Name ____________________

Underline any words you don't know. Answer the questions on the last page once you finish reading.

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Why Do Math?

Dear Meg,

As you probably anticipated, I was very glad to hear you're thinking of studying mathematics, not least because it means all those weeks you spent reading and rereading *A Wrinkle in Time* a few summers ago were not wasted, nor all the hours I spent explaining tesseracts and higher dimensions to you. Rather than deal with your questions in the order you asked them, let me take the most practical one first: does anyone besides me actually make a living doing math?

The answer is different from what most people think. My home university did a survey of its alumni a few years back, and they discovered that out of all the various degree subjects, the one that led to the highest average income was . . . mathematics. Mind you, that was before they opened the new medical school, but it demolishes one myth: that mathematics can't lead to a well-paying job.

The truth is that we encounter mathematicians everywhere, every day, but we hardly ever know it. Past students of mine have managed breweries, started their own electronics companies, designed automobiles, written software for computers, and traded futures on the stock market. It simply doesn't occur to us that our bank manager might have a degree in math, or that the people who invent or manufacture DVDs and MP3 players employ large numbers of mathematicians, or that the technology that transmits those stunning pictures of the moons of Jupiter relies heavily on math. We know that our doctor has a medical degree, and our lawyer has a law degree, because those are specific, well-defined professions that require equally specific training. But you don't find brass plaques on buildings advertising a licensed mathematician within, who, for a large fee, will solve any math problems that you need help with.

Our society consumes an awful lot of math, but it all happens behind the scenes. The reason is straightforward: that's where it belongs. When you drive a car, you don't want to have to worry about all the complicated mechanical things that make it work; you want to get in and drive away. Sure, it helps you to be a better driver if you're aware of the basics of car mechanics, but even that is not essential. It's the same with math. You want your car navigation system to give you directions *without* your having to do the math yourself. You want your

phone to work without your having to understand signal processing and error-correcting codes.

Some of us, however, need to know how to do the math, or none of these wonders could function. It would be great if the rest of us were aware of just how strongly we rely on mathematics in our daily lives; the problem with putting math so far behind the scenes is that many people have no idea it's there at all.

I sometimes think that the best way to change the public attitude to math would be to stick a red label on everything that uses mathematics. "Math inside." There would be a label on every computer, of course, and I suppose if we were to take the idea literally, we ought to slap one on every math teacher. But we should also place a red math sticker on every airline ticket, every telephone, every car, every airplane, every traffic light, every vegetable . . .

Vegetable?

Yes. The days when farmers simply planted what their fathers had planted, and their fathers before them, are long gone. Virtually any plant you can buy is the outcome of a long and complicated commercial breeding program. The whole topic of "experimental design," in the mathematical sense, was invented in the early 1900s to provide a systematic way to assess new breeds of plants, not to mention the newer methods of genetic modification.

Wait. Isn't this biology?

Biology, sure. But math, too. Genetics was one of the first parts of biology to go mathematical. The Human Genome Project succeeded because of a lot of clever work by biologists, but a vital feature of the entire project was the development of powerful mathematical methods to analyze the experimental results and reconstruct accurate genetic sequences from very fragmentary data.

So, vegetables get a red sticker. Just about everything there is gets a red sticker.

You go to movies? Do you like the special effects? *Star Wars*, *Lord of the Rings*? Mathematics. The first full-length computer-animated movie, *Toy Story*, led to the publication of about twenty research papers on math. "Computer graphics" isn't just computers making pictures; it's the mathematical methods that make those pictures look realistic. To do that, you need three-dimensional geometry, the mathematics of light, "in-betweening" to interpolate a smooth series of images between a start and a finish, and lots more. "Interpolation" is a mathematical idea. Computers are clever engineering, but they don't do anything useful without a lot of clever math. Red sticker.

And then, of course, there's the Internet. If anything makes use of math, it's the Internet. The main search engine at the moment, Google, was founded on a mathematical method for working out which web pages are most likely to contain the information required by a

user. It's based on matrix algebra, probability theory, and the combinatorics of networks.

But the math of the Internet is much more fundamental than that. The telephone network relies on math. It's not like the old days when switchboard operators literally connected calls by plugging phone lines in by hand. Today those lines have to carry millions of messages at once. There are so many of us, all wanting to talk to our friends or send faxes or access the Internet, that we have to share the phone lines and the suboceanic cables and the satellite relays, or the network wouldn't be able to carry all that traffic. So each conversation is broken up into thousands upon thousands of short pieces, and only one piece in a hundred is actually transmitted. At the other end, the missing ninety-nine pieces are restored by filling in the gaps as smoothly as possible (it works because the samples, though short, are very frequent, so that the sounds you make when you speak change much more slowly than the interval between samples). Oh, and the entire signal is coded so that any transmission errors can not only be detected, they can be put right at the receiving end.

Modern communications systems simply would not work without a huge quantity of math. Coding theory, Fourier analysis, signal processing . . .

Anyway, you go onto the Internet to get a plane ticket, book your flight and turn up at the airport, hop on the plane, and away you go. The plane flies because the engineers who designed it used the mathematics of fluid flow, aerodynamics, to make sure it would stay up. It navigates using a global positioning system (GPS), a system of satellites whose signals, analyzed mathematically, can tell you where you are to within a few feet. The flights have to be scheduled so that each plane is in the right place when it is next needed, rather than somewhere on the far side of the globe, and that, again, requires yet other areas of math.

And so, Meg, my dear, it goes. You asked me whether mathematicians are all shut away in universities, or whether some of them do work related to real life. Your entire life bobs like a small boat on a vast ocean of mathematics.

But hardly anyone notices. Hiding the math away makes us all feel comfortable, but it devalues mathematics. That is a shame. It makes people think that math isn't useful, that it doesn't matter, that it's just intellectual games without any true significance. Which is why I'd like to see those red stickers. In fact, the best reason not to use them is that most of the planet would be covered with them.

Your third question was the most important, and the saddest. You asked me whether you would have to give up your sense of beauty to study mathematics, whether everything would become just numbers and equations to you, laws and formulas. Rest assured, Meg, I don't blame you for asking this, since it's unfortunately a very com-

mon idea, but it couldn't be more wrong. It's exactly the opposite of the truth.

What math does for me is this: It makes me aware of the world I inhabit in an entirely new way. It opens my eyes to nature's laws and patterns. It offers an entirely new experience of beauty.

When I see a rainbow, for instance, I don't just see a bright, multicolored arc across the sky. I don't just see the effect of raindrops on sunlight, splitting the white light from the sun into its constituent colors. I still find rainbows beautiful and inspiring, but I appreciate that there's more to a rainbow than mere refraction of light. The colors are, so to speak, a red (and blue and green) herring. What require explanation are the shape and the brightness. Why is a rainbow a circular arc? Why is the light from the rainbow so bright?

You may not have thought about those questions. You know that a rainbow appears when sunlight is refracted by tiny droplets of water, with each color of light being diverted through a slightly different angle and bouncing back from the raindrops to meet the observing eye. But if that's all there is to a rainbow, why don't the billions of differently colored light rays from billions of raindrops just overlap and smear out?

The answer lies in the geometry of the rainbow. When the light bounces around inside a raindrop, the spherical shape of the drop causes the light to emerge with a very strong focus along a particular direction.

Each drop in effect emits a bright cone of light, or, rather, each color of light forms its own cone, and the angle of the cone is slightly different for each color. When we look at a rainbow, our eyes detect only the cones that come from raindrops lying in particular directions, and for each color, those directions form a circle in the sky. So we see lots of concentric circles, one for each color.

The rainbow that you see and the rainbow that I see are created by different raindrops. Our eyes are in different places, so we detect different cones, produced by different drops.

Rainbows are personal.

Some people think that this kind of understanding "spoils" the emotional experience. I think this is rubbish. It demonstrates a depressing sort of aesthetic complacency. People who make such statements often like to pretend they are poetic types, wide open to the world's wonders, but in fact they suffer from a serious lack of curiosity: they refuse to believe the world is more wonderful than their own limited imaginations. Nature is always deeper, richer, and more interesting than you thought, and mathematics gives you a very powerful way to appreciate this. The ability to *understand* is one of the most important differences between human beings and other animals, and we should value it. Lots of animals emote, but as far as we know, only humans think rationally. I'd say that my understanding of the geometry of the rain-

bow adds a new dimension to its beauty. It doesn't take anything away from the emotional experience.

The rainbow is just one example. I also look at animals differently, because I'm aware of the mathematical patterns that underlie their movements. When I look at a crystal, I am aware of the beauties of its atomic lattice as well as the charm of its colors. I see mathematics in waves and sand dunes, in the rising and the setting of the sun, in raindrops splashing in a puddle, even in birds sitting on telephone cables. And I'm aware—dimly, as if looking out over a foggy ocean—of the infinity of things we *don't* know about these everyday wonders.

Then there's the inner beauty of mathematics, which should not be underrated. Math done "for its own sake" can be exquisitely beautiful and elegant. Not the "sums" we all do at school; as individuals those are mostly ugly and formless, although the general principles that govern them have their own kind of beauty. It's the ideas, the generalities, the sudden flashes of insight, the realization that trying to trisect an angle with straightedge and compass is like trying to prove that 3 is an even number, that it makes perfect sense that you can't construct a regular seven-sided polygon but you can construct one with seventeen sides, that there is no way to untie an overhand knot, and why some infinities are bigger than others whereas some that ought to be bigger are actually equal, that the *only* square number (other than 1, if you want to be picky) that is the sum of consecutive squares, 1 + 4 + 9 + ···, is the number 4900.

You, Meg, have the potential to become an accomplished mathematician. You have a logical mind and also an inquiring one. You're not convinced by vague arguments; you want to see the details and check them out for yourself. You don't just want to know how to make things work, you want to know *why* they work. And your letter made me hope that you'll come to see mathematics as I see it, as something fascinating and beautiful, a way of seeing the world that is like no other.

I hope this sets the scene for you.

Yours,
Ian

1. On p. 6, Stewart writes, "Your entire life bobs like a small boat on a vast ocean of mathematics." After reading this letter, does this seem true to you? Why or why not?

2. What is one example that Stewart gives of something that is mathematical that surprised you?

3. On p. 8, Stewart writes, "Nature is always deeper, richer, and more interesting than you thought, and mathematics gives you a very powerful way to appreciate this." Has this been your experience? Explain.