 3$. Will the paintball hit the baseball? If so, when? At what height will the baseball be?

23. a) Will the parabola defined by $f(x) = x^2 - 6x + 9$ intersect the line $g(x) = -3x - 5$? Justify your answer.

b) Change the slope of the line so that it will intersect the parabola in two locations.

3 Chapter Self-Test

1. You are given $f(x) = -5x^2 + 10x - 5$.
   - **a)** Express the function in factored form and determine the vertex.
   - **b)** Identify the zeros, the axis of symmetry, and the direction of opening.
   - **c)** State the domain and range.
   - **d)** Graph the function.
2. For each function, state whether it will have a maximum or a minimum value. Describe the method you would choose to calculate the maximum or minimum value.
   - **a)** $f(x) = -2x^2 - 8x + 3$ **b)** $f(x) = 3(x - 1)(x + 5)$
3. You can choose whether you are provided the equation of a quadratic function in standard form, factored form, or vertex form. If you needed to know the information listed, which form would you choose and why?
   - **a)** the vertex
   - **b)** the y-intercept
   - **c)** the zeros
   - **d)** the axis of symmetry
   - **e)** the domain and range
4. Determine the maximum area of a rectangular field that can be enclosed by 2400 m of fencing.
5. Determine the equation of the inverse of $f(x) = 2(x - 1)^2 - 3$.
6. **a)** Simplify $(2 - \sqrt{8})(3 + \sqrt{2})$.
   - **b)** Simplify $(3 + \sqrt{5})(5 - \sqrt{10})$.
   - **c)** Explain why the answer to part (a) has fewer terms than the answer to part (b).
7. Calculate the value of k such that $kx^2 - 4x + k = 0$ has one root.
8. Does the linear function $g(x) = 6x - 5$ intersect the quadratic function $f(x) = 2x^2 - 3x + 2$? How can you tell? If it does intersect, determine the point(s) of intersection.
9. Determine the equation in standard form of the parabola shown below.



Chapter Task

Baseball Bonanza

Vernon Wells hits a baseball that travels for 142 m before it lands. The flight of the ball can be modelled by a quadratic function in which x is the horizontal distance the ball has travelled away from Vernon, and $h(x)$ is the height of the ball at that distance.

There are many quadratic equations you could use to model the distance and height, but you want to find one that is close to reality.

? What is the function that will model the height of Vernon's ball accurately over time?

A. Assume that the ball was between 0.6 m and 1.5 m above the ground when it was hit.

- **a)** What would $h(142)$ be?
- **b)** What happens when $x = 0$?
- **c)** What are the possible values for $h(x)$ when $x = 0$?
- **d)** What would be a good range of values for the height of the ball? Are some values for the height unreasonable?

B. Explain why each function is not a good model of the situation, and support your claim with reasons and a well-labelled sketch.

- **a)** $h(x) = -0.5x(x - 142)$
- **b)** $h(x) = -0.5x^2 + 71x + 1$
- **c)** $h(x) = -0.0015x^2 + 0.213x + 1.2$

C. Determine an equation that models the path of the ball, given this additional information:

- **a)** The ball was 1.2 m off the ground when it was hit.
- **b)** The ball reached a maximum height of 17 m when it was approximately 70 m away from Vernon.

Explain the method you are using to get the equation, and show all of your steps. Why did you approach the problem this way?

D. Use the model you created to graph the flight of Vernon's ball.

Task *Checklist*

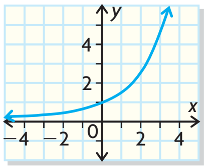
- ✓ Did you state your reasons that the given models were not reasonable?
- ✓ Did you draw a well-labelled graph, including some values?
- ✓ Did you show your work in your choice of method for part C?
- ✓ Did you support your choice of method in part C?

Chapters 1–3 Cumulative Review

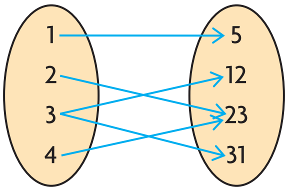
Multiple Choice

1. Identify the relation that is not a function.

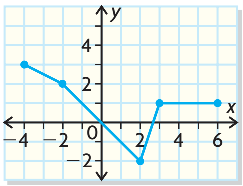
a)



c)



b)



d) $\{(8, 9), (3, 2), (5, 7), (1, 0), (4, 6)\}$

2. For the graph of $f(x) = \sqrt{x}$, identify the transformation that would *not* be applied to $f(x)$ to obtain the graph of $y = 2f(-2x) + 3$.

a) vertical stretch by factor of 2
b) reflection in x-axis
c) vertical translation up 3 units
d) horizontal compression by factor of $\frac{1}{2}$

3. An American visitor to Canada uses this function to convert from temperature in degrees Celsius into degrees Fahrenheit: $f(x) = 2x + 30$.
Identify $f^{-1}(x)$.

a) $f^{-1}(x) = \dfrac{x + 30}{2}$ **c)** $f^{-1}(x) = \dfrac{x - 2}{30}$
b) $f^{-1}(x) = \dfrac{x - 30}{2}$ **d)** $f^{-1}(x) = \dfrac{x + 2}{30}$

4. The range of $f(x) = -|x - 2| + 3$ is

a) $\{y \in \mathbf{R} \mid y \leq 3\}$ **c)** $\{y \in \mathbf{R} \mid 2 \leq y \leq 3\}$
b) $\{y \in \mathbf{R} \mid y \geq 3\}$ **d)** $\{y \in \mathbf{R} \mid 0 \leq y \leq 2\}$

5. Which pairs of functions are equivalent?

a) i **b)** i and ii **c)** i and iii **d)** iii and iv

i) $h(x) = (x + 6)(x + 3)(x - 6)$ and $b(x) = (x + 3)(x^2 - 36)$
ii) $b(t) = (3t + 2)^3$ and $c(t) = 27t^3 + 54t^2 + 36t + 8$
iii) $h(t) = (4 - x)^3$ and $h(t) = (x - 4)^3$
iv) $f(x) = (x^2 - 4x) - (2x^2 + 2x - 4) - (x^2 + 1)$ and $b(x) = (2x - 5)(2x - 1)$

6. Which expression has the restrictions $y \neq -1, 0, \frac{1}{2}$ on its variable?

a) $\dfrac{3y}{y - 2} \times \dfrac{4(y - 2)}{6y}$
b) $\dfrac{5y(y + 3)}{4y} \times \dfrac{(y - 5)}{(y + 3)}$
c) $\dfrac{(3y + 1)}{(2y - 1)} \div \dfrac{3y(y + 1)}{2y - 1}$
d) $\dfrac{10y}{y + 2} \div \dfrac{5}{2(y + 2)}$

7. Identify the correct product of $\dfrac{x^2 - 5x + 6}{x^2 - 1} \times \dfrac{x^2 - 4x - 5}{x^2 - 4}$.

a) $\dfrac{(x + 3)(x - 5)}{(x - 1)(x + 2)}$ **c)** $\dfrac{(x - 3)(x - 5)}{(x - 1)(x - 2)}$
b) $\dfrac{(x - 3)(x + 5)}{(x + 1)(x + 2)}$ **d)** $\dfrac{(x - 3)(x - 5)}{(x - 1)(x + 2)}$

8. Identify the correct sum of $\dfrac{5x - 6}{x + 1} + \dfrac{3x}{x - 4}$.

a) $\dfrac{2x^2 + 23x + 24}{(x + 1)(x - 4)}$ **c)** $\dfrac{15x^2 - 18x}{(x + 1)(x - 4)}$
b) $\dfrac{8x^2 - 23x + 24}{(x + 1)(x - 4)}$ **d)** $\dfrac{8x^2 - 29x + 24}{(x + 1)(x - 4)}$

9. Given the quadratic function $f(x) = 3x^2 - 6x + 15$, identify the coordinates of the vertex.

a) $(1, 12)$ **c)** $(12, 1)$
b) $(-1, -12)$ **d)** $(12, -1)$

10. When the equation of a quadratic function is in factored form, which feature is most easily determined?

a) y-intercepts **c)** vertex
b) x-intercepts **d)** maximum value

11. The height, h, in metres, of a baseball after Bill hits it with a bat is described by the function $h(t) = 0.8 + 29.4t - 4.9t^2$, where t is the time in seconds after the ball is struck. What is the maximum height of the ball?

a) 4.9 m **b)** 29.4 m **c)** 44.9 m **d)** 25 m

12. It costs a bus company \$225 to run a minibus on a ski trip, plus \$30 per passenger. The bus has seating for 22 passengers, and the company charges \$60 per fare if the bus is full. For each empty seat, the company has to increase the ticket price by \$5. How many empty seats should the bus run with to maximize profit from this trip?

a) 8 **b)** 6 **c)** 10 **d)** 2

13. Without drawing the graph, identify the function that has two zeros.

a) $n(x) = -x^2 - 6x - 9$
b) $m(x) = 4(x + 1)^2 + 0.5$
c) $h(x) = -5(x + 1.3)^2$
d) $g(x) = -2(x + 3.6)^2 + 4.1$

14. The graph of function $f(x) = x^2 - kx + k + 8$ touches the x-axis at one point. What are the possible values of k?

a) $k = 1$ or $k = 8$ **c)** $k = 0$ or $k = 1$
b) $k = -4$ or $k = 8$ **d)** $k = -8$ or $k = 4$

15. For $f(x) = 2(x - 3)^2 + 5, x \geq 3$, determine the equation for f^{-1}.

a) $y = 3 + \sqrt{\frac{x - 5}{2}}, x \geq 5$

b) $y = 3 - \sqrt{\frac{x - 5}{2}}, x \geq 5$

c) $y = 3 \pm \sqrt{\frac{x - 5}{2}}$

d) $y = 3 + \sqrt{\frac{x + 5}{2}}, x \leq 5$

16. The relation that is also a function is

a) $x^2 + y^2 = 25$ **c)** $x^2 = y$
b) $y^2 = x$ **d)** $x^2 - y^2 = 25$

17. Given $f(x) = x^2 - 5x + 3$, then

a) $f(-1) = -3$ **c)** $f(-1) = -1$
b) $f(-1) = 7$ **d)** $f(-1) = 9$

18. Which of the following statements is not true?

a) The horizontal line test can be used to show that a relation is a function.
b) The set of all possible input values of a function is called the domain.
c) The equation $y = 3x + 5$ describes a function.
d) This set of ordered pairs describes a function: $\{(0, 1), (1, 2), (3, -3), (4, -1)\}$.

19. The range that best corresponds to $f(x) = \frac{3}{x}$ is

a) $\{y \in \mathbf{R}\}$ **c)** $\{y \in \mathbf{R} \mid y < 0\}$
b) $\{y \in \mathbf{R} \mid y > 0\}$ **d)** $\{y \in \mathbf{R} \mid y \neq 0\}$

20. If $f(x) = 5x - 7$, then

a) $f^{-1}(x) = 7x - 5$ **c)** $f^{-1}(x) = \frac{x - 5}{7}$
b) $f^{-1}(x) = x - 7$ **d)** $f^{-1}(x) = \frac{x + 7}{5}$

21. The inverse of $g(x) = x^2 - 5x - 6$ is

a) $g^{-1}(x) = \left(x - \frac{1}{2}\right)^2 - \frac{49}{4}$

b) $g^{-1}(x) = \frac{1}{2} \pm \sqrt{x - \frac{49}{4}}$

c) $g^{-1}(x) = \frac{1}{2} \pm \sqrt{x + \frac{49}{4}}$

d) $g^{-1}(x) = \left(x + \frac{1}{2}\right)^2 + \frac{49}{4}$

22. Which of the following statements is false?

a) The domain of f is the range of f^{-1}.
b) The graph of the inverse can be found by reflecting $y = f(x)$ in the line $y = x$.
c) The domain of f^{-1} is the range of f.
d) To determine the equation of the inverse, interchange x and y and solve for x.

23. If $f(x) = 3(x + 2)^2 - 5$, the domain must be restricted to which interval if the inverse is to be a function?

a) $x \geq -5$ **c)** $x \geq 2$
b) $x \geq -2$ **d)** $x \geq 5$

24. The inverse of $f(x) = \sqrt{x - 1}$ is

a) $f^{-1}(x) = x^2 + 1, x \leq 1$
b) $f^{-1}(x) = x^2 - 1, x \geq 1$
c) $f^{-1}(x) = x^2 + 1, x \geq 1$
d) $f^{-1}(x) = x^2 - 1, x \leq 1$

25. What transformations are applied to $y = f(x)$ to obtain the graph of $y = af(x - p) + q$, if $a < 0$, $p < 0$, and $q < 0$?

a) Vertical stretch by a factor of $|a|$, followed by a translation $|p|$ units to the left and $|q|$ units down
b) Reflection in the x-axis, vertical stretch by a factor of $|a|$, followed by a translation $|p|$ units to the right and $|q|$ units down
c) Reflection in the x-axis, vertical stretch by a factor of $|a|$, followed by a translation $|p|$ units to the left and $|q|$ units down
d) Reflection in the x-axis, vertical stretch by a factor of $|a|$, followed by a translation $|p|$ units to the right and $|q|$ units up

26. The vertex form of the equation $y = -2x^2 - 12x - 19$ is

a) $y = -2x(x + 6) - 19$
b) $y = -2(x - 3)(x + 6)$
c) $y = -2(x + 3)^2 - 1$
d) $y = -2(x - 3)^2 + 1$

27. The coordinates of the vertex for the graph of $y = (x + 2)(x - 3)$ are

a) $(-2, 3)$
b) $\left(-\frac{1}{2}, -\frac{21}{4}\right)$
c) $(2, 3)$
d) $\left(\frac{1}{2}, -\frac{25}{4}\right)$

28. The profit function for a new product is given by $P(x) = -4x^2 + 28x - 40$, where x is the number sold in thousands. How many items must be sold for the company to break even?

a) 2000 or 5000
b) 2000 or 3500
c) 5000 or 7000
d) 3500 or 7000

29. Which of the following statements is not true for the equation of a quadratic function?

a) In standard form, the y-intercept is clearly visible.
b) In vertex form, the break-even points are clearly visible.
c) In factored form, the x-intercepts are clearly visible.
d) In vertex form, the coordinates of the vertex are clearly visible.

30. State the value of the discriminant, D, and the number of roots for $7x^2 + 12x + 6 = 0$.

a) $D = 312, n = 2$
b) $D = 24, n = 2$
c) $D = 312, n = 1$
d) $D = -24, n = 0$

31. The simplified form of $\frac{7}{ab} - \frac{2}{b} + \frac{1}{3a^2}$ is

a) $\frac{6}{ab - b + 3a^2}, a, b \neq 0$
b) $\frac{21a - 6a^2 + b}{3a^2b}, a, b \neq 0$
c) $\frac{7a - 2a^2 + b}{3a^2b}, a, b \neq 0$
d) $\frac{7a - 2b + ab}{3a^3b^2}, a, b \neq 0$

32. The simplified form of $\frac{x^2 - 4}{x + 3} \div \frac{2x + 4}{x^2 - 9}$ is

a) $\frac{2(x - 2)(x + 2)^2}{(x + 3)^2(x - 3)}$
b) $\frac{(x^2 - 4)(x - 3)}{2x + 4}$
c) $\frac{(x - 2)(x - 3)}{2}$
d) $\frac{2(x - 3)}{x - 2}$

Investigations

33. Studying Functions

Analyze two of the following functions in depth.

a) $f(x) = 3x^2 - 24x + 50$

b) $g(x) = 5 - 2\sqrt{3x + 6}$

c) $h(x) = \dfrac{1}{\frac{1}{3}(x - 6)} - 2$

Include:

i) the domain and range

ii) the relationship to the parent function, including all applied transformations

iii) a sketch of the function

34. Charity Walk

Sacha and Jill set off at the same time on a 30 km walk for charity. Sacha, who has trained all year for this event, walks 1.4 km/h faster than Jill, but sees a friend on the route and stops to talk for 20 min. Even with this delay, Sacha finishes the walk 2 h ahead of Jill.
How fast was each person walking, and how long did it take for each person to finish the walk?

35. Ski Trip

Josh is running a ski trip over March Break. Last year he had 25 students go and each paid \$550. This year he will increase the price and knows that for each \$50 price increase, 2 fewer students will go on the trip. The bus costs a flat fee of \$5500, and hotel and lift tickets cost \$240 per person.
Determine

a) the number of students who must go for Josh to break even

b) the cost of the trip that will maximize his profit

2. **a)** $72a^2 - 198a + 70$ **c)** $20x^3 - 57x^2 - 11x + 6$
b) $-7x^3y^3 + 10x^4y^2 - 12x^2y^4$ **d)** $9p^4 + 6p^3 - 11p^2 - 4p + 4$

3. no

4. **a)** $24n^2 + 48n + 26$ **b)** 866

5. **a)** $m(m - 1)$ **d)** $(x + 3)(3x - 2y - 1)$
b) $(x - 3)(x - 24)$ **e)** $(y - 2)(5x - 3)$
c) $(5x + y)(3x - 2y)$ **f)** $(p - m + 3)(p + m - 3)$

6. $x = -1, 1, 4$

7. **a)** $\frac{14}{3b^2}, a \neq 0, b \neq 0$
b) $\frac{2}{(x-2)(x+3)}, x \neq -3, 2, 4$
c) $\frac{6t - 49}{(t+2)(t-9)}, t \neq -2, 9$
d) $\frac{-x^2 - 18x}{(3x+2)(2x+3)(2x-3)}, x \neq -\frac{3}{2}, -\frac{2}{3}, \frac{3}{2}$

8. yes (as long as there are no restrictions that were factored out)

9. yes

Chapter 3

Getting Started, p. 138

1. **a)** 0 **c)** 0 **e)** $-3k^2 + 4k - 1$
b) -21 **d)** -1 **f)** $-3k^2 - 4k - 1$

2. **a)** $f(x) = x^2 + 2x - 15$ **c)** $f(x) = -3x^2 - 12x - 9$
b) $f(x) = 2x^2 + 12x$ **d)** $f(x) = x^2 - 2x + 1$

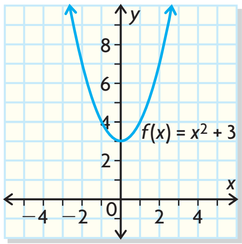
3. **a)** vertex $(-3, -4)$, $x = -3$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq -4\}$
b) vertex $(5, 1)$, $x = 5$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 1\}$

4. **a)** vertex $(0, 4)$, $x = 0$, opens up
b) vertex $(4, 1)$, $x = 4$, opens up
c) vertex $(-7, -3)$, $x = -7$, opens down
d) vertex $(1.5, 36.75)$, $x = 1.5$, opens down

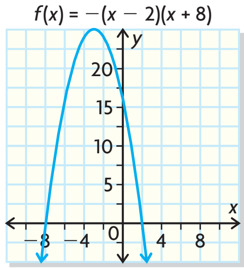
5. **a)** $x = 3$ or 8 **c)** $x = -1$ or 1.67
b) $x = 0.55$ or 5.45 **d)** $x = 0.5$ or 3

6. **a)** $(-3, 0), (3, 0)$ **c)** $(1.33, 0), (2, 0)$
b) $(-1.83, 0), (9.83, 0)$ **d)** $(0, 0), (3, 0)$

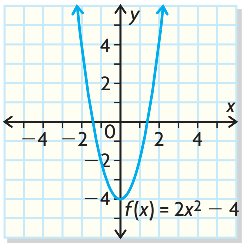
7. **a)**



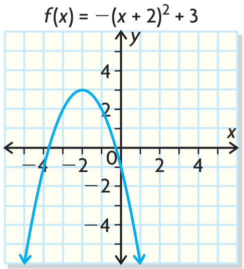
c)



b)



d)



8.

Definition: equation is of form $y = ax^2 + bx + c$ or equivalent	**Characteristics:** graph is a parabola function has two, one, or no zeros second differences are constant
Examples: $y = x^2$ $y = -4(x + 3)^2 - 5$	**Non-examples:** $y = 5 - 4x$ $y = 2\sqrt{x} - 5$

Quadratic Function

Lesson 3.1, pp. 145–147

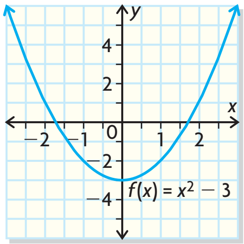
1. **a)** linear, first differences are constant
b) quadratic, second differences are constant
c) linear, first differences are constant
d) quadratic, second differences are constant

2. **a)** opens up **b)** opens down **c)** opens down **d)** opens up

3. **a)** zeros $x = 2$ or -6 **b)** opens down **c)** $x = -2$

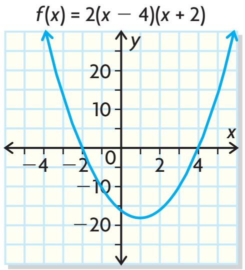
4. **a)** vertex $(-2, 3)$ **b)** $x = -2$
c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 3\}$

5. **a)**



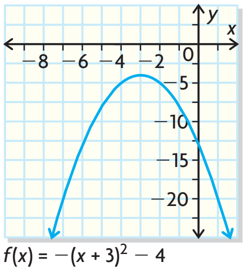
opens up, vertex $(0, -3)$, $x = 0$

c)



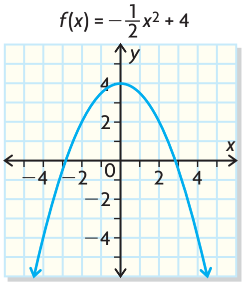
opens up, vertex $(1, -18)$, $x = 1$

b)



opens down, vertex $(-3, -4)$, $x = -3$

d)



opens down, vertex $(0, 4)$, $x = 0$

6. **a)** $f(x) = -3x^2 + 6x + 3$, $(0, 3)$
b) $f(x) = 4x^2 + 16x - 84$, $(0, -84)$

7. **a)** opens down
b) vertex $(-1, 8)$
c) $(-3, 0), (1, 0)$
d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 8\}$
e) negative; parabola opens down
f) $f(x) = -2(x + 1)^2 + 8$ or $f(x) = -2(x + 3)(x - 1)$

8. **a)** opens up
b) vertex $(1, -3)$
c) $x = 1$
d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -3\}$
e) positive; parabola opens up

9. **a)** $x = 0$ **c)** $x = 12$ **e)** $x = -1.5$

b) $x = -7$ **d)** $x = -2$ **f)** $x = -\frac{5}{16}$

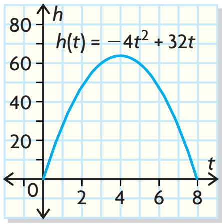
10. **a)**

x	−2	−1	0	1	2
f(x)	3	4	3	0	−5

b) First differences: 1, −1, −3, −5; Second differences: −2; parabola opens down

c) $f(x) = -(x + 1)^2 + 4$

11. **a)**



b) 8 s; height starts at 0 m and is 0 m again after 8 s.

c) $h(3) = 60$ m

d) 64 m

12. $y = 30$

13. Similarities: both are quadratic; both have axis of symmetry $x = 1$. Differences: $f(x)$ opens up, $g(x)$ opens down; $f(x)$ has vertex $(1, -2)$, $g(x)$ has vertex $(1, 2)$

14.

x	−2	−1	0	1	2	3
f(x)	19	9	3	1	3	9
First Differences		−10	−6	−2	2	6
Second Differences			4	4	4	4

15. $56 250

16. $y = -\frac{1}{105}(x + 60)(x - 35)$ $-\frac{1}{105}(x+12.5)^2 + \frac{1805}{84}$

Lesson 3.2, pp. 153–154

1. a) and c); (a) is negative.

2. **a)** vertex $(-5, -2)$, minimum value -2

b) vertex $(4, 8)$, maximum value 8

3. **a)** maximum: 6 **c)** maximum: 8

b) minimum: 0 **d)** minimum: -7

4. **a)** complete the square; minimum: -5

b) factor or complete the square; minimum: -4

c) factor or complete the square; minimum: -18

d) factor or complete the square; maximum: 27

e) use partial factoring; minimum: 2

f) use vertex form; maximum: -5

5. **a)** **i)** $R(x) = -x^2 + 5x$ **ii)** maximum revenue: $6250

b) **i)** $R(x) = -4x^2 + 12x$ **ii)** maximum revenue: $9000

c) **i)** $R(x) = -0.6x^2 + 15x$ **ii)** maximum revenue: $93 750

d) **i)** $R(x) = -1.2x^2 + 4.8x$ **ii)** maximum revenue: $4800

6. **a)** minimum: -2.08 **b)** maximum: 1.6

7. **a)** **i)** $P(x) = -x^2 + 12x - 28$ **ii)** $x = 6$

b) **i)** $P(x) = -2x^2 + 18x - 45$ **ii)** $x = 4.5$

c) **i)** $P(x) = -3x^2 + 18x - 18$ **ii)** $x = 3$

d) **i)** $P(x) = -2x^2 + 22x - 17$ **ii)** $x = 5.5$

8. **a)** 70 m **b)** 2 s **c)** 50 m

9. $562 500

10. Minimum value is 2, therefore $3x^2 - 6x + 5$ cannot be less than 1.

11. **a)** $5 450 000

b) Maximum profit occurs when $40 000 is spent on advertising.

c) between $22 971 and $57 029

12. Is possible, because maximum rectangular area occurs when rectangle is 125 m by $\frac{125}{\pi}$ m.

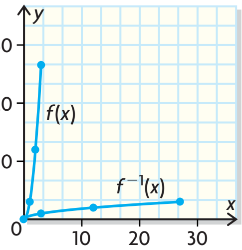
13. Possible response: Function is in standard form, so to find the minimum, we must find the vertex. Completing the square would result in fractions that are more difficult to calculate than whole numbers. Since this function will factor, putting the function in factored form and averaging the zeros to find the *x*-intercept of the vertex would be possible; however, there would still be fractions to work with. Using the graphing calculator to graph the function, then using CALC to find the minimum, would be the easiest method for this function.

14. $t = \frac{v_0}{9.8}$ seconds

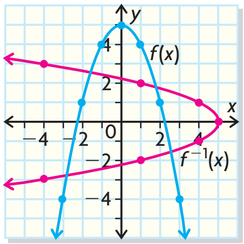
15. $9

Lesson 3.3, pp. 160–162

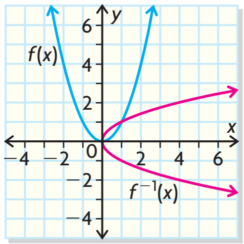
1. **a)**



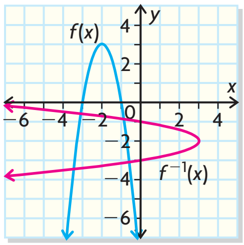
b)



2. **a)**



b)

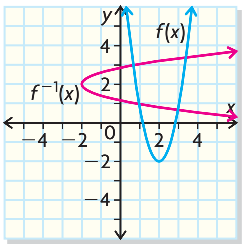


3. $f^{-1}(x) = \pm\sqrt{\frac{x + 1}{2}}$

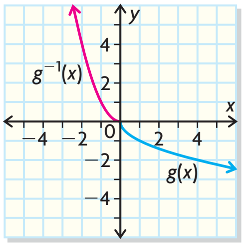
4. **a)** -1 **c)** 0 or 2

b) $1 \pm \sqrt{\frac{7 - x}{2}}$ **d)** $1 \pm \sqrt{-a}$

5. **a), b)**



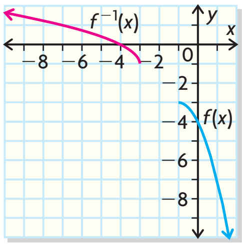
6. **a), b)**



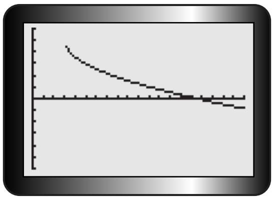
c) domain $= \{x \in \mathbf{R} \mid x \leq 0\}$; range $= \{y \in \mathbf{R} \mid y \geq 0\}$

d) $g^{-1}(x) = (-x)^2$ or $g^{-1}(x) = x^2, x \leq 0$

7. $f^{-1}(x) = -1 + \sqrt{-x - 3}$



8. $f^{-1}(x) = 5 - \sqrt{2x - 6}, x \geq 3$



9. **a)** domain $= \{x \in \mathbf{R} \mid -2 < x < 3\}$;
range $= \{y \in \mathbf{R} \mid -3 \leq y < 24\}$

b) $f^{-1}(x) = 1 + \sqrt{\dfrac{x + 3}{3}}, -3 \leq x \leq 24$

10. **a)** $h(t) = -5(t - 1)^2 + 40$

b) domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 3.83\}$;
range $= \{h \in \mathbf{R} \mid 0 \leq h \leq 40\}$

c) $$t = \begin{cases} 1 - \sqrt{\dfrac{40 - h}{5}}, & 35 < h \leq 40 \\ 1 + \sqrt{\dfrac{40 - h}{5}}, & 0 < h \leq 40 \end{cases}$$

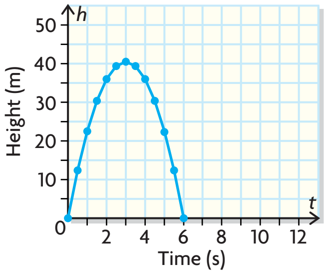
d) domain $= \{h \in \mathbf{R} \mid 0 \leq h \leq 40\}$;
range $= \{t \in \mathbf{R} \mid 0 \leq t \leq 3.83\}$

11. **a)**

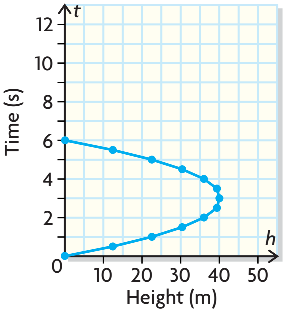
Time (s)	0	0.5	1	1.5	2	2.5
Height (m)	0	12.375	22.5	30.375	36.0	39.375

Time (s)	3	3.5	4	4.5	5	5.5	6
Height (m)	40.5	39.375	36.0	30.375	22.5	12.375	0

b)



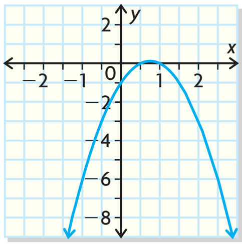
c)



d) The inverse is not a function. It does not pass the vertical-line test.

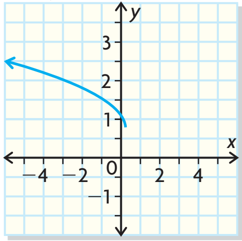
12. **a)** (0.75, 0.125)

b)



$f(x) = -2x^2 + 3x - 1$

c) $y = \sqrt{\dfrac{0.125 - x}{2}} + 0.75$

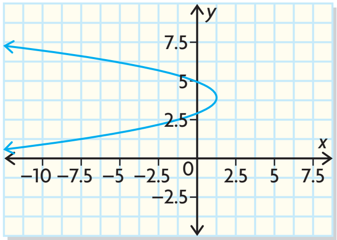


d) domain $= \{x \in \mathbf{R} \mid x \leq 0.125\}$; range $= \{y \in \mathbf{R} \mid y \geq 0.75\}$

e) The y-values were restricted to ensure $f^{-1}(x)$ is a function.

13. **a)** **i)** $f(x) = -(x - 4)^2 + 1$

ii)

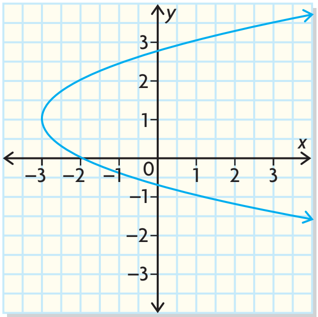


iii) Domain of f should be restricted to $\{x \in \mathbf{R} \mid x \geq 4\}$ or $\{x \in \mathbf{R} \mid x \leq 4\}$

iv) f^{-1} is $y = \pm\sqrt{-x + 1} + 4$

b) i) $f(x) = (x - 1)^2 - 3$

ii)

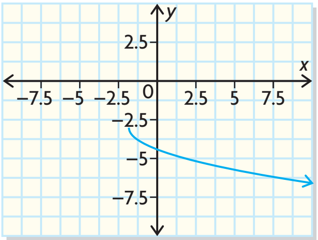


iii) Domain of f should be restricted to $\{x \in \mathbf{R} \mid x \leq 1\}$ or $\{x \in \mathbf{R} \mid x \geq 1\}$

iv) f^{-1} is $y = \pm\sqrt{x + 3} + 1$

c) i) $f(x) = (x + 3)^2 - 2$ where $x \leq -3$

ii)

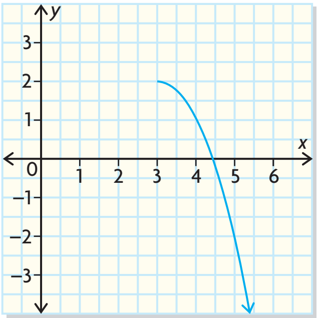


iii) No restrictions necessary.

iv) $f^{-1}(x)$ is $y = -\sqrt{x + 2} - 3$

d) i) $f(x) = 3 + \sqrt{2 - x}$

ii)



iii) No restrictions necessary.

iv) f^{-1} is $y = -(x - 3)^2 + 2$ where $x \geq 3$

14. The original function must be restricted so that only one branch of the quadratic function is admissible. For example, if $f(x) = x^2$ had its domain restricted to $x \geq 0$, the inverse of $f(x)$ would be a function.

15. a) Possible response: Switch x and y and solve resulting quadratic equation for y, either by completing the square or by using the quadratic formula.

b) No, because the original function assigns some y-values to two x-values, so the inverse assigns two y-values to some x-values.

16. a) $P(x) = (x - 3.21)(14\,700 - 3040x)$

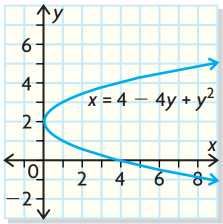
b) $P^{-1}(x) = 4.02 \pm \sqrt{\dfrac{-x + 2008}{3040}}$. This equation will take the total profit and determine the price per kilogram.

c) $$P^{-1}(1900) = 4.02 \pm \sqrt{\frac{-1900 + 2008}{3040}}$$
$$= 4.21 \text{ or } 3.83$$

If the meat manager charges either \$4.21/kg or \$3.83/kg, she will make a profit of \$1900.

d) \$4.02/kg **e)** \$3.97/kg. Profit would be about \$2289.

17. a)



b) domain = $\{x \in \mathbf{R} \mid x \geq 0\}$; range = $\{y \in \mathbf{R}\}$

c) $y = (x - 2)^2$

d) Yes, the inverse is a function. Its graph will be a parabola, so it will pass the vertical-line test.

Lesson 3.4, pp. 167–168

1. **a)** $3\sqrt{3}$ **b)** $5\sqrt{2}$ **c)** $7\sqrt{2}$ **d)** $4\sqrt{2}$

2. **a)** $\sqrt{35}$ **b)** $\sqrt{66}$ **c)** $10\sqrt{6}$ **d)** $-32\sqrt{39}$

3. **a)** $7\sqrt{5}$ **b)** $5\sqrt{7}$ **c)** $-\sqrt{3} + 19\sqrt{2}$ **d)** $-\sqrt{2}$

4. **a)** $6\sqrt{3}$ **c)** $20\sqrt{10}$ **e)** $2\sqrt{5}$
b) $-25\sqrt{5}$ **d)** $-\sqrt{15}$ **f)** $-18\sqrt{3}$

5. **a)** $2\sqrt{3} - \sqrt{15}$ **c)** 32 **e)** $36\sqrt{2}$
b) $2\sqrt{14} + 6\sqrt{6}$ **d)** $-24\sqrt{3}$ **f)** -140

6. **a)** $-2\sqrt{2}$ **c)** $-9\sqrt{2}$ **e)** $2\sqrt{13}$
b) $-\sqrt{3} + 8\sqrt{2}$ **d)** $15\sqrt{2}$ **f)** $16\sqrt{3} - 4\sqrt{7}$

7. **a)** $18 + 12\sqrt{10} - 3\sqrt{5} - 10\sqrt{2}$
b) $31 + 12\sqrt{3}$
c) -3
d) $-7 - 2\sqrt{6}$
e) $83 - 12\sqrt{35}$
f) $4 + 3\sqrt{6} - 8\sqrt{3} - 13\sqrt{2}$

8. $4\sqrt{2}$ cm

9. $15\sqrt{2}$ cm

10. $3\sqrt{10}$ cm

11. $6\sqrt{2}$

12. Perimeter = $8\sqrt{2} + 4\sqrt{5}$, Area = 12

13. $(\sqrt{a} + \sqrt{b})^2$

14. Possible response: $2\sqrt{50}$, $5\sqrt{8}$, $10\sqrt{2}$; The last one is in simplest radical form because the number under the radical sign cannot be simplified any further.

15. **a)** $a\sqrt{a}$ **c)** $3n^3\sqrt{n}$
b) $x^2y^3\sqrt{x}$ **d)** $-p + 2q - \sqrt{pq}$

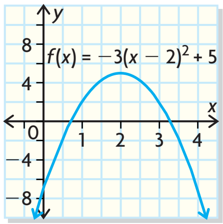
16. $2\sqrt{2}$

17. $x = 16$

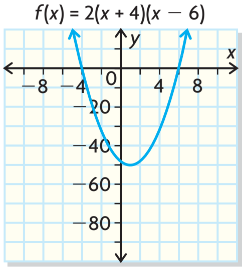
Mid-Chapter Review, p. 170

1. **a)** second differences = −4; quadratic
b) second differences = 2; quadratic

2. **a)**



b)



3. **a)** vertex (2, 5), $x = 2$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 5\}$
b) vertex (1, −50), $x = 1$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -50\}$

4. **a)** $f(x) = -3x^2 + 12x - 7$
b) $f(x) = 2x^2 - 4x - 48$

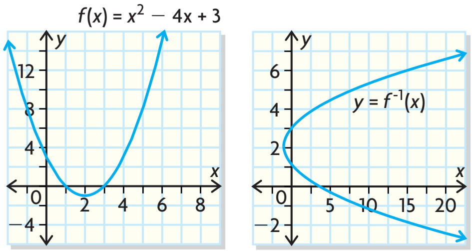
5. **a)** Minimum value of −7
b) Minimum value of −50
c) Maximum value of 12.5
d) Minimum value of −24.578 125

6. Maximum profit is $9000 when 2000 items are sold.

7. 2000 items/h

8. 64

9. **a)** $f^{-1}(x) = 2 \pm \sqrt{x + 1}$
b) domain of $f(x) = \{x \in \mathbf{R}\}$, range of $f(x) = \{y \in \mathbf{R} \mid y \geq -1\}$; domain of $f^{-1}(x) = \{x \in \mathbf{R} \mid x \geq -1\}$, range of $f^{-1}(x) = \{y \in \mathbf{R}\}$
c)



10. $x = 10 \pm \sqrt{\dfrac{R - 15}{-2.8}}$

11. Usually, the original function assigns some y-values to two x-values, so the inverse assigns two y-values to some x-values.

12. **a)** $\{x \in \mathbf{R} \mid x \geq -3\}$ $\{y \in \mathbf{R} \mid y \leq 0\}$
b) $f^{-1}(x) = x^2 - 3, x \leq 0\}$

13. **a)** $4\sqrt{3}$ **b)** $2\sqrt{17}$ **c)** $6\sqrt{5}$ **d)** $-15\sqrt{3}$ **e)** $35\sqrt{2}$ **f)** $-16\sqrt{3}$

14. **a)** $7\sqrt{2}$ **b)** $30\sqrt{3}$ **c)** $-5\sqrt{3}$ **d)** $9\sqrt{7} - 19\sqrt{2}$ **e)** $14 + 3\sqrt{3}$ **f)** $70 + 55\sqrt{2}$

Lesson 3.5, pp. 177–178

1. **a)** $x = -1$ or -4 **b)** $x = 2$ or 9 **c)** $x = \pm\dfrac{3}{2}$ **d)** $x = -\dfrac{1}{2}$ or 4

2. **a)** $x = 5.61$ or -1.61
b) $x = 1.33$ or -2
c) no real roots
d) $x = -1.57$ or 5.97

3. **a)** $x = -1$ or -0.25 **b)** $x = 1$ or 4.5

4. **a)** **i)** Solve by factoring, function factors **ii)** $x = 0$ or 10
b) **i)** Quadratic formula, function does not factor
ii) $x = \dfrac{-3 \pm \sqrt{5}}{4}$
c) **i)** Quadratic formula, function does not factor
ii) $x = -2 \pm \sqrt{7}$
d) **i)** Quadratic formula, function does not factor
ii) $x = -4 \pm \sqrt{7}$
e) **i)** Solve by factoring, function factors
ii) $x = -1$ or 10
f) **i)** Quadratic formula, function does not factor
ii) $x = 2 \pm \sqrt{19}$

5. **a)** about (2.59, 0), about (−0.26, 0) **b)** (1, 0), $\left(\dfrac{21}{4}, 0\right)$

6. **a)** 14 000 **b)** 4000 or 5000 **c)** 836 or 10 163 **d)** 900 or 11 099

7. 1.32 s

8. **a)** 50 000 **b)** 290 000 **c)** 2016

9. 15 m by 22 m

10. −19, −18 or 18, 19

11. base = 8 cm, height = 24 cm

12. about 2.1 m

13. **a)** after 1about 1.68 s and again at about 17.09 s
b) The rocket will be above 150 m for 17.09 − 1.68 = 15.41 s.

14. $2.75 (It is unreasonable to raise the fare to $14.25.)

15. Factoring the function and finding the zeros; substituting the values of a, b, and c into the quadratic formula; graphing the function on a graphing calculator and using **CALC** to find the zeros

16. 10 cm, 24 cm, 26 cm

17. $x = 0$ or $-\dfrac{2}{3}$

Lesson 3.6, pp. 185–186

1. **a)** vertex (0, −5), up, 2 zeros
b) vertex(0, 7), down, 2 zeros
c) vertex (0, 3), up, no zeros
d) vertex (−2, 0), up, 1 zero
e) vertex (−3, −5), down, no zeros
f) vertex (4, −2), up, 2 zeros

2. **a)** 2 zeros **b)** 2 zeros **c)** 2 zeros **d)** 1 zero

3. **a)** 2 zeros **b)** no zeros **c)** 1 zero **d)** 1 zero

4. **a)** 2 zeros **b)** 2 zeros **c)** 2 zeros **d)** no zeros

5. **a)** 2 break-even points
b) Cannot break even
c) 1 break-even point
d) Cannot break even

6. $k = \dfrac{4}{3}$

7. $k < -2$ or $k > 2$

8. $k > \dfrac{4}{3}$, $k = \dfrac{4}{3}$, $k < \dfrac{4}{3}$

9. $k = -4$ or 8

10. No, resulting quadratic has no solutions.

11. Answers may vary. For example,
a) $y = -2(x + 1)(x + 2)$
b) $y = 2x^2 + 1$
c) $y = -2(x - 2)^2$

12. A: break-even at $x = 4.8$
B: break-even at about $x = 0.93$ or about 5.22
C: break-even at about $x = 2.24$ or about 6.19
Buy Machine B. It has the earliest break-even point.

13. **a)** no effect
b) no effect
c) no effect
d) change from 1 to 2 zeros
e) change from 1 to no zeros
f) change from 1 to 2 zeros

14. 10.5

15. $f(x) = -(x - 3)(3x + 1) + 4$ is a vertical translation of 4 units up of the function $g(x) = -(x - 3)(3x + 1)$. Function $g(x)$ opens down and has 2 zeros. Translating this function 4 units up will have no effect on the number of zeros, so $f(x)$ has 2 zeros.

16. **a)** If the vertex is above the x-axis, the function will have 2 zeros if it opens down and no zeros if it opens up. If the vertex is below the x-axis, there will be 2 zeros if the function opens up and no zeros if it opens down. If the vertex is on the x-axis, there is only 1 zero.
b) If the linear factors are equal or multiples of each other, there is 1 zero; otherwise, there are 2 zeros.
c) If possible, factor and determine the number of zeros as in part (b). If not, use the value of $b^2 - 4ac$. If $b^2 - 4ac > 0$, there are 2 zeros, if $b^2 - 4ac = 0$, 1 zero, and if $b^2 - 4ac < 0$, no zeros.

17.
$$\begin{aligned}(x^2 - 1)k &= (x - 1)^2\\ kx^2 - k &= x^2 - x - x + 1\\ kx^2 - k &= x^2 - 2x + 1\\ 0 &= x^2 - kx^2 - 2x + 1 + k\\ 0 &= x^2(1 - k) - 2x + (1 + k)\end{aligned}$$

The equation will have one solution when the discriminant is equal to zero.

$$\begin{aligned} b^2 - 4ac &= 0 \\ (-2)^2 - 4(1 - k)(1 + k) &= 0 \\ 4 - 4(1 - k^2) &= 0 \\ 4 - 4 + 4k^2 &= 0 \\ 4k^2 &= 0 \\ k^2 &= 0 \\ k &= 0 \end{aligned}$$

Therefore, the function will have one solution when $k = 0$.

18. Function has 2 zeros for all values of k.

Lesson 3.7, pp. 192–193

1. Same zeros, 3 and -4

2. Same vertex, $(2, -4)$; stretched vertically by different factors, opening in different directions

3. $(0, -7)$ (the y-intercept)

4. **a)** $f(x) = -\frac{7}{6}(x + 4)(x - 3)$ **c)** $f(x) = \frac{1}{6}(x^2 - 7)$

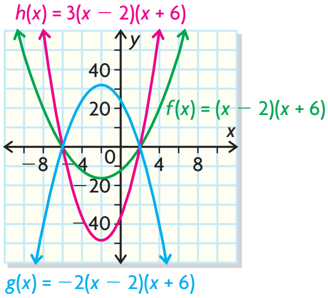
b) $f(x) = -\frac{6}{33}x(x - 8)$ **d)** $f(x) = -4(x^2 - 2x - 1)$

5. **a)** $f(x) = -\frac{13}{36}(x + 2)^2 + 5$ **c)** $f(x) = \frac{2}{25}(x - 4)^2 - 5$

b) $f(x) = -13(x - 1)^2 + 6$ **d)** $f(x) = \frac{8}{49}(x - 4)^2$

6. $f(x) = 5.5x^2 - 6x - 7$

7. **a)–c)**

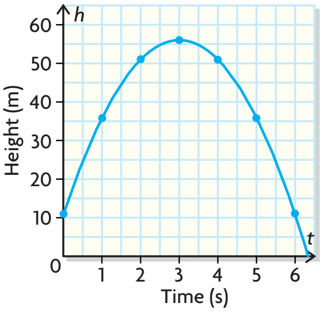


8. $f(x) = -\frac{6}{7}(x - 4)(x + 4)$

9. $f(x) = \frac{5}{33}(x^2 - 4x + 1)$

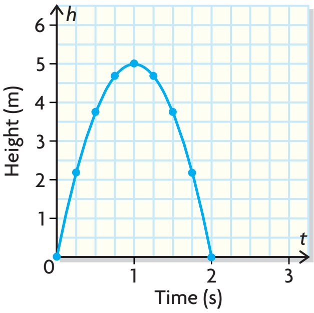
10. $f(x) = -\frac{3}{16}x(x - 12)$ Yes, because at a height of 5 m the bridge is 6.11 m wide.

11. **a), b)**



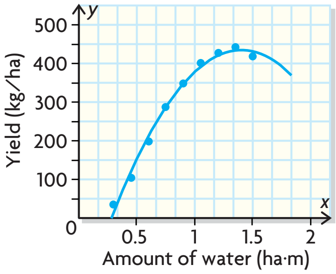
c) $h(t) = -5(t - 3)^2 + 56$

12. **a), b)**



c) $h(t) = -5t^2 + 10t$

13. **a)**

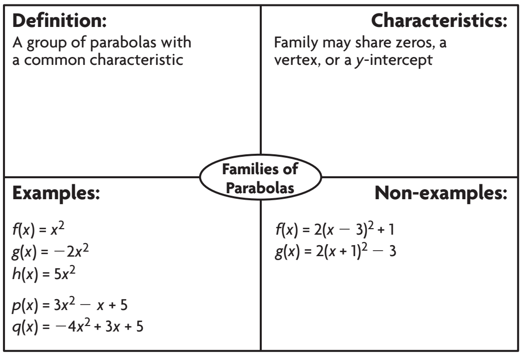


b) approximately $(1.35, 442)$

c) possible function (using $(0.60, 198)$ and vertex): $f(x) = -443(x - 1.35)^2 + 442$

14. $f(x) = -3(x + 3)(x + 1)$ or $f(x) = -3x^2 - 12x - 9$

15. Sample response:



Definition:
A group of parabolas with a common characteristic

Characteristics:
Family may share zeros, a vertex, or a y-intercept

Families of Parabolas

Examples:
$f(x) = x^2$
$g(x) = -2x^2$
$h(x) = 5x^2$
$p(x) = 3x^2 - x + 5$
$q(x) = -4x^2 + 3x + 5$

Non-examples:
$f(x) = 2(x - 3)^2 + 1$
$g(x) = 2(x + 1)^2 - 3$

16. 15.36 m

17. $f(x) = -\frac{1}{4}(x + 3)(x - 1)(x - 5)$

Lesson 3.8, pp. 198–199

1. **a)** $(3, 9)$ $(-2, 4)$ **b)** $(0, 3)$ $(-0.25, 2.875)$ **c)** no solutions

2. **a)** $(4, 3)$ $(6, -5)$ **b)** $(2, 7)$ $(-0.5, -0.5)$ **c)** no solutions

3. one solution

4. **a)** $(1.5, 8)$ $(-7, -43)$

b) about $(1.91, 8.91)$, about $(-1.57, 5.43)$

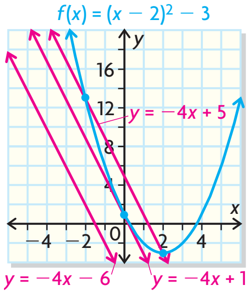
c) no solutions

d) about $(-0.16, 3.2)$, about $(-1.59, -3.95)$

5. 3 and 5 or -1 and 1

6. either \$3.00 or \$4.00

7. **a)** Answers may vary. For example,



b) $y = -4x - 6$, $y = -4x + 1$, $y = -4x + 5$

c) y-intercepts are all less than 1

8. $k = -5$

9. $k > \frac{73}{12}$

10. about 7.20 s

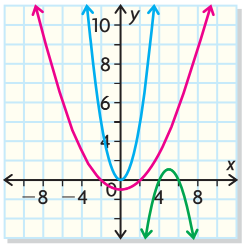
11. 10, −2

12. Yes, at about 0.18 s after kick at (0.18, 4.0)

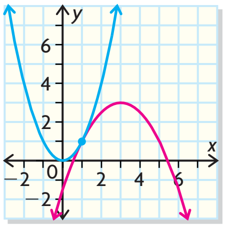
13. Plot graphs of functions and count points of intersection; calculate $b^2 - 4ac$, since there are two points of intersection when $b^2 - 4ac > 0$, one when $b^2 - 4ac = 0$, and none when $b^2 - 4ac < 0$

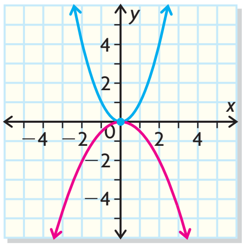
14. $(0, -2), (4, -\frac{14}{3})$

15. Zero points of intersection:

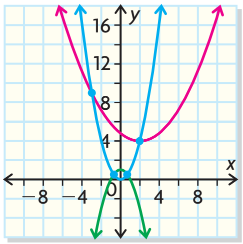


One point of intersection:





Two points of intersection:



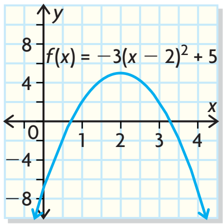
16. $y = 0.5x - 1$

Chapter Review, pp. 202–203

1. **a)** down, vertex (2, 5), $x = 2$

b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 5\}$

c)

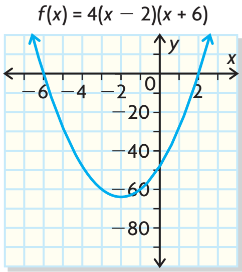


2. **a)** up, zeros 2 and −6

b) vertex (−2, −64)

c) domain $= \{x \in \mathbf{R}\}$; range $= \{y \in \mathbf{R} \mid y \geq -64\}$

d)



3. $x = -1$

4. **a)** Maximum value of 7 at $x = 4$

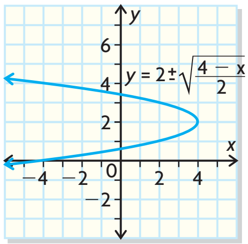
b) Minimum value of −36 at $x = -3$

5. 42 m after about 2.9 s

6. $g(x)$ and $h(x)$ are the two branches of the inverse of $f(x) = x^2$.

7. The inverse of a quadratic function is not a function, because it has two y-values for every x-value. It can be a function only if the domain of the original function has been restricted to a single branch of the parabola.

8. **a)**



b) Domain $= \{x \in \mathbf{R} \mid x \leq 4\}$; Range $= \{y \in \mathbf{R}\}$

c) The inverse relation is not a function; it does not pass the vertical-line test.

9. **a)** $7\sqrt{2}$ **c)** $-4\sqrt{3}$
b) $-20\sqrt{2}$ **d)** $37 - 12\sqrt{7}$

10. $2\sqrt{66}$

11. $(9 + 3\sqrt{5})$ cm

12. $\left(\frac{5}{2}, 0\right), (-3, 0)$

13. **a)** 52 428 **b)** 2124

14. 55.28 m by 144.72 m

15. Yes, because $14t - 5t^2 = 9$ has $b^2 - 4ac = 16 > 0$, so there are two roots. Because parabola opens down and is above t-axis for small positive t, at least one of these roots is positive.

16. $k < -0.5$ or $k > 3.5$

17. 4408 bikes

18. $f(x) = -\frac{5}{3}x^2 + \frac{20}{3}x - \frac{5}{3}$

19. The family of parabolas will all have vertex $(-3, -4)$;
$f(x) = 10(x + 3)^2 - 4$

20. **a)** $f(x) = -\frac{7}{9}x^2 + 15$ **b)** about 8.8 m

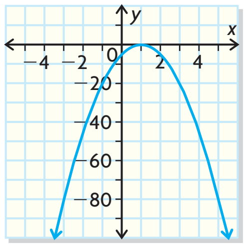
21. $(-5, 19), (1.5, -0.5)$

22. Yes, after 4 s. Height is 15 m.

23. **a)** No, they will not intersect. The discriminant of $f(x) - g(x)$ is -47. There are no real solutions for $f(x) - g(x)$, meaning that $f(x)$ and $g(x)$ do not intersect.
b) Answers will vary. For example, $g(x) = 3x - 5$.

Chapter Self-Test, p. 204

1. **a)** $f(x) = -5(x - 1)^2$, vertex $(1, 0)$
b) zero at $x = 1$, axis of symmetry $x = 1$, opens down
c) domain $= \{x \in \mathbf{R}\}$; range $= \{y \in \mathbf{R} \mid y \leq 0\}$
d)



$f(x) = -5x^2 + 10x - 5$

2. **a)** Maximum value; complete the square.
b) Minimum value; average the zeros.

3. **a)** Vertex form; vertex is visible in equation.
b) Standard form; y-intercept is visible in equation.
c) Factored form; zeros are visible in equation.
d) Vertex form; use x-coordinate of vertex.
e) Vertex form.; use vertex and direction of opening.

4. 360 000 m^2

5. $f^{-1} = 1 \pm \sqrt{\frac{x + 3}{2}}$

6. **a)** $2 - 4\sqrt{2}$
b) $15 - 3\sqrt{10} + 5\sqrt{5} - 5\sqrt{2}$
c) $\sqrt{8}$ can be simplified to $2\sqrt{2}$. This resulted in like radicals that could be combined.

7. $k = -2$ or 2

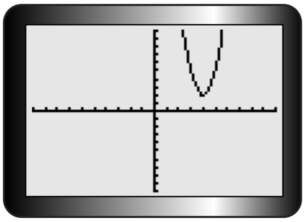
8. Intersects in 2 places, since $2x^2 - 3x + 2 = 6x - 5$ has $b^2 - 4ac > 0$; (1, 1), (3.5, 16)

9. $f(x) = -x^2 + 8x - 13$

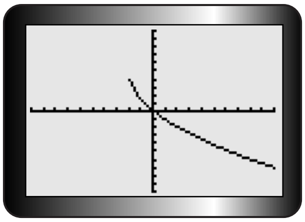
Cumulative Review Chapters 1–3, pp. 206–209

1. (c)
2. (b)
3. (b)
4. (a)
5. (b)
6. (c)
7. (d)
8. (b)
9. (a)
10. (b)
11. (c)
12. (a)
13. (d)
14. (b)
15. (a)
16. (c)
17. (d)
18. (a)
19. (d)
20. (d)
21. (c)
22. (d)
23. (b)
24. (c)
25. (b)
26. (c)
27. (d)
28. (a)
29. (b)
30. (d)
31. (b)
32. (c)

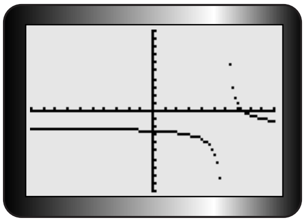
33. **a)** Domain: $\{x \in \mathbf{R}\}$, Range: $\{y \in \mathbf{R} \mid y \geq 2\}$; Parent function: $y = x^2$; Transformations: Vertical stretch by a factor of 3, horizontal translation 4 right, vertical translation 2 up; Graph:



b) Domain: $\{x \in \mathbf{R} \mid x \geq -2\}$, Range: $\{y \in \mathbf{R} \mid y \leq 5\}$; Parent function: $y = \sqrt{x}$; Transformations: Vertical stretch by a factor of 2, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 2 left, vertical translation 5 up; Graph:



c) Domain: $\{x \in \mathbf{R} \mid x \neq 6\}$, Range: $\{y \in \mathbf{R} \mid y \neq -2\}$; Parent function: $y = \frac{1}{x}$; Transformations: Horizontal stretch by a factor of 3, horizontal translation 6 to the right, vertical translation 2 down; Graph:



34. Jill: 3.6 km/h, 8 h 20 min; Sacha: 5 km/h; 6 h 20 min (including time to stop and talk with friend)

35. **a)** 8 or 30 students **b)** about \$700

Chapter 4

Getting Started, p. 212

1. **a)** 49 **c)** $\frac{1}{5}$ **e)** 10 000
b) 32 **d)** 1 **f)** $\frac{1}{8}$

2. **a)** 9 **c)** -16 **e)** -125
b) -27 **d)** 16 **f)** -125

3. $(-5)^{120}$ will result in a positive answer since the exponent is an even number.

4. **a)** 81 **c)** 4096 **e)** 256
b) 5 764 801 **d)** $-1\,000\,000$ **f)** -1

5. **a)** 49 **b)** 24 **c)** 8 **d)** $\frac{1}{3}$

6. **a)** $\frac{55}{24}$ **c)** $\frac{21}{16}$ **e)** $\frac{62}{75}$
b) $-\frac{25}{24}$ **d)** $-\frac{3}{10}$ **f)** $-\frac{39}{16}$

7. **a)** a^7 **b)** b^4 **c)** c^{12} **d)** d^{10}

8. **a)** $x = 2$ **b)** $m = \frac{8}{3}$ **c)** $a = 3$ **d)** $r = 8$

9. **a)** $V \doteq 94.25\text{ cm}^3$ **b)** $V \doteq 65.45\text{ cm}^3$

10. **a)** first differences are all -5; linear function
b) first differences are 1, 2, 3, 4, 5; second differences are all 1; quadratic function

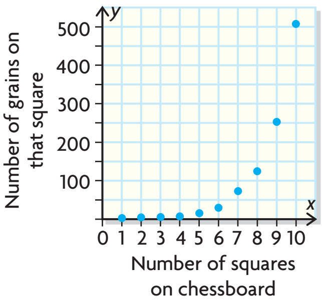
Lesson 4.1, p. 216

1. **a)** Both graphs decrease rapidly at the beginning, then continue to decrease less rapidly before levelling off.
b) 85 °C
c) 20 °C

2. **a)**

Number of Squares on the Chessboard	Number of Grains on that Square	First Differences
1	1	1
2	2	2
3	4	4
4	8	8
5	16	16
6	32	32
7	64	64
8	128	128
9	256	256
10	512	

b)



c) They are similar in that both first difference tables show a multiplicative pattern. They are different in that in the first case, the values decrease sharply and then level off while in the second case, the values start level and then increase sharply.

Lesson 4.2, pp. 221–223

1. **a)** $\frac{1}{5^4}$ **c)** 2^4 **e)** $\left(\frac{11}{3}\right)^1$
b) $(-10)^3$ **d)** $-\left(\frac{5}{6}\right)^3$ **f)** $\frac{8^1}{7^2}$

2. **a)** $(-10)^0 = 1$ **c)** 2^{13} **e)** $-\frac{1}{9^4}$
b) $\frac{1}{6^2}$ **d)** $\frac{1}{11^8}$ **f)** $\frac{1}{7^{12}}$

3. $2^{-5} = \frac{1}{2^5}$ is less than $\left(\frac{1}{2}\right)^{-5} = 2^5$

4. **a)** $2^4 = 16$ **c)** $5^{-2} = \frac{1}{25}$ **e)** $4^3 = 64$
b) $(-8)^0 = 1$ **d)** $3^{-2} = \frac{1}{9}$ **f)** $7^{-2} = \frac{1}{49}$

5. **a)** $3^1 = 3$ **c)** $12^0 = 1$ **e)** $3^{-2} = \frac{1}{9}$
b) $9^0 = 1$ **d)** $5^0 = 1$ **f)** $9^1 = 9$

6. **a)** $10^3 = 1000$ **c)** $6^{-1} = \frac{1}{6}$ **e)** $2^{-3} = \frac{1}{8}$
b) $8^{-1} = \frac{1}{8}$ **d)** $4^2 = 16$ **f)** $13^1 = 13$

7. **a)** $-\frac{3}{16}$ **c)** 1 **e)** $\frac{1}{1000}$
b) $\frac{1}{2}$ **d)** 9 **f)** $-\frac{1}{12}$

8. **a)** $\frac{1}{400}$ **c)** $-\frac{1}{3}$ **e)** $\frac{1}{16}$
b) 2 **d)** $\frac{1}{9}$ **f)** 125

9. **a)** $-\frac{1}{64}$ **c)** $-\frac{1}{125}$ **e)** $-\frac{1}{216}$
b) $\frac{1}{16}$ **d)** $-\frac{1}{25}$ **f)** $-\frac{1}{36}$

10. 5^{-2}, 10^{-1}, 3^{-2}, 2^{-3}, 4^{-1}, $(0.1)^{-1}$; If the numerators of the numbers are all the same (1), then the larger the denominator, the smaller the number.

11. **a)** $\frac{1}{36}$ **b)** $-\frac{9}{2}$ **c)** $-\frac{2}{3}$ **d)** $\frac{1}{576}$

12. **a)** Erik: $3^{-1} \neq -\frac{1}{3}$ (negative exponents do not make numbers negative)
Vinn: $3 = 3^1$ and he did not add the exponents correctly.

b) Correct solution:

$$\boxed{\begin{aligned} &3^{-2} \times 3 \\ &= \frac{1}{3^2} \times 3 \\ &= \frac{1}{9} \times 3 \\ &= \frac{3}{9} \\ &= \frac{1}{3} \end{aligned}}$$