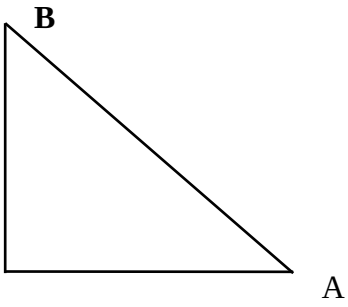
Practice Problems for Related Rates - AP Calculus BC

1. A circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min. At what rate is the area of the plate increasing when the radius is 50 cm?

2. Two commercial jets at 40,000 ft are flying at 520 mi/hr along straight line courses that cross at right angles. How fast is the distance between them closing when the airplane A is 5 mi from the intersection point and airplane B is 12 mi from the intersection point? How fast is the distance closing at any time *t*?



3. A spherical tootsie roll pop that you are enjoying is giving up volume at a steady rate of 0.08 ml/min. How fast will the radius of the pop be decreasing when the tootsie pop is 20mm across?

4. The mechanics at Lincoln Automotive are reboring a 6-in. deep cylinder to fit a new piston. The machine that they are using increases the cylinder's radius one-thousandth of an inch every 3 minutes. How rapidly is the volume of the cylinder increasing when the bore (diameter) is 3.80 inches?

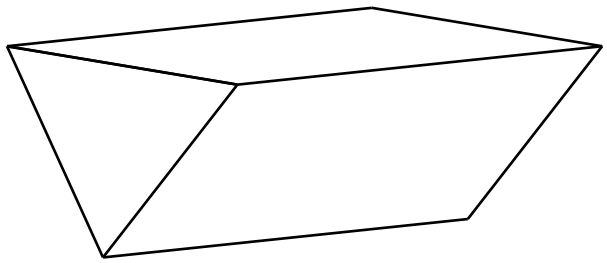
5. Water is flowing out at the rate of 50 cubic meters/min from a shallow conical reservoir (vertex downward) of base radius 45 m and height 6m.
    a) How fast is the water level falling when the water is 5m deep?
    b) How fast is the radius of the water's surface changing then?

6. [1977 AB 6] A rectangle has a constant area of 200 square meters and its length *L* is increasing at the rate of 4 meters per second.
    a) Find the width *W* at the instant the width is decreasing at the rate of 0.5 meters per second.
    b) At what rate is the diagonal *D* of the rectangle changing at the instant when the width *W* is 10 meters?

7. [1982 AB 4] A ladder 15 feet long is leaning against a building so that end *X* is on level ground and end *Y* is on the wall. *X* is moved away from the building at the constant rate of 0.5 foot per second.
    a) Find the rate in feet per second at which the height of the ladder above the ground is changing when *X* is 9 feet from the building.
    b) find the rate of change in square feet per second of the area of the triangle formed by the building, the ground, and the ladder when *X* is 9 feet from the building.

8. [1984 AB5, BC2] A balloon is in the shape of a cylinder and has hemispherical ends of the same radius as that of the cylinder. (i.e., it looks like a medicine capsule). The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant that the radius of the cylinder is 3 cm, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.
    a) At this instant, what is the height of the cylinder?
    b) At this instant, how fast is the height of the cylinder changing?

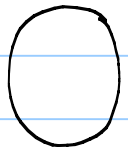
9. [1987 AB5] The trough has an inverted isosceles triangle as a base. This isosceles triangle has a base of 2 feet and a height of 3 feet. The trough is 5 feet long. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time *t*, let *H* be the depth and *V* be the volume of water in the trough.
    a) Find the volume of water in the trough when it is full.
    b) What is the rate of change in *H* at the instant when the trough is ¼ full by volume?
    c) What is the rate of change in the area of the surface of the water, at the instant when the trough is ¼ full by volume?



10. [1990 AB 4] The radius *r* of a sphere is increasing at the constant rate of 0.04 centimeters per second.
    a) At the time when the radius of the sphere is 10 cm., what is the rate of increase of its volume?
    b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?
    c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

Related Rates Worksheet Solutions

Note Title 05/02/2012

1) 

$\frac{dr}{dt} = 0.01$ cm/min

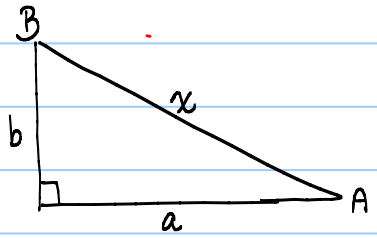
find $\frac{dA}{dt}$ when $r = 50$ cm

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(50)\left(\frac{1}{100}\right)$

$\boxed{\frac{dA}{dt} = \pi \text{ cm}^2/\text{min}}$

2) 

Find $\frac{dx}{dt}$ when $\frac{da}{dt} = -520$ mi/hr

$\frac{db}{dt} = -520$ miles/hr and $a = 5$ miles

$b = 12$ miles

$x = 13$ miles

$x^2 = a^2 + b^2$

$2x\frac{dx}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$

$2(13)\frac{dx}{dt} = 2(5)(-520) + 2(12)(-520)$

$26\frac{dx}{dt} = -17680$; $\boxed{\frac{dx}{dt} = -680 \text{ miles/hr}}$

$\frac{dx}{dt} = \frac{2a(-520) + 2b(-520)}{2x}$

$\boxed{\frac{dx}{dt} = \frac{-520(a+b)}{\sqrt{a^2+b^2}} \text{ mi/hr}}$

3) 

$\frac{dV}{dt} = -0.08$ mℓ/min $= 80$ mm³/min

find $\frac{dr}{dt}$ when $r = 10$ mm.

$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

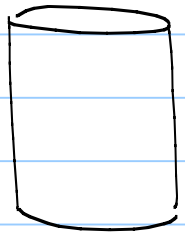
$-80 = 4\pi(10)^2\frac{dr}{dt}$

$\frac{-80}{400\pi} = \frac{dr}{dt}$

$\boxed{-0.2/\pi \text{ mm/min}}$

-0.064 mm/min

4)



$\frac{dr}{dt} = \frac{1}{1000}$ / 3 min or $\quad h = 6$ inches

$= \frac{1}{3000}$ in/min

find $\frac{dV}{dt}$ when $r = \frac{3.8}{2} = 1.9$ inches

$V = \pi r^2 h$

$V = 6\pi r^2$

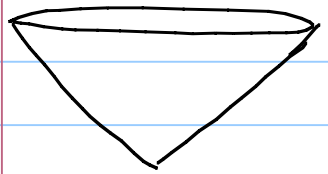
$\frac{dV}{dt} = 12\pi r \frac{dr}{dt}$

$\frac{dV}{dt} = 12\pi(1.9)\left(\frac{1}{3000}\right)$

$= \frac{22.8\pi}{3000}$ in^3/min

$\boxed{\frac{dV}{dt} = 0.024 \text{ in}^3/\text{min}}$

5a)



$\frac{dV}{dt} = -50$ m^3/min

find $\frac{dh}{dt}$ when $h = 5$m

$\frac{r}{h} = \frac{45}{6}$

$6r = 45h$

$2r = 15h$

$r = \frac{15}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$V = \frac{225}{12}\pi h^3$

$\frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi(5)^2 \frac{dh}{dt}$

$\frac{-8}{225\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -0.0113 \text{ m/min}}$

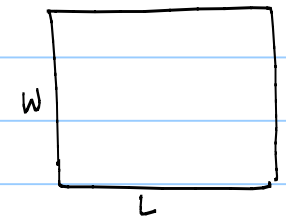
b) $r = \frac{15}{2}h$

$\frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt}$

$= \frac{15}{2}\left(\frac{-8}{225\pi}\right)$

$\frac{dr}{dt} = \frac{-4}{15\pi}$

$\boxed{\frac{dr}{dt} = -0.085 \text{ m/min}}$

6a) 

$A = 200\ m^2$ find w when $\frac{dw}{dt} = -0.5\ m/s$

$\frac{dL}{dt} = 4\ m/s.$

$A = LW$

$200 = LW$

$0 = L\frac{dw}{dt} + \frac{dL}{dt}W$

$0 = \left(\frac{200}{w}\right)\left(-\frac{1}{2}\right) + (4)(w)$

$0 = \frac{-100}{w} + 4w$

$L \cdot W = 200$

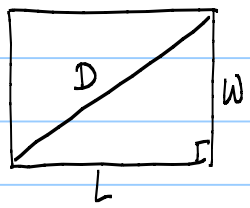
$L = \frac{200}{w}$

$4w = \frac{100}{w}$

$4w^2 = 100$

$w^2 = 25$

$\boxed{w = 5\ m}$

b) 

$\frac{dL}{dt} = 4\ m/s$ find $\frac{dD}{dt}$ when $\begin{cases} w = 10\ m \\ L = 20\ m \\ D = 10\sqrt{5}\ m \end{cases}$

$D^2 = L^2 + w^2$

$2D\frac{dD}{dt} = 2L\frac{dL}{dt} + 2w\frac{dw}{dt}$

$2(10\sqrt{5})\frac{dD}{dt} = 2(20)(4) + 2(10)(-2)$

$20\sqrt{5}\frac{dD}{dt} = 120$

$\boxed{\frac{dD}{dt} = \frac{6}{\sqrt{5}} \doteq 2.683\ m/s}$

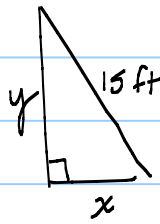
$200 = LW$

$0 = L\frac{dw}{dt} + W\frac{dL}{dt}$

$0 = (20)\frac{dw}{dt} + (10)(4)$

$-\frac{40}{20} = \frac{dw}{dt}$; $\frac{dw}{dt} = -2\ m/s$

7a)



$\frac{dx}{dt} = 0.5$ ft/sec

find $\frac{dy}{dt}$ when $\begin{cases} x = 9 \text{ ft} \\ y = 12 \text{ ft} \end{cases}$

$15^2 = x^2 + y^2$

$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$0 = 2(9)(\frac{1}{2}) + 2(12)\frac{dy}{dt}$

$0 = 9 + 24\frac{dy}{dt}$

$\frac{dy}{dt} = -\frac{9}{24}$ ft/sec

$\boxed{\frac{dy}{dt} = \frac{-3}{8} \text{ or } 0.375 \text{ ft/sec}}$

b) $A = \frac{1}{2}(x)(y)$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}$

$\frac{dA}{dt} = \frac{1}{2}(\frac{1}{2})(12) + \frac{1}{2}(9)(\frac{-3}{8})$

$\boxed{\frac{dA}{dt} = \left(3 - \frac{27}{16}\right) = \frac{21}{16} \text{ ft}^2/\text{sec} \text{ or } 1.3125 \text{ ft}^2/\text{sec}}$

8)

$\frac{dV}{dt} = 261\pi \text{ cm}^3/\text{min}$

When $r = 3$ cm, $V = 144\pi$ cm^3 and $\frac{dr}{dt} = 2$ cm/min

find a) h b) $\frac{dh}{dt}$

a) $V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$144\pi = \pi(3)^2 h + \frac{4}{3}\pi(3)^3$

$144\pi = 9\pi h + 36\pi$

$109\pi = 9\pi h$

$\boxed{12 \text{ cm} = h}$

b) $V = \pi r^2 h + \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt} + 4\pi r^2 \frac{dr}{dt}$$

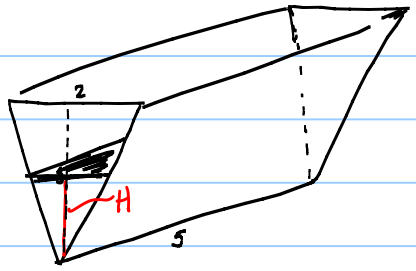
$$261\pi = 2\pi(3)(2)(12) + \pi(3)^2 \frac{dh}{dt} + 4\pi(3)^2(2)$$

$$261\pi = 144\pi + 9\pi \frac{dh}{dt} + 72\pi$$

$$261\pi = 216\pi + 9\pi \frac{dh}{dt} \quad \rightarrow \quad 45\pi = 9\pi \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = 5 \text{ cm/min}}$$

9)



$\frac{dV}{dt} = -2 \text{ ft}^3/\text{min}$

find $\frac{dH}{dt}$ when $V = \frac{1}{4}(15) = 3.75$

a) $V = \frac{1}{2}(b)(h)(\text{length})$

$V = \frac{1}{2}(2)(3)(5)$

$\boxed{V = 15 \text{ ft}^3}$

b) $V = \frac{1}{2} bH(5)$

$V = \frac{5}{2} bH$

$V = \frac{5}{2}\left(\frac{2}{3}H\right)(H)$

$V = \frac{5}{3}H^2$

$\frac{2}{3} = \frac{b}{H}$

$2H = 3b$

$\frac{2}{3}H = b$

$3.75 = \frac{5}{3}H^2$

$2.25 = H^2$

$H = 1.5$

$\frac{dV}{dt} = \frac{10}{3} H \frac{dH}{dt}$

$-2 = \frac{10}{3}(1.5)\frac{dH}{dt}$

$-2 = 5 \frac{dH}{dt}$

$\boxed{\frac{dH}{dt} = \frac{-2}{5} = -0.4 \text{ ft/min}}$

c) $A = b(5)$

$\frac{dA}{dt} = 5\frac{db}{dt}$; $3b = 2H$

$3\frac{db}{dt} = 2\frac{dH}{dt}$

$3\frac{db}{dt} = 2\left(\frac{-2}{5}\right)$

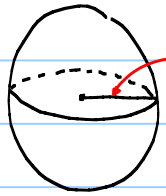
$\frac{db}{dt} = \frac{-4}{15}$

$\frac{dA}{dt} = 5\left(\frac{-4}{15}\right)$

$\boxed{\frac{dA}{dt} = \frac{-4}{3}}$

$\boxed{\frac{dA}{dt} = -1.\overline{3} \text{ ft}^2/\text{min}}$

10a)



$\frac{dr}{dt} = 0.04 \text{ cm/sec}$

find $\frac{dV}{dt}$ when $r = 10 \text{ cm}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(10)^2(0.04)$

$\boxed{\frac{dV}{dt} = 16\pi \text{ cm}^3/\text{sec}}$

b) find $\frac{dA}{dt}$ when $V = 36\pi$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3$

$\boxed{r = 3}$

$\frac{dA}{dt} = 2\pi(3)(0.04)$

$\boxed{\frac{dA}{dt} = \frac{6}{25}\pi \text{ cm}^2/\text{cm} = 0.754 \text{ cm}^2/\text{cm}}$

c) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

When $\frac{dV}{dt} = \frac{dr}{dt}$

$1 = 4\pi r^2$

$\frac{1}{4\pi} = r^2$

$\boxed{\frac{1}{2\sqrt{\pi}} = r}$

$\boxed{r = 0.282 \text{ cm}}$