

# Mathematical Notation

Mathematics is a language; I imagine you wouldn't hand in an essay with no punctuation, or with words and sentence fragments scattered across a page...well, how you communicate in math shouldn't be any different. I know you're super busy with lots of work, but try to take the time to at least skim-read this sometime during the term (preferably before the 1<sup>st</sup> test, at least ☺). Better yet, you should probably refresh your memory of this advice before each test, as some of it refers to theorems and concepts you'll learn later on in the course, and I'll add more notation advice as we go along, specific to some of the material.

This is a compilation of advice that was given out to first year students at University of Toronto, as well as UOIT. This is not a complete list of everything you need to know, but I think it's a good starting point. Hope this helps!

## *THE BASICS*

### **MAKE IT CLEAR WHAT YOU'RE DOING**

If you differentiate a function  $y=x^2$ , then writing

$$\begin{aligned}y &= x^2 \\ &= 2x\end{aligned}$$

is horribly **WRONG**, even if we can guess what you're doing.

You must write:

$$\begin{aligned}y &= x^2 \\ \Rightarrow y' &= 2x\end{aligned}$$

### **EQUALITY VERSUS IMPLICATION NOTATION**

Going back to the last example, you'll note that if two things are equal, we use "=". Do not confuse this with the implication sign, " $\Rightarrow$ ", which means "if...then..."

### **SHOW YOUR WORK**

Unless you're performing a simple step, you should show your work. First of all, if you don't, you're losing out on part marks if your answer is wrong (close doesn't count when you don't show your work!). Second, it's difficult to follow your solution if important steps are missing.

### **USE WORDS TOO**

If you have time, then it is often a good idea to explain what you're doing if a computation is long/complicated. For example, when you do mean value theorem questions, you should explain in words what you're doing at each step. If you're trying to prove something false by contradiction, then say so.

### **ROUGH WORK**

If you do rough work calculations that aren't part of the formal solution, then these should be written to the side of the page, and clearly labelled as "Aside" or "Rough Work". As an example, if you're factoring something and it requires the quadratic formula, then that should be written as part of the main solution. If, on the other hand, you're factoring something simple by writing down all the possible factors and trying to determine which ones work, then that's rough work.

## SCATTERED MATH

You should not have math calculations scattered all over the place, with no explanation of which calculation implies the next, or why you're doing what you're doing.

e.g. If you're asked to evaluate  $f'(1)$  for  $f(x)=3x^2$ , then

$$\begin{aligned}f(x) &= 3x^2 \\ \Rightarrow f'(x) &= 6x \\ \text{So } f'(1) &= 6\end{aligned}$$

is fine, but writing

$$\begin{aligned}6x \\ 6\end{aligned}$$

with no other explanations as part of your solution is really bad. As another example, the quadratic formula is an EQUATION, so don't just write it's left-hand-side...show that the quantity you're solving for is given by this formula. i.e. include the right-hand-side as well.

## CHECKING YOUR WORK

IF you finish your work early (e.g. quiz, midterm, final), then there's plenty you can do to check your work before you leave. For antiderivative/integration questions, differentiate to make sure you found the right antiderivative...if it is indeed correct, then differentiating your answer should give you the function you started with. For first principles questions, you may be able to check your answer by using one of your known rules, e.g. power rule, once you're done. Use second derivative test (if not too tedious) to verify results of 1<sup>st</sup> derivative test. Use Fundamental Theorem of Calculus (part 2) to verify results of doing Riemann sums, etc.

## SIMPLIFYING YOUR WORK

At the very least, you must collect like terms

e.g. a final answer of  $3y5z+9x+2x$ ,

will not receive full marks...you should write  $15yz+11x$  for the above example.

## STATE FORMULAS

If you're using a formula such as the mean value theorem formula, or the linear approximation formula, or the definition of the derivative, etc, start the question by writing the formula down BEFORE plugging things into the formula. First, it makes it clear what you're doing, and if you make a mistake in the formula, we'll at least know what you're trying to do and you might get part marks. In some instances, you may even get marks for doing this (i.e. you will lose marks if the formula is not stated). In fact, you often get part marks just for writing the relevant formula down.

## ANSWER WHAT THE QUESTION IS ASKING

Make sure you've answered the question that is being asked...yes, this may seem obvious, but as an example of what I mean, consider the following:

e.g. if you're asked for the tangent line to a curve at (1,1), then make sure that the answer you give is for the equation of the line at that point, not just at general (x,y). Also, make sure you evaluate the slope at the appropriate point.

e.g. If asked for the max/min of a function, state what the max/min is (y value), not just where it occurs. Make sure your final answer is clearly indicated. Do NOT just find all the  $f(x)$  values and stop. E.g.  $f(1)=2$ ,  $f(3)=7$ ,  $f(5)=1$ . Then your final answer should be that the absolute min is 1 and occurs at  $x=5$ , and the absolute max is 7 and occurs at  $x=3$ .

e.g. If doing word problems and asked for TWO numbers with certain sum and max product, state BOTH numbers in the final answer, etc.

## **REFER TO THEOREMS**

For the main theorems you've learned that have names, (e.g. Squeeze Theorem, Intermediate Value Theorem, Mean Value Theorem, Rolle's Theorem, and any others I may have missed), state when you're using them. If you come to some conclusion that is the result of using one of these theorems, say so. Also, you would have to make sure that the conditions of the theorem are satisfied. E.g. If you plan to use Mean Value Theorem, then you better make sure your function is continuous on the given interval  $[a,b]$  and differentiable on  $(a,b)$ , and state that it is. If you're using Intermediate Value Theorem, then make sure that your function is continuous, and state that it is.

## **KNOWING THE RIGHT ANSWER**

For "first principles" questions (either derivatives by first principles or integration using Riemann sums), the answer is usually very easy to check just by differentiating or integrating as usual. In some cases, if you get stuck using the first principles approach, but you know what the answer is supposed to be using a different method, it is a good idea to write it down (e.g. so you might get part marks for final answer and can continue to do the rest of the question if, for example, you're also asked for the domain of the derivative function). HOWEVER, do NOT just jump to this answer as if it somehow follows from what you've done...if it doesn't follow because you got stuck in the first principles approach, it'll look like you guessed, or copied, or are confused as to what you're doing. Instead, make a note clearly stating that you're stuck, but that you know from e.g. quotient rule, etc. that the answer is supposed to be \_\_\_\_\_ and you can then use this to answer the rest of the question, e.g. domain of derivative. P.S. As a reminder, if you have no trouble finishing the question using first principles, doing a check using usual differentiation or integration rules is great, but make sure it is clearly labelled as a "check" (again, to avoid confusion as to what you're doing).