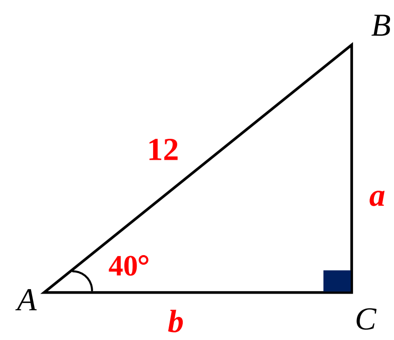
Section 2.3 – Solving Right Triangle Trigonometry

Example

In the right triangle *ABC*, *A* = 40° and c = 12 cm. Find *a*, *b*, and *B*.

<u>*Solution*</u>



$$\sin 40^\circ = \frac{a}{c} = \frac{a}{12}$$

$$a = 12\sin 40^\circ$$

$$= 7.7cm$$

$$\cos 40^\circ = \frac{b}{c}$$

$$= \frac{b}{12}$$

$$b = 12\cos 40^\circ$$

$$= 9.2cm$$

$B = 90° - A$

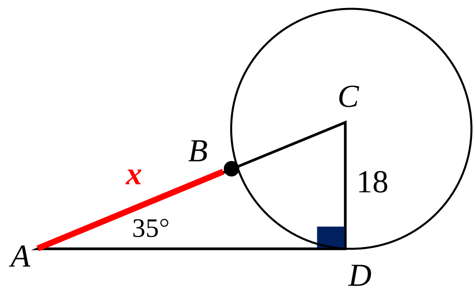
$= 90° - 40°$

$\underline{\approx 50^\circ|}$

Example

A circle has its center at *C* and a radius of 18 inches. If triangle *ADC* is a right triangle and *A* = 35°. Find x, the distance from *A* to *B*.

<u>*Solution*</u>



$$\sin 35^\circ = \frac{18}{x+18}$$

$$(x+18)\sin 35^\circ = 18$$

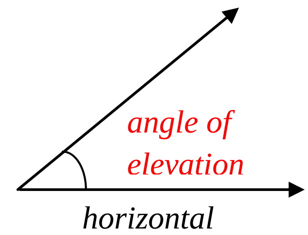
$$x+18 = \frac{18}{\sin 35^\circ}$$

$$x = \frac{18}{\sin 35^\circ} - 18$$

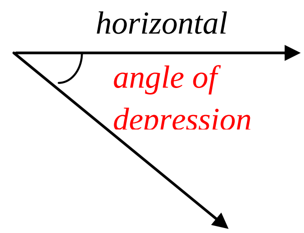
$$= 13\ in$$

Definitions

An angle measured from the horizontal up is called an ***angle of elevation***.



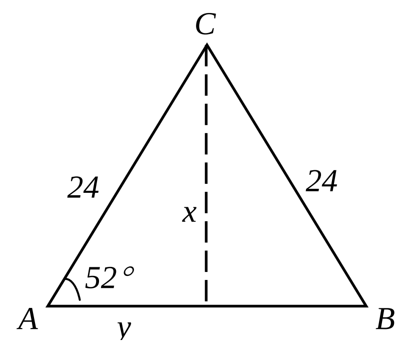
An angle measured from the horizontal down is called an ***angle of depression***.



Example

The two equal sides of an isosceles triangle are each 24 cm. If each of the two equal angles measures 52°, find the length of the base and the altitude.

Solution



$$\sin 52° = \frac{x}{24}$$

$$x = 24\sin 52°$$

$$x = 19\ cm$$

$$\cos 52° = \frac{y}{24}$$

$$y = 24\cos 52°$$

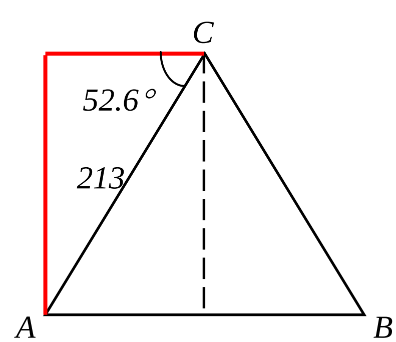
$$y = 15\ cm$$

$$\Rightarrow \boxed{AB = 2y = 30\ cm}$$

Example

A man climbs 213 meters up the side of a pyramid. Find that the angle of depression to his starting point is 52.6°. How high off of the ground is he?

Solution



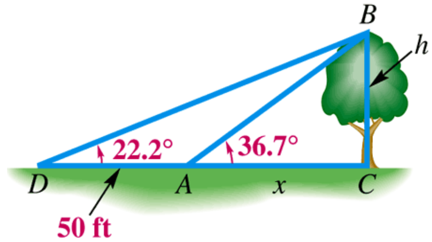
$$\sin 52.6° = \frac{h}{213}$$

$$h = 213\sin 52.6°$$

$$\underline{h = 169\ m}$$

Example

From a given point on the ground, the angle of elevation to the top of a tree is 36.7°. From a second point, 50 feet back, the angle of elevation to the top of the tree is 22.2°. Find the height of the tree to the nearest foot.



<u>*Solution*</u>

Triangle *DCB*

$$\Rightarrow \tan 22.2^\circ = \frac{h}{50+x}$$

$$h = (50+x)\tan 22.2^\circ$$

Triangle *ACB*

$$\Rightarrow \tan 36.7^\circ = \frac{h}{x}$$

$$h = x\tan 36.7^\circ$$

$$x\tan 36.7^\circ = (50+x)\tan 22.2^\circ$$

$$x\tan 36.7^\circ = 50\tan 22.2^\circ + x\tan 22.2^\circ$$

$$x\tan 36.7^\circ - x\tan 22.2^\circ = 50\tan 22.2^\circ$$

$$x(\tan 36.7^\circ - \tan 22.2^\circ) = 50\tan 22.2^\circ$$

$$x = \frac{50\tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}$$

$$h = x\tan 36.7^\circ$$

$$= \left(\frac{50\tan 22.2^\circ}{\tan 36.7^\circ - \tan 22.2^\circ}\right)\tan 36.7^\circ$$

$$\approx 45\ ft$$

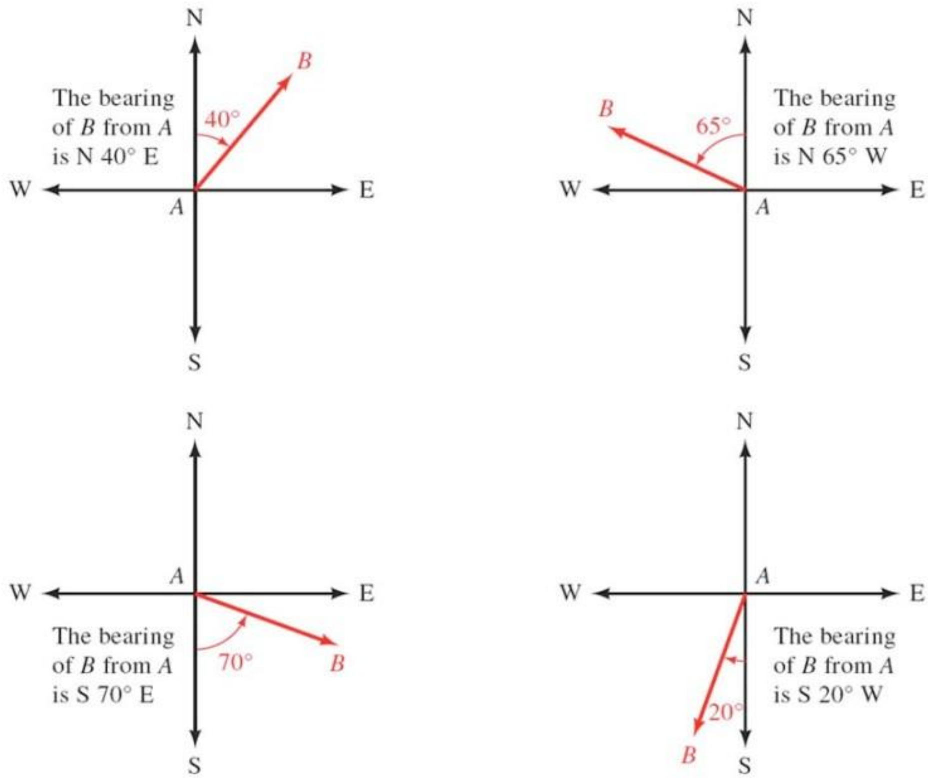
The tree is about 45 feet tall.

Bearing

Definition

The *bearing of a line* ℓ is the acute angle formed by the *north-south* line and the line ℓ.

The notation used to designate the bearing of a line begins with N (for north) or S (for south), followed by the number of degrees in the angle, and ends with E (for east) or W (for west).



Example

A boat travels on a course of bearing N 52° 40′ E for distance of 238 miles. How many miles north and how many miles east have the boat traveled?

<u>*Solution*</u>

$$52°40' = 52° + 40'\frac{1°}{60'}$$

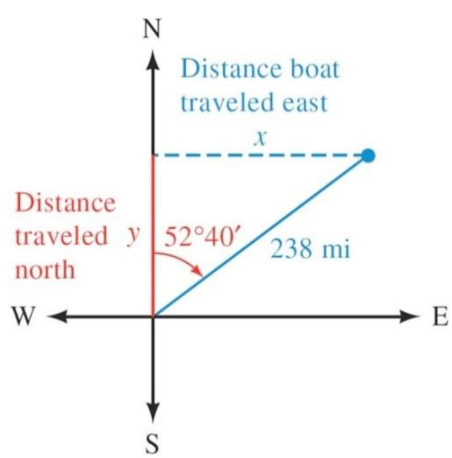
$$= 52.6667$$

$$\sin 52.6667 = \frac{x}{238}$$

$$x = 238 \sin 52.6667 \ \underline{= 189\ mi}$$

$$\cos 52.6667 = \frac{y}{238}$$

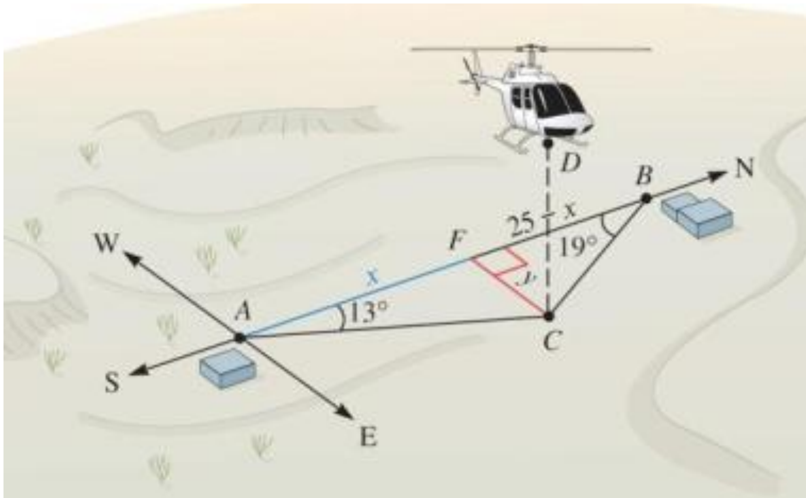
$$y = 238 \cos 52.6667 \ \underline{= 144\ mi}$$



Example

A helicopter is hovering over the desert when it develops mechanical problems and is forced to land. After landing, the pilot radios his position to a pair of radar station located 25 miles apart along a straight road running north and south. The bearing of the helicopter from one station is N 13° E, and from the other it is S 19° E. After doing a few trigonometric calculations, one of the stations instructs the pilot to walk due west for 3.5 miles to reach the road. Is this information correct?

<u>Solution</u>



In triangle AFC

$$\tan 13° = \frac{y}{x}$$

$$y = x\tan 13°$$

In triangle BFC

$$\tan 19° = \frac{y}{25-x}$$

$$y = (25-x)\tan 19°$$

$$y = y$$

$$(25-x)\tan 19° = x\tan 13°$$

$$25\tan 19° - x\tan 19° = x\tan 13°$$

$$25\tan 19° = x\tan 13° + x\tan 19°$$

$$25\tan 19° = x(\tan 13° + \tan 19°)$$

$$\frac{25\tan 19°}{\tan 13° + \tan 19°} = x$$

$$x = 14.966$$

$$y = x\tan 13°$$

$$= 14.966\tan 13°$$

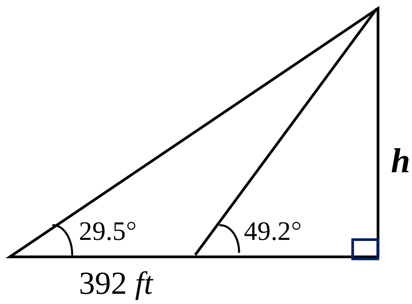
$$\underline{= 3.5\ mi}$$

Exercises *Section* 2.3 – Solving Right Triangle Trigonometry

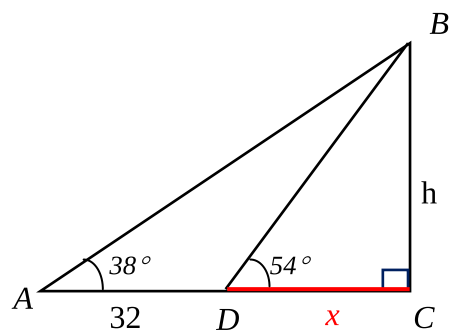
1. In the right triangle *ABC*, $a = 29.43$ and $c = 53.58$. Find the remaining side and angles.

2. In the right triangle *ABC*, $a = 2.73$ and $b = 3.41$. Find the remaining side and angles.

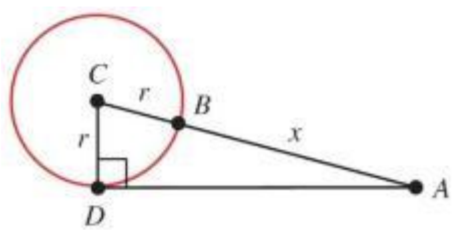
3. Find h as indicated in the figure.



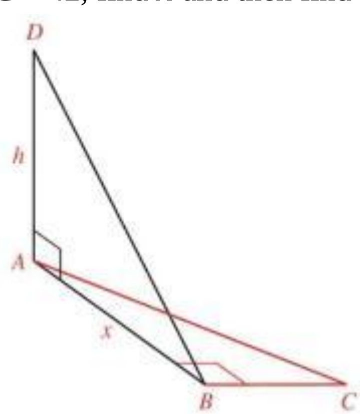
4. The distance from *A* to *D* is 32 feet. Use the information in figure to solve *x*, the distance between *D* and *C*.



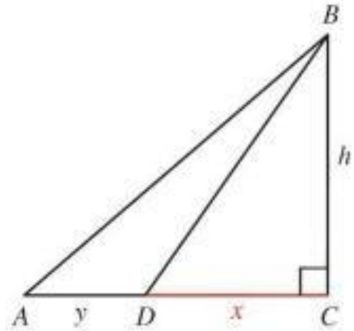
5. If C = 26° and $r = 19$, find x.



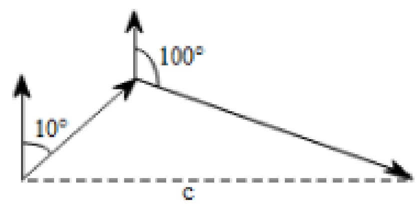
6. If ∠ABD = 53°, C = 48°, and BC = 42, find $\boldsymbol{x}$ and then find $\boldsymbol{h}$.



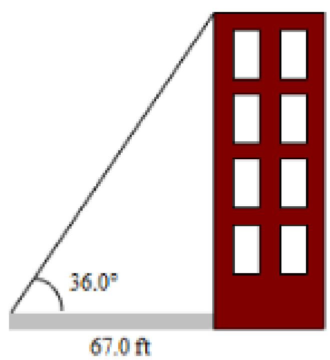
7. If A = 41°, ∠BDC = 58°, and AB = 28, find ***h***, then ***x*.**



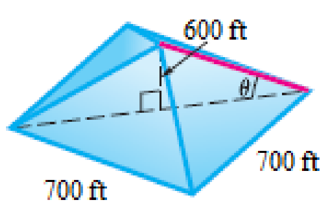
8. A plane flies 1.7 hours at 120 mph on a bearing of 10°. It then turns and flies 9.6 hours at the same speed on a bearing of 100°. How far is the plane from its starting point?



9. The shadow of a vertical tower is 67.0 *ft* long when the angle of elevation of the sun is 36.0°. Find the height of the tower.

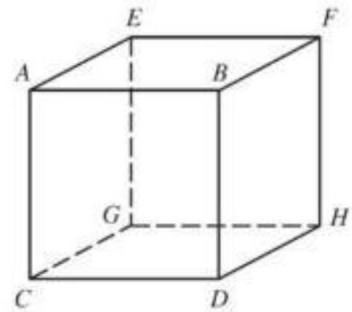


10. The base of a pyramid is square with sides 700 ft long, and the height of the pyramid is 600 ft. Find the angle of elevation of the edge indicated in the figure to two significant digits. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.)

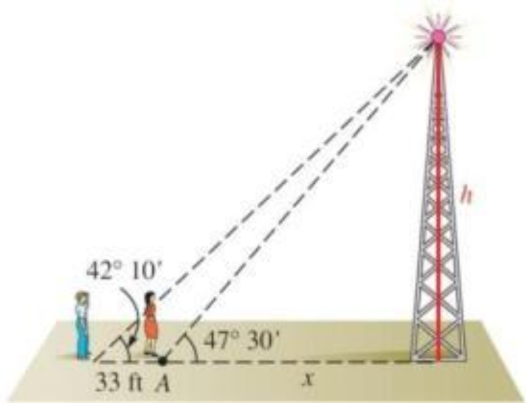


11. If a 73-foot flagpole casts a shadow 51 feet long, what is the angle of elevation of the sun (to the nearest tenth of a degree)?

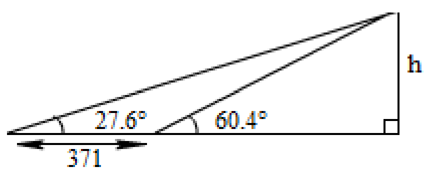
12. Suppose each edge of the cube is 3.00 inches long. Find the measure of the angle formed by diagonals DE and DG. *Round your answer to the nearest tenth of a degree*.



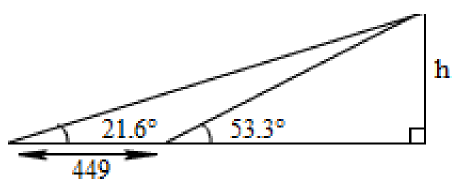
13. A person standing at point A notices that the angle of elevation to the top of the antenna is 47° 30′. A second person standing 33.0 feet farther from the antenna than the person at A finds the angle of elevation to the top of the antenna to be 42° 10′. How far is the person at A from the base of the antenna?



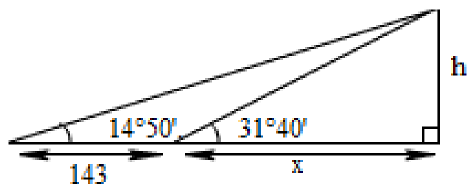
14. Find ***h*** as indicated in the figure.



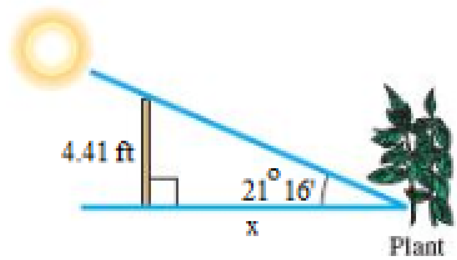
15. Find ***h*** as indicated in the figure.



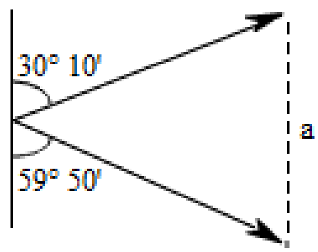
16. The angle of elevation from a point on the ground to the top of a pyramid is 31° 40′. The angle of elevation from a point 143 *ft* farther back to the top of the pyramid is 14° 50′. Find the height of the pyramid.



17. In one area, the lowest angle of elevation of the sun in winter is 21° 16′. Find the minimum distance, ***x***, that a plant needing full sun can be placed from a fence 4.41 *ft* high.

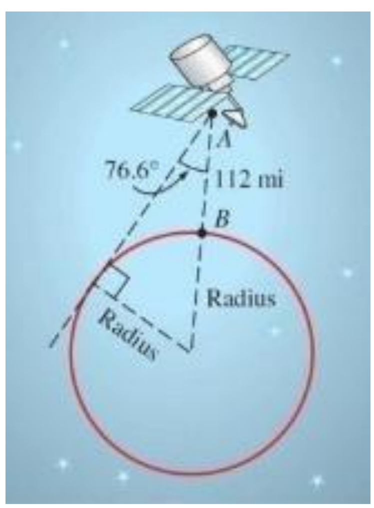


18. A ship leaves its port and sails on a bearing of N 30° 10′ E, at speed 29.4 mph. Another ship leaves the same port at the same time and sails on a bearing of S 59° 50′ E, at speed 17.1 mph. Find the distance between the two ships after 2 hrs.

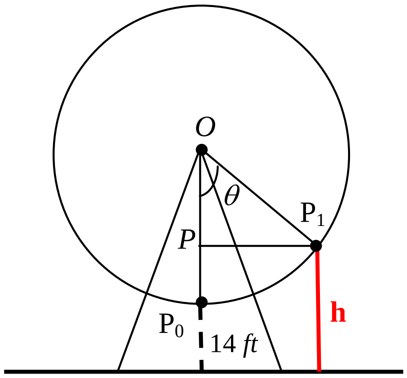


19. Radar stations *A* and *B* are on the east-west line, 3.7 km apart. Station *A* detects a place at *C*, on a bearing of 61°. Station *B* simultaneously detects the same plane, on a bearing of 331°. Find the distance from *A* to *C*.

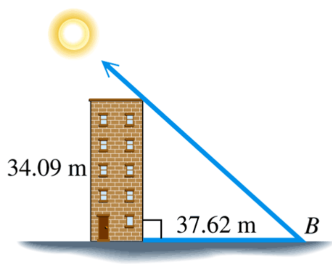
20. Suppose the figure below is exaggerated diagram of a plane flying above the earth. If the plane is 4.55 miles above the earth and the radius of the earth is 3,960 miles, how far is it from the plane to the horizon? What is the measure of angle A?



21. The Ferry wheel has a 250 feet diameter and 14 feet above the ground. If θ is the central angle formed as a rider moves from position P_0 to position P_1, find the rider's height above the ground h when θ is 45°.



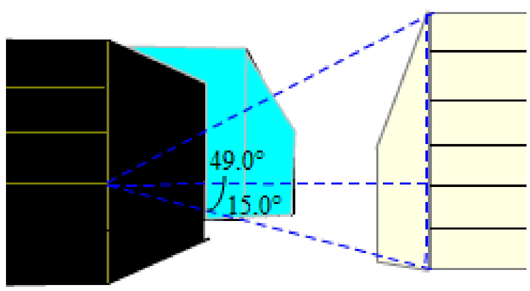
22. The length of the shadow of a building 34.09 *m* tall is 37.62 *m*. Find the angle of the elevation of the sun.



23. San Luis Obispo, California is 12 miles due north of Grover Beach. If Arroyo Grande is 4.6 miles due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

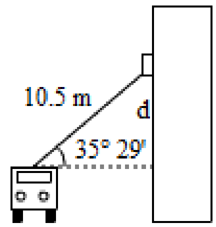
24. The bearing from *A* to *C* is S 52° E. The bearing from *A* to *B* is N 84° E. The bearing from *B* to *C* is S 38° W. A plane flying at 250 mph takes 2.4 hours to go from *A* to *B*. Find the distance from *A* to *C*.

25. From a window 31.0 *ft*. above the street, the angle of elevation to the top of the building across the street is 49.0° and the angle of depression to the base of this building is 15.0°. Find the height of the building across the street.

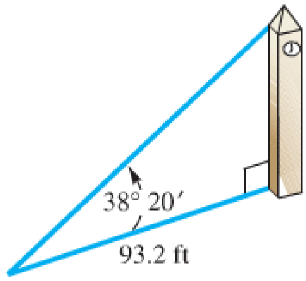


26. A man wondering in the desert walks 2.3 miles in the direction S 31° W. He then turns 90° and walks 3.5 miles in the direction N 59° W. At that time, how far is he from his starting point, and what is his bearing from his starting point?

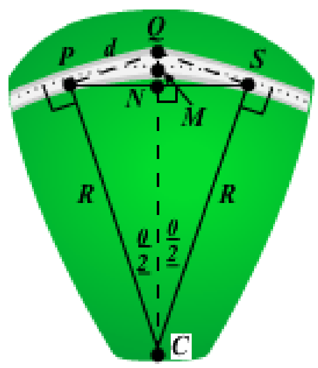
27. A 10.5-*m* fire truck ladder is leaning against a wall. Find the distance ***d*** the ladder goes up the wall (above the fire truck) if the ladder makes an angle of 35° 29′ with the horizontal.



28. The angle of elevation from a point 93.2 ft from the base of a tower to the top of the tower is 38° 20′. Find the height of the tower.



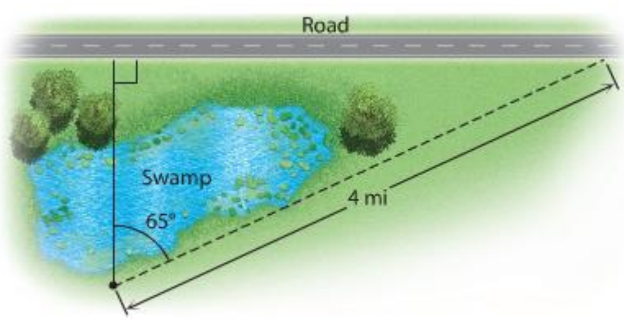
29. A basic curve connecting two straight sections of road is often circular. In the figure, the points ***P*** and ***S*** mark the beginning and end of the curve. Let ***Q*** be the point of intersection where the two straight sections of highway leading into the curve would meet if extended. The radius of the curve is ***R***, and the central angle denotes how many degrees the curve turns.



a) If ***R*** = 965 ft. and θ = 37°, find the distance d between P and ***Q***.

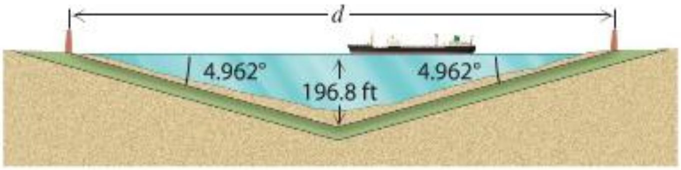
b) Find an expression in terms of ***R*** and θ for the distance between points ***M*** and ***N***.

30. Jane was hiking directly toward a long straight road when she encountered a swamp. She turned 65° to the right and hiked 4 mi in that direction to reach the road. How far was she form the road when she encountered the swamp?

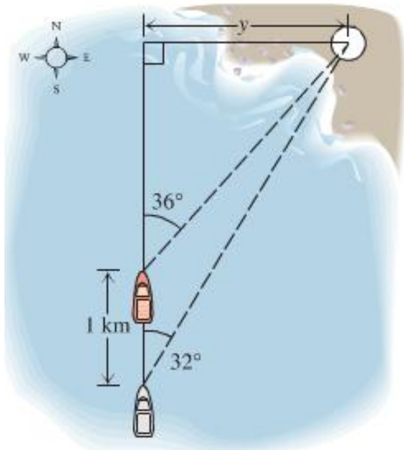


31. From a highway overpass, 14.3 m above the road, the angle of depression of an oncoming car is measured at 18.3°. How far is the car from a point on the highway directly below the observer?

32. A tunnel under a river is 196.8 ft. below the surface at its lowest point. If the angle of depression of the tunnel is 4.962° , then how far apart on the surface are the entrances to the tunnel? How long is the tunnel?

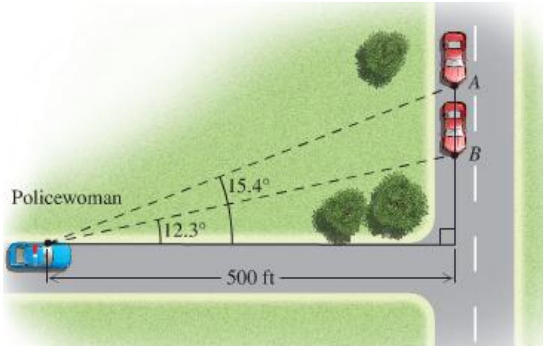


33. A boat sailing north sights a lighthouse to the east at an angle of 32° from the north. After the boat travels one more kilometer, the angle of the lighthouse from the north is 36°. If the boat continues to sail north, then how close will the boat come to the lighthouse?

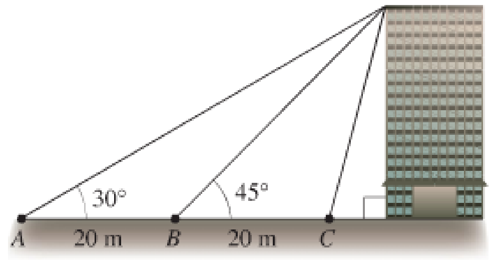


34. The angle of elevation of a pedestrian crosswalk over a busy highway is 8.34°, as shown in the drawing. If the distance between the ends of the crosswalk measured on the ground is 342 ft., then what is the height h of the crosswalk at the center?

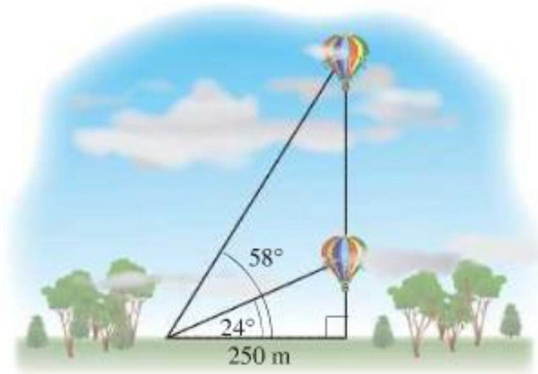
35. A policewoman has positioned herself 500 ft. from the intersection of two roads. She has carefully measured the angles of the lines of sight to points A and B. If a car passes from A to B is 1.75 sec and the speed limit is 55 mph, is the car speeding? (Hint: Find the distance from B to A and use R = D/T)



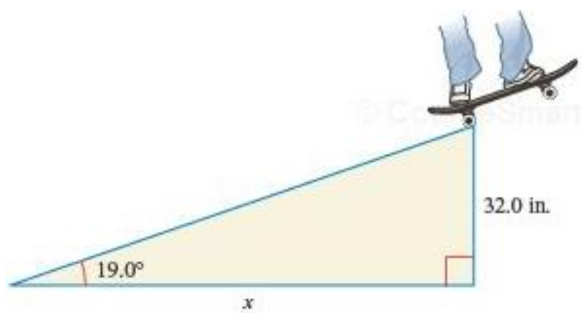
36. From point A the angle of elevation to the top of the building is 30°. From point B, 20 meters closer to the building, the angle of elevation is 45°. Find the angle of elevation of the building from point C, which is another 20 meters closer to the building.



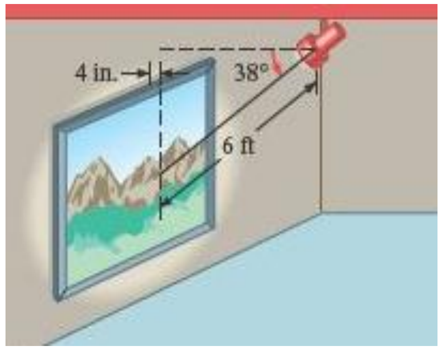
37. A hot air balloon is rising upward from the earth at a constant rate. An observer 250 m away spots the balloon at an angle of elevation of 24°. Two minutes later the angle of elevation of the balloon is 58°. At what rate is the balloon ascending?



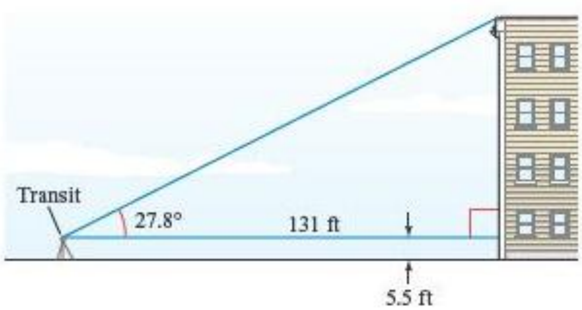
38. A skateboarder wishes to build a jump ramp that is inclined at a 19° angle and that has a maximum height of 32.0 inches. Find the horizontal width *x* of the ramp.



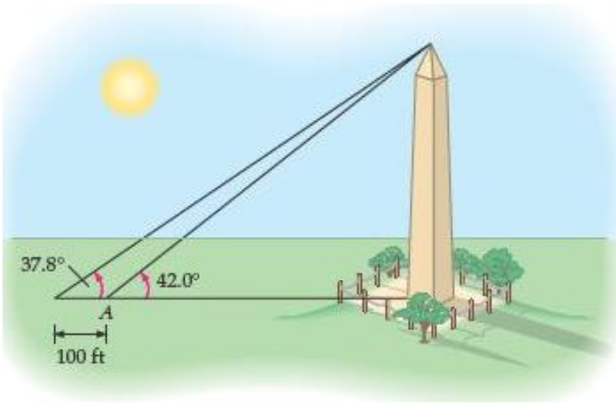
39. For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 ft from the piece of art and that the angle of depression of the light be 38°. How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 inches from the wall.



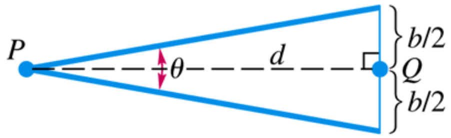
40. A surveyor determines that the angle of elevation from a transit to the top of a building is 27.8°. The transit is positioned 5.5 feet above ground level and 131 feet from the building. Find the height of the building to the nearest tenth of a foot.



41. From a point A on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is 42.0°. From a point 100 ft away from A and on the same line, the angle to the top is 37.8°. Find the height, to the nearest foot, of the Monument.



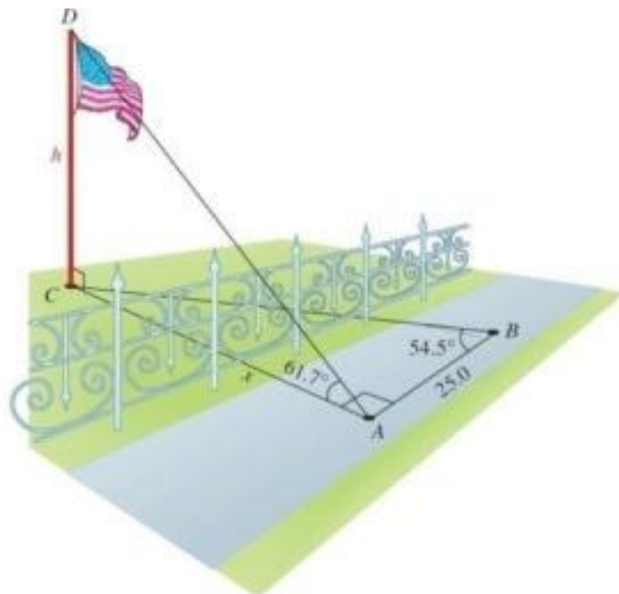
42. A method that surveyors use to determine a small distance d between two points P and Q is called the ***subtense bar method***. The subtense bar with length b is centered at Q and situated perpendicular to the line of sight between P and Q. Angle θ is measured, then the distance d can be determined.



a) Find d with $\theta = 1° \ 23' \ 12''$ and $b = 2.000 \ cm$

b) Angle θ usually cannot be measured more accurately than to the nearest 1″. How much change would there be in the value of d if θ were measured 1″ larger?

43. A diagram that shows how Diane estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way. So she stands at point A facing the pole and finds the angle of elevation from point A to the top of the pole to be 61.7°. Then she turns 90° and walks 25.0 ft to point B, where she measures the angle between her path and a line from *B* to the base of the pole. She finds that angle is 54.5°. Use this information to find the height of the pole.



Solution ***Section* 2.3 – Solving Right Triangle Trigonometry**

Exercise

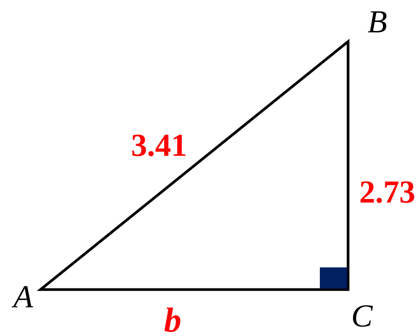
In the right triangle *ABC*, $a = 29.43$ and $c = 53.58$. Find the remaining side and angles.

Solution

$$c^2 = a^2 + b^2$$

$$b^2 = c^2 - a^2$$

$$\boxed{b = \sqrt{53.58^2 - 29.43^2} \approx 44.77}$$



$$\sin A = \frac{a}{c}$$

$$= \frac{29.43}{53.58}$$

$$A = \sin^{-1}\left(\frac{29.43}{53.58}\right)$$

$$\boxed{\approx 33.32^\circ}$$

$$B = 90^\circ - A$$

$$= 90^\circ - 33.32^\circ$$

$$\boxed{\approx 56.68^\circ}$$

Exercise

In the right triangle *ABC*, $a = 2.73$ and $b = 3.41$. Find the remaining side and angles.

Solution

$$c^2 = a^2 + b^2$$

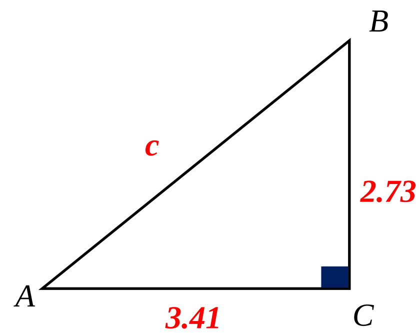
$$c = \sqrt{2.73^2 + 3.41^2} = 4.37$$

$$\tan A = \frac{a}{b} \qquad \textit{or} \qquad \sin A = \frac{a}{c}$$

$$= \frac{2.73}{3.41} \qquad\qquad = \frac{2.73}{4.37}$$

$$A = \tan^{-1}\left(\frac{2.73}{3.41}\right) \qquad A = \sin^{-1}\left(\frac{2.73}{4.37}\right)$$

$$= 38.7^\circ \qquad\qquad = 38.7^\circ$$



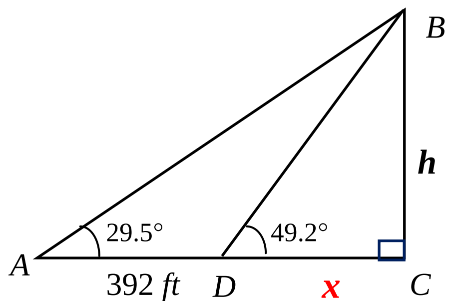
$B = 90° - A$

$= 90° - 38.7°$

Exercise

Find h as indicated in the figure.

Solution



Triangle $DCB \Rightarrow \tan 49.2° = \frac{h}{x}$

$h = x\tan 49.2°$

Triangle $ACB \Rightarrow \tan 29.5° = \frac{h}{x+392}$

$h = (x+392)\tan 29.5°$

$h = x\tan 49.2° = (x+392)\tan 29.5°$

$x\tan 49.2° = x\tan 29.5° + 392\tan 29.5°$

$x\tan 49.2° - x\tan 29.5° = 392\tan 29.5°$

$x\left(\tan 49.2° - \tan 29.5°\right) = 392\tan 29.5°$

$x = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°}$

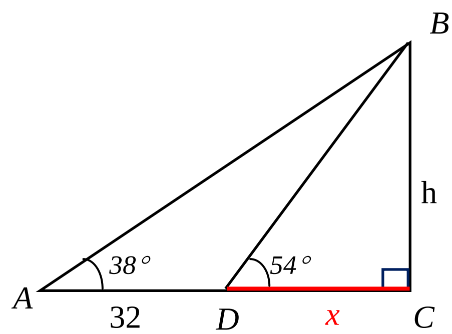
$h = \frac{392\tan 29.5°}{\tan 49.2° - \tan 29.5°}\tan 49.2°$

$= 433.5\ ft$

Exercise

The distance from A to D is 32 feet. Use the figure to solve x, the distance between D and C.

Solution



Triangle $DCB \Rightarrow \tan 54° = \frac{h}{x}$

$h = x\tan 54°$

Triangle $ACB \Rightarrow \tan 38° = \frac{h}{x+32}$

$h = (x+32)\tan 38°$

$h = x\tan 54° = (x+32)\tan 38°$

$x\tan 54° = x\tan 38° + 32\tan 38°$

$x\tan 54° - x\tan 38° = 32\tan 38°$

$x(\tan 54° - \tan 38°) = 32\tan 38°$

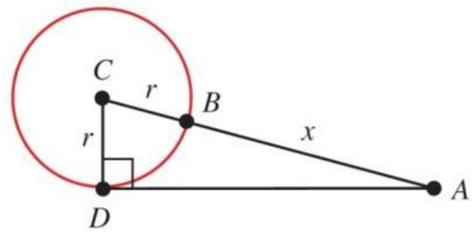
$x = \frac{32\tan 38°}{\tan 54° - \tan 38°}$

$= 42\ ft$

Exercise

If C = 26° and r = 19, find x.

Solution



$$\cos 26° = \frac{r}{r+x} = \frac{19}{19+x}$$

$$(19+x)\cos 26° = 19$$

$$19\cos 26° + x\cos 26° = 19$$

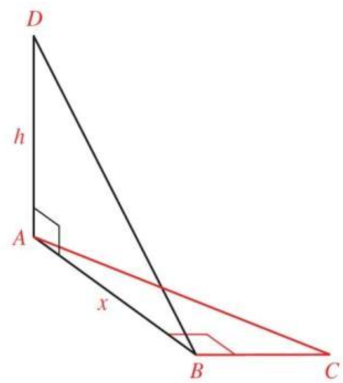
$$x\cos 26° = 19 - 19\cos 26°$$

$$x = \frac{19-19\cos 26°}{\cos 26°} \underline{\approx 2.14|}$$

Exercise

If ∠ABD = 53°, C = 48°, and BC = 42, find x and then find h.

Solution



$$\tan 48° = \frac{x}{42}$$

$$x = 42\tan 48° = 46.65 \approx 47$$

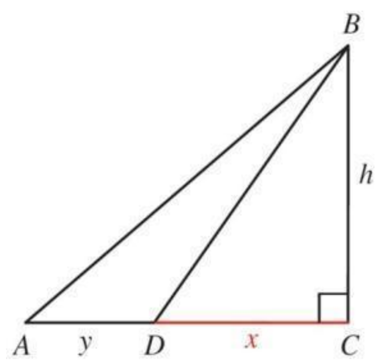
$$\tan 53° = \frac{h}{x}$$

$$\Rightarrow h = 47\tan 53° \underline{\approx 62|}$$

Exercise

If A = 41°, ∠BDC = 58°, and AB = 28, find h, then x.

Solution



$$\sin 41° = \frac{h}{AB}$$

$$\Rightarrow h = 28\sin 41° \underline{\approx 18|}$$

$$\tan 58° = \frac{h}{x}$$

$$\Rightarrow x = \frac{18}{\tan 58°} \approx 11$$

Exercise

A plane flies 1.7 hours at 120 mph on a bearing of 10°. It then turns and flies 9.6 hours at the same speed on a bearing of 100°. How far is the plane from its starting point?

Solution

$$b = 120\,\frac{mi}{hr}\,1.7\,hrs = 204\ mi$$

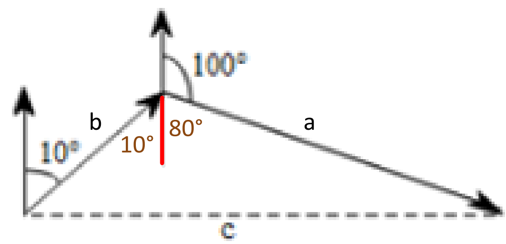
$$a = 120\,\frac{mi}{hr}\,9.6\,hrs = 1152\ mi$$

The triangle is right triangle.

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{1152^2 + 204^2}$$

$$\approx 1170\ mi$$



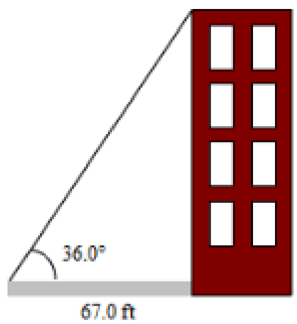
Exercise

The shadow of a vertical tower is 67.0 *ft* long when the angle of elevation of the sun is 36.0°. Find the height of the tower.

Solution

$$\tan 36° = \frac{h}{67}$$

$$h = 67 \tan 36° \approx 48.7\ ft$$



Exercise

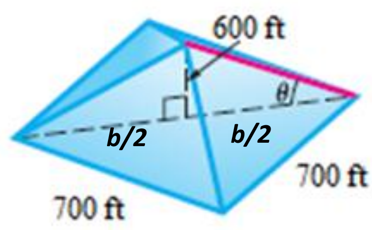
The base of a pyramid is square with sides 700 *ft*. long, and the height of the pyramid is 600 *ft*. Find the angle of elevation of the edge indicated in the figure to two significant digits. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.)

Solution

$$b^2 = 700^2 + 700^2 \Rightarrow b = \sqrt{2\left(700^2\right)} = 700\sqrt{2}$$

$$\tan\theta = \frac{600}{b/2} = \frac{600}{\frac{700\sqrt{2}}{2}} = 600\,\frac{2}{700\sqrt{2}} = \frac{12}{7\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{12}{7\sqrt{2}}\right) \approx 50.48°$$



Exercise

If a 73-foot flagpole casts a shadow 51 feet long, what is the angle of elevation of the sun (to the nearest tenth of a degree)?

Solution

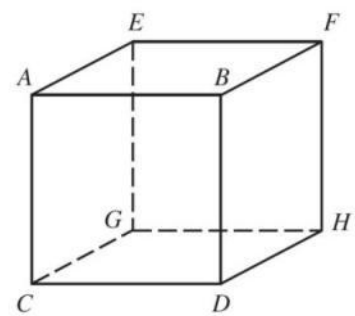
$$\tan\theta = \frac{73}{51}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{73}{51}\right) \underline{= 55.1^\circ}$$

Exercise

Suppose each edge of the cube is 3.00 inches long. Find the measure of the angle formed by diagonals DE and DG. *Round your answer to the nearest tenth of a degree*.

Solution



$$|DG| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

$$\tan(EDG) = \frac{EG}{GD} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$$

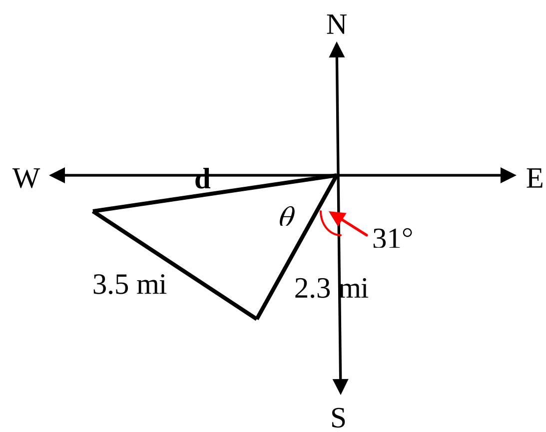
$$EDG = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{EDG = 45^\circ}$$

Exercise

A man wondering in the desert walks 2.3 miles in the direction S 31° W. He then turns 90° and walks 3.5 miles in the direction N 59° W. At that time, how far is he from his starting point, and what is his bearing from his starting point?

Solution



$$d = \sqrt{2.3^2 + 3.5^2} = 4.2$$

$$\cos\theta = \frac{2.3}{4.2} = .55$$

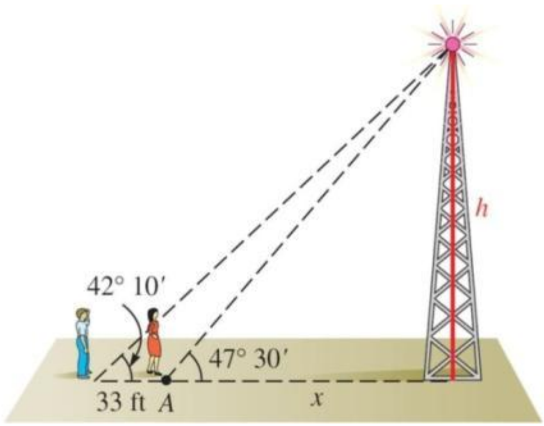
$$\theta = \cos^{-1} 0.55 \approx 57^\circ$$

S (57°+31°) W

$\rightarrow$ Bearing $S\ 88^\circ\ W$

Exercise

A person standing at point A notices that the angle of elevation to the top of the antenna is 47° 30′. A second person standing 33.0 feet farther from the antenna than the person at A finds the angle of elevation to the top of the antenna to be 42° 10′. How far is the person at A from the base of the antenna?



Solution

$$47°\ 30' = 47 + 30\frac{1}{60} = 47.5°$$

$$\tan 47.5° = \frac{h}{x}$$

$$\Rightarrow h = x\tan 47.5° \quad (1)$$

$$42°\ 10' = 42 + 10\frac{1}{60} = 42.167°$$

$$\tan 42.167° = \frac{h}{33+x}$$

$$\Rightarrow h = (33+x)\tan 42.167° \quad (2)$$

$$\boldsymbol{h =} (33+x)\tan 42.167° = x\tan 47.5°$$

$$33\tan 42.167° + x\tan 42.167° = x\tan 47.5° \qquad 29.88 + 0.906x = 1.09x$$

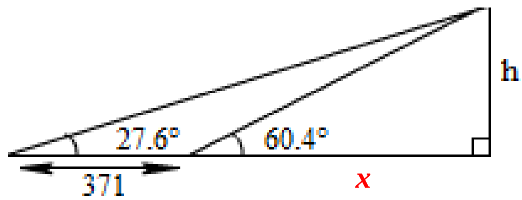
$$33\tan 42.167° = x\tan 47.5° - x\tan 42.167° \qquad 29.88 = 1.09x - .906x$$

$$\frac{33\tan 42.167°}{\tan 47.5° - \tan 42.167°} = x \qquad 29.88 = 0.184x$$

$$x = \frac{29.88}{0.18} \ \underline{=161|}$$

Exercise

Find ***h*** as indicated in the figure.



Solution

Outside triangle:

$\tan 27.6° = \dfrac{h}{371+x} \Rightarrow h = (371+x)\tan 27.6°$

Inside triangle: $\tan 60.4° = \dfrac{h}{x} \Rightarrow h = x\tan 60.4°$

Both triangles have the same ***h***, therefore:

$x\tan 60.4° = 371\tan 27.6° + x\tan 27.6°$

$x\tan 60.4° - x\tan 27.6° = 371\tan 27.6°$

$x(\tan 60.4° - \tan 27.6°) = 371\tan 27.6°$

$x = \dfrac{371\tan 27.6°}{\tan 60.4° - \tan 27.6°}$

$x \approx 157$

$\Rightarrow h = x\tan 60.4° \approx 276$

Exercise

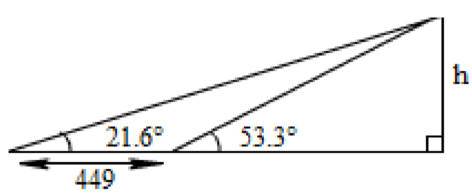
Find ***h*** as indicated in the figure.

Solution

Outside triangle: $\tan 21.6° = \dfrac{h}{449+x} \Rightarrow h = (449+x)\tan 21.6°$

Inside triangle: $\tan 53.5° = \dfrac{h}{x} \Rightarrow h = x\tan 53.5°$

Both triangles have the same ***h***, therefore:



$x\tan 53.5° = 449\tan 21.6° + x\tan 21.6°$

$x\tan 53.5° - x\tan 21.6° = 449\tan 21.6°$

$x(\tan 53.5° - \tan 21.6°) = 449\tan 21.6°$

$x = \dfrac{449\tan 21.6°}{\tan 53.5° - \tan 21.6°}$

$x \approx 186$

$\Rightarrow h = x\tan 53.5°$

$= 186\tan 53.5°$

≈ 252

Exercise

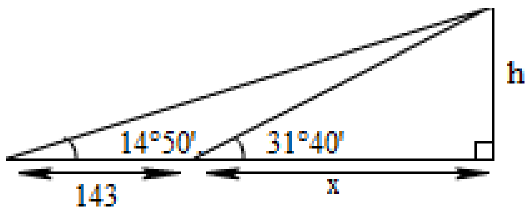
The angle of elevation from a point on the ground to the top of a pyramid is 31° 40′. The angle of elevation from a point 143 *ft* farther back to the top of the pyramid is 14° 50′. Find the height of the pyramid.

Solution

$$14°50' = 14° + \frac{50}{60}° = 14.833° \quad \textit{and} \quad 31°40' = 31° + \frac{40}{60}° = 31.667°$$

$$\tan 14.833° = \frac{h}{143+x} \Rightarrow h = (143+x)\tan 14.833°$$

$$\tan 31.667° = \frac{h}{x} \Rightarrow h = x\tan 31.667°$$



Both triangles have the same ***h***, therefore:

$$\Rightarrow h = x\tan 31.667° = (143+x)\tan 14.833°$$

$$x\tan 31.667° = 143\tan 14.833° + x\tan 14.833°$$

$$x\tan 31.667° - x\tan 14.833° = 143\tan 14.833°$$

$$x(\tan 31.667° - \tan 14.833°) = 143\tan 14.833°$$

$$x = \frac{143\tan 14.833°}{\tan 31.667° - \tan 14.833°}$$

$$\Rightarrow h = x\tan 31.667°$$

$$= \frac{143\tan 14.833°}{\tan 31.667° - \tan 14.833°}\tan 31.667°$$

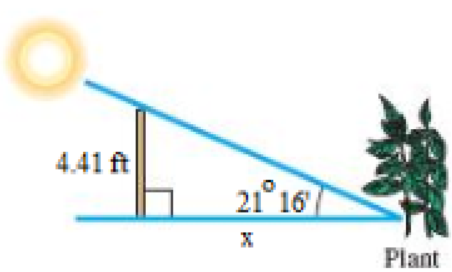
$$\approx 66$$

Exercise

In one area, the lowest angle of elevation of the sun in winter is 21° 16′. Find the minimum distance, ***x***, that a plant needing full sun can be placed from a fence 4.41 *ft* high.

Solution

$$\tan(21°16') = \frac{4.41}{x}$$



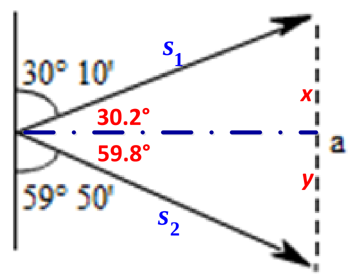
$$x = \frac{4.41}{\tan\left(21° + \frac{16°}{60}\right)}$$

$$\approx 11.33\ ft$$

Exercise

A ship leaves its port and sails on a bearing of N 30° 10′ E, at speed 29.4 mph. Another ship leaves the same port at the same time and sails on a bearing of S 59° 50′ E, at speed 17.1 mph. Find the distance between the two ships after 2 hrs.

Solution



$$\begin{cases} 30°10' = 30° + \frac{10°}{60} \approx 30.16667° \\ 59°50' = 59° + \frac{50°}{60} \approx 59.8333° \end{cases}$$

After 2 hours:

$$\begin{cases} s_1 = 29.4 \frac{mi}{hr}.(2)hr = 58.8 \\ s_2 = 17.1 \frac{mi}{hr}.(2)hr = 34.2 \end{cases}$$

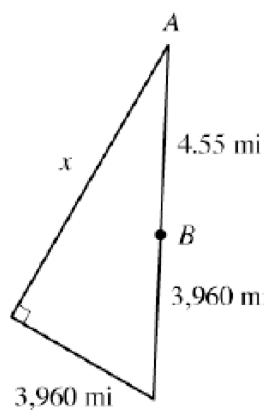
$$\begin{cases} \tan 30.2° = \frac{x}{s_1} \Rightarrow x = 58.8 \tan 30.2° \\ \tan 59.8° = \frac{y}{s_2} \Rightarrow y = 34.2 \tan 59.8° \end{cases}$$

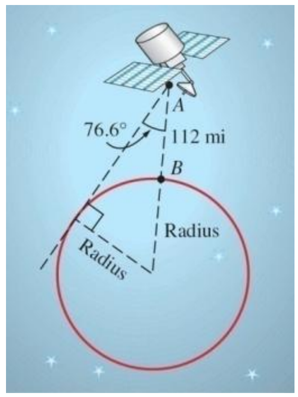
$$\begin{aligned} a &= x + y \\ &= 58.8 \tan 30.2° + 34.2 \tan 59.8° \\ &\approx 93 \text{ miles} \end{aligned}$$

Exercise

Suppose the figure below is exaggerated diagram of a plane flying above the earth. If the plane is 4.55 miles above the earth and the radius of the earth is 3,960 miles, how far is it from the plane to the horizon? What is the measure of angle A?

Solution





$x^2 + 3960^2 = 3964.55^2$

$x^2 = 3964.55^2 - 3960^2$

$x = \sqrt{3964.55^2 - 3960^2}$

$x \approx 190$

The plane is 190 miles from the horizon.

$\sin A = \frac{3960}{3964.55} \approx 0.9989$

$A = \sin^{-1}(0.9989) \approx 87.3°$

Exercise

The Ferry wheel has a 250 feet diameter and 14 feet above the ground. If θ is the central angle formed as a rider moves from position P_0 to position P_1, find the rider's height above the ground h when θ is 45°.

Solution

Distance between O *and* $P_0 = radius = \frac{250}{2} = 125\,ft$

$$\cos\theta = \frac{OP}{OP_1}$$

$$\cos 45^\circ = \frac{OP}{125}$$

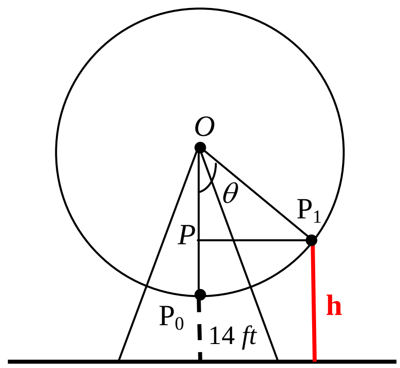
$$OP = 125\cos 45^\circ$$

$$h = PP_0 + 14$$

$$= OP_0 - OP + 14$$

$$= 125 - 125\cos 45^\circ + 14$$

$$\boxed{= 51\,ft}$$



Exercise

If a 75-foot flagpole casts a shadow 43 *ft* long, to the nearest 10 minutes what is the angle of elevation of the sum from the tip of the shadow?

Solution

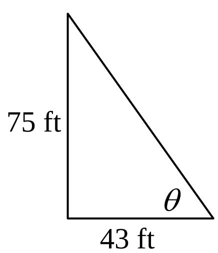
$$\tan\theta = \frac{75}{43}$$

$$\theta = \tan^{-1}\left(\frac{75}{43}\right)$$

$$\theta = 60.17^\circ$$

$$\theta = 60^\circ\ \ 0.17^\circ\left(\frac{60'}{1^\circ}\right)$$

$$\boxed{\theta = 60^\circ\ \ 10'}$$



Exercise

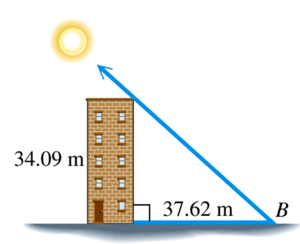
The length of the shadow of a building 34.09 *m* tall is 37.62 *m*. Find the angle of the elevation of the sun.

Solution

$$\tan B = \frac{34.09}{37.62}$$

$$B = \tan^{-1}\left(\frac{34.09}{37.62}\right)$$

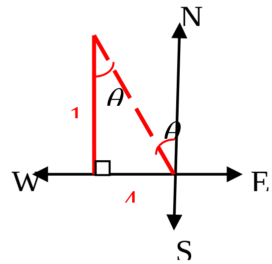
$\boxed{\approx 42.18^\circ}$ $\Rightarrow$ The angle of elevation is $\approx 42.18^\circ$



Exercise

San Luis Obispo, California is 12 miles due north of Grover Beach. If Arroyo Grande is 4.6 miles due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

Solution



$$\tan\theta = \frac{4.6}{12} = 0.3833$$

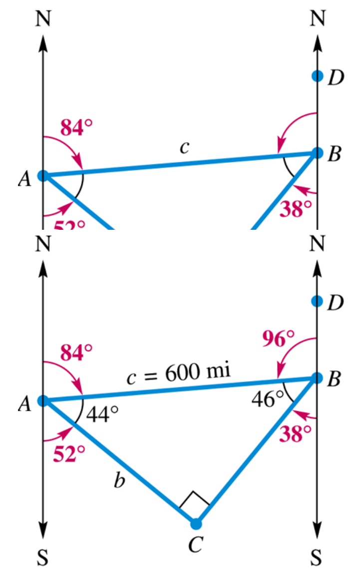
$$\theta = \tan^{-1} 0.3833 = 21^\circ$$

The bearing of San Luis Obispo from Arroyo Grande is N 21° W

Exercise

The bearing from A to C is S 52° E. The bearing from A to B is N 84° E. The bearing from B to C is S 38° W. A plane flying at 250 mph takes 2.4 hours to go from A to B. Find the distance from A to C.

Solution



$$\angle ABD = 180^\circ - 84^\circ = \underline{96^\circ}$$

$$\angle ABC = 180^\circ - (96^\circ + 38^\circ) = \underline{46^\circ}$$

$$\angle C = 180^\circ - (46^\circ + 44^\circ) = \underline{90^\circ}$$

$$\begin{aligned} c &= rate \times time \\ &= 250(2.4) \\ &= 600\ mi. \end{aligned}$$

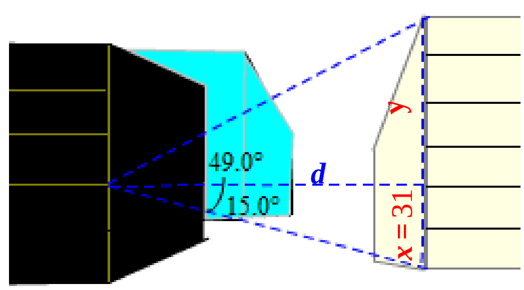
$$\sin 46^\circ = \frac{b}{c} = \frac{b}{600}$$

$$b = 600 \sin 46^\circ \approx \underline{430\ mi}$$

Exercise

From a window 31.0 *ft*. above the street, the angle of elevation to the top of the building across the street is 49.0° and the angle of depression to the base of this building is 15.0°. Find the height of the building across the street.

Solution



$$\tan 15^\circ = \frac{31}{d} \Rightarrow d = \frac{31}{\tan 15^\circ}$$

$$\tan 49^\circ = \frac{y}{d} \Rightarrow y = \frac{31}{\tan 15^\circ} \tan 49^\circ$$

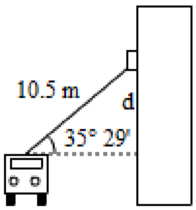
$$h = x + y = 31 + \frac{31}{\tan 15^\circ} \tan 49^\circ$$

$$\underline{= 164\ ft}$$

Exercise

A 10.5-*m* fire truck ladder is leaning against a wall. Find the distance ***d*** the ladder goes up the wall (above the fire truck) if the ladder makes an angle of 35° 29′ with the horizontal.

Solution



$$\sin(35°29') = \frac{d}{10.5}$$

$$d = 10.5\sin\left(35° + \frac{29°}{60}\right)$$

$$\boxed{d = 6.1\ m}$$

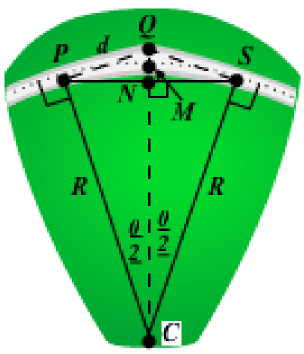
Exercise

A basic curve connecting two straight sections of road is often circular. In the figure, the points ***P*** and ***S*** mark the beginning and end of the curve. Let ***Q*** be the point of intersection where the two straight sections of highway leading into the curve would meet if extended. The radius of the curve is ***R***, and the central angle denotes how many degrees the curve turns.

a) If ***R*** = 965 ft. and θ = 37°, find the distance d between P and ***Q***.

b) Find an expression in terms of ***R*** and θ for the distance between points ***M*** and ***N***.

Solution



a) $\sin\frac{\theta}{2} = \frac{|PN|}{R} \Rightarrow |PN| = 965\sin\left(\frac{37°}{2}\right) \approx 306.2$

$$\angle CPN = 90° - \frac{\theta}{2} = 71.5°$$

$$\angle NPQ = 90° - \angle CPN = 90° - 71.5° = 18.5° = \frac{\theta}{2}$$

$$\cos(NPQ) = \frac{|PN|}{d}$$

$$\Rightarrow \underline{\left|d = \frac{|PN|}{\cos 18.5°} = \frac{306.2}{\cos 18.5°} \approx 322.9\right|}$$

b) $\cos\frac{\theta}{2} = \frac{|CN|}{R}$

$$|CN| = R\cos\frac{\theta}{2}$$

$$R = |CQ| = |CM| + 2|NM|$$

$$2|NM| = R - |CM|$$

$$2|NM| = R - R\cos\frac{\theta}{2}$$

$$|NM| = \frac{1}{2}R\left(1 - \cos\frac{\theta}{2}\right)$$

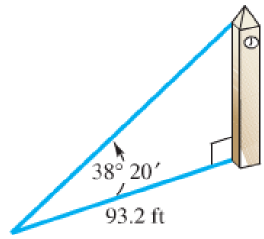
Exercise

The angle of elevation from a point 93.2 ft from the base of a tower to the top of the tower is 38° 20′. Find the height of the tower.

Solution

$$\tan\left(38^\circ\ 20'\right) = \frac{h}{93.2}$$

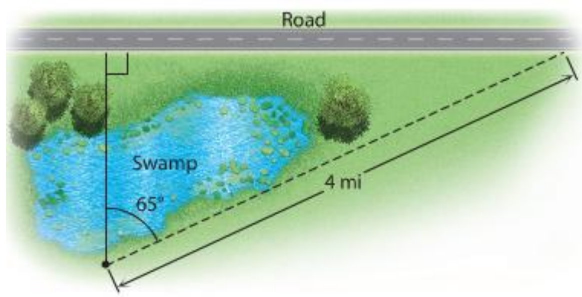
$$\boxed{h = 93.2\tan\left(38^\circ\ 20'\right) = 73.7}$$



Exercise

Jane was hiking directly toward a long straight road when she encountered a swamp. She turned 65° to the right and hiked 4 mi in that direction to reach the road. How far was she forms the road when she encountered the swamp?

Solution



$$\cos 65^\circ = \frac{d}{4}$$

$$d = 4\cos 65^\circ$$

$$\boxed{\approx 1.7\ miles}$$

Exercise

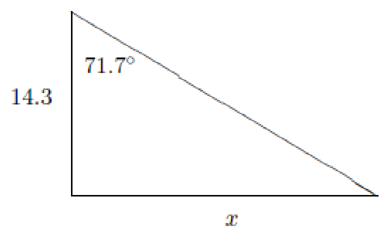
From a highway overpass, 14.3 m above the road, the angle of depression of an oncoming car is measured at 18.3°. How far is the car from a point on the highway directly below the observer?

Solution

$$\alpha = 90^\circ - 18.3^\circ = 71.7^\circ$$

$$\tan(71.7^\circ) = \frac{x}{14.3}$$

$$\boxed{x = 14.3\tan(71.7^\circ) \approx 43.2\ m}$$

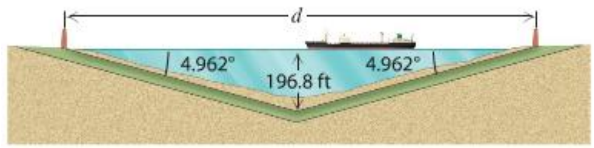


Exercise

A tunnel under a river is 196.8 ft. below the surface at its lowest point. If the angle of depression of the tunnel is 4.962° , then how far apart on the surface are the entrances to the tunnel? How long is the tunnel?

Solution

$$\tan 4.962^\circ = \frac{196.8}{x}$$



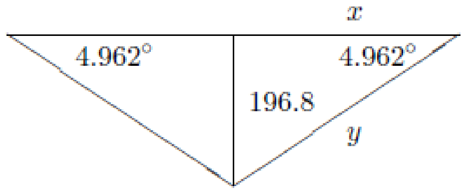
$$x = \frac{196.8}{\tan 4.962°} \approx 2266.75$$

$$\underline{|d} = 2x \ \underline{= 4533\ ft|}$$

$$\sin 4.962° = \frac{196.8}{y}$$

$$y = \frac{196.8}{\sin 4.962°} \approx 2275.3$$

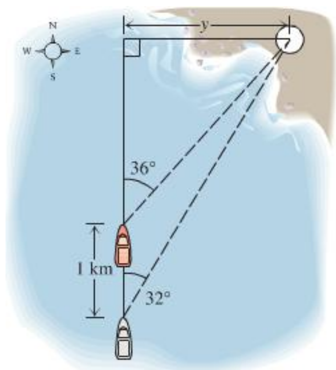
The tunnel length: $2y = 4551\ ft$



Exercise

A boat sailing north sights a lighthouse to the east at an angle of 32° from the north. After the boat travels one more kilometer, the angle of the lighthouse from the north is 36°. If the boat continues to sail north, then how close will the boat come to the lighthouse?

Solution



$$\tan 36° = \frac{x}{y} \Rightarrow x = y\tan 36°$$

$$\tan 32° = \frac{x}{y+1} \Rightarrow x = (y+1)\tan 32°$$

$$x = y\tan 36° = (y+1)\tan 32°$$

$$y\tan 36° = y\tan 32° + \tan 32°$$

$$y\tan 36° - y\tan 32° = \tan 32°$$

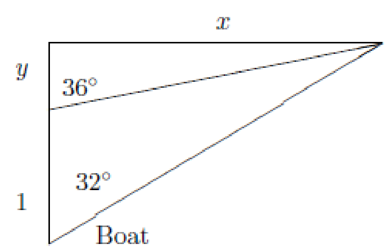
$$y(\tan 36° - \tan 32°) = \tan 32°$$

$$y = \frac{\tan 32°}{\tan 36° - \tan 32°}$$

$$\Rightarrow x = y\tan 36°$$

$$= \frac{\tan 32°}{\tan 36° - \tan 32°}\tan 36°$$

$$\underline{\approx 4.5\ km|}$$



The closest will the boat come to the lighthouse is 4.5 km.

Exercise

The angle of elevation of a pedestrian crosswalk over a busy highway is 8.34°, as shown in the drawing. If the distance between the ends of the crosswalk measured on the ground is 342 ft., then what is the height h of the crosswalk at the center?

Solution

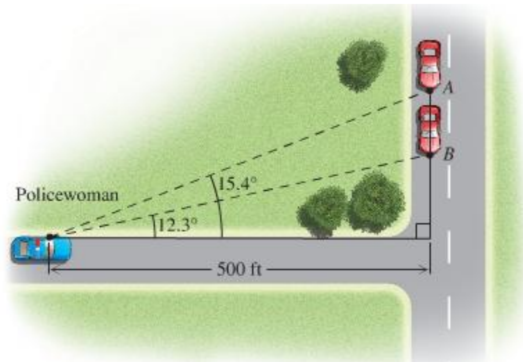
$$\tan 8.34° = \frac{h}{171}$$

$$\underline{|h =}\ 171\tan 8.34° \ \underline{\approx 25.1\ ft|}$$

Exercise

A policewoman has positioned herself 500 ft. from the intersection of two roads. She has carefully measured the angles of the lines of sight to points A and B. If a car passes from A to B is 1.75 sec and the speed limit is 55 mph, is the car speeding? (Hint: Find the distance from B to A and use R = D/T)

Solution



$$\tan 12.3° = \frac{b}{500} \Rightarrow b = 500\tan 12.3°$$

$$\tan 15.4° = \frac{b+a}{500} \Rightarrow b + a = 500\tan 15.4°$$

$$\begin{aligned} \Rightarrow a &= 500\tan 15.4° - b \\ &= 500\tan 15.4° - 500\tan 12.3° \\ &= 28.7\ ft \frac{1\ mi}{5280\ ft} \\ &\approx 0.0054356\ mi \end{aligned}$$

The speed is: $= 0.0054356\ mi \frac{1}{1.75\ \text{sec}} \frac{3600\ \text{sec}}{1\ hr}$ $\boxed{= 11.2\ mph}$

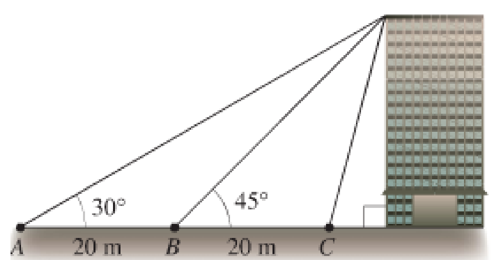
$\Rightarrow$ The car is not speeding.

Exercise

From point A the angle of elevation to the top of the building is 30°. From point B, 20 meters closer to the building, the angle of elevation is 45°. Find the angle of elevation of the building from point C, which is another 20 meters closer to the building.

Solution

Let x be the distance between C and the building.



$$\tan 30° = \frac{h}{40+x} \Rightarrow h = (40+x)\tan 30° = (40+x)\frac{1}{\sqrt{3}}$$

$$\tan 45° = \frac{h}{20+x} \Rightarrow h = (20+x)\tan 45° = (20+x)(1)$$

$$\Rightarrow h = \frac{1}{\sqrt{3}}(40+x) = 20 + x$$

$$40 + x = 20\sqrt{3} + x\sqrt{3}$$

$$x - x\sqrt{3} = 20\sqrt{3} - 40$$

$$x\left(1-\sqrt{3}\right) = 20\sqrt{3} - 40$$

$$x = \frac{20\sqrt{3}-40}{1-\sqrt{3}} \approx 7.32$$

$$\Rightarrow h = (40+7.32)\frac{1}{\sqrt{3}} \approx 27.32$$

$$\tan C = \frac{h}{x} = \frac{27.32}{7.32} \qquad \Rightarrow \boxed{C = \tan^{-1}\left(\frac{27.32}{7.32}\right) \approx 75°}$$

Exercise

A hot air balloon is rising upward from the earth at a constant rate. An observer 250 m away spots the balloon at an angle of elevation of 24°. Two minutes later the angle of elevation of the balloon is 58°. At what rate is the balloon ascending?

Solution

$$\tan 24° = \frac{h_1}{250} \to h_1 = 250\tan 24°$$

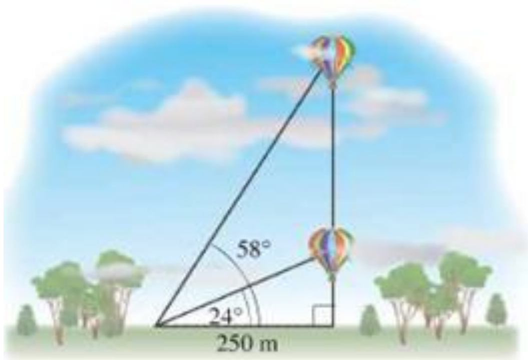
$$\tan 58° = \frac{h_2}{250} \to h_2 = 250\tan 58°$$

It took 2 minutes to get from h_1 *to* h_2

$$rate = \frac{h_2 - h_1}{2}$$

$$= \frac{250\tan 58° - 250\tan 24°}{2}$$

$$\approx 144.4 \quad m/\min$$



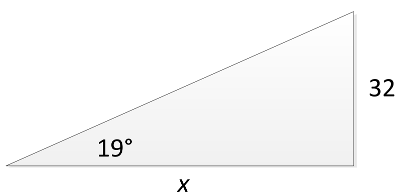
Exercise

A skateboarder wishes to build a jump ramp that is inclined at a 19° angle and that has a maximum height of 32.0 inches. Find the horizontal width x of the ramp.

Solution

$$\tan 19° = \frac{32}{x}$$

$$x = \frac{32}{\tan 19°} = 92.9 \ in$$



Exercise

For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 6 ft from the piece of art and that the angle of depression of the light be 38°. How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 4 inches from the wall.

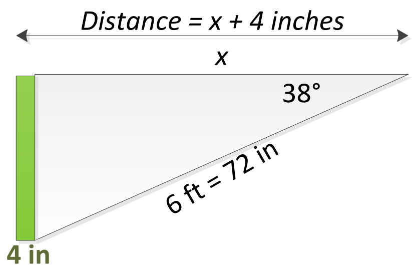
Solution

$$\cos 38° = \frac{x}{6}$$

$$x = 6\cos 38° = 4.7 \ feet$$

$$distance = 4.7\ ft \frac{12\ in}{1\ ft} + 4in = 60.7\ in$$

$$distance = \frac{60.7}{12} = 5.1 \ \ ft$$



Exercise

A surveyor determines that the angle of elevation from a transit to the top of a building is 27.8°. The transit is positioned 5.5 feet above ground level and 131 feet from the building. Find the height of the building to the nearest tenth of a foot.

Solution

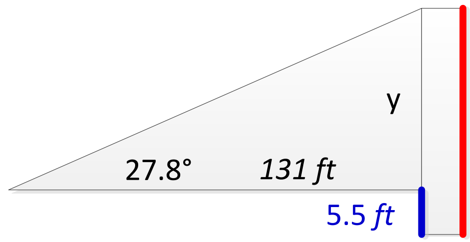
$$\tan 27.8^\circ = \frac{y}{131}$$

$$y = 131 \tan 27.8^\circ$$

$$h = y + 5.5$$

$$= 131 \tan 27.8^\circ + 5.5$$

$$= 74.6 \ \ ft$$



Exercise

From a point A on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is 42.0°. From a point 100 ft away from A and on the same line, the angle to the top is 37.8°. Find the height, to the nearest foot, of the Monument.

Solution

Triangle ACB: $\tan 37.8^\circ = \dfrac{h}{x+100} \Rightarrow h = (x+100)\tan 37.8^\circ$

Triangle DCB: $\tan 42^\circ = \dfrac{h}{x} \Rightarrow h = x \tan 42^\circ$

$$\Rightarrow h = x \tan 42^\circ = (x+100)\tan 37.8^\circ$$

$$x \tan 42^\circ = x \tan 37.8^\circ + 100 \tan 37.8^\circ$$

$$x \tan 42^\circ - x \tan 37.8^\circ = 100 \tan 37.8^\circ$$

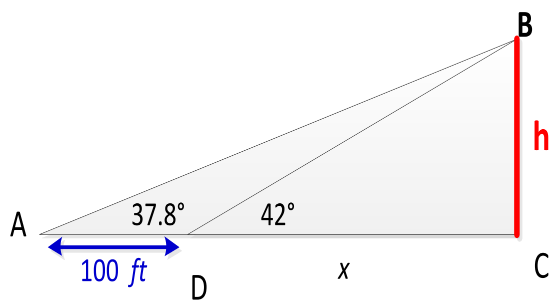
$$x(\tan 42^\circ - \tan 37.8^\circ) = 100 \tan 37.8^\circ$$

$$x = \frac{100 \tan 37.8^\circ}{\tan 42^\circ - \tan 37.8^\circ}$$

$$\Rightarrow \boldsymbol{h} = x \tan 42^\circ$$

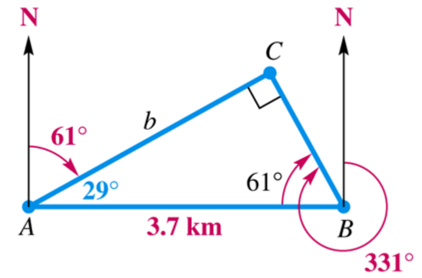
$$= \frac{100 \tan 37.8^\circ}{\tan 42^\circ - \tan 37.8^\circ} \tan 42^\circ$$

$$= 560 \ \ ft$$



Exercise

Radar stations A and B are on the east-west line, 3.7 km apart. Station A detects a place at C, on a bearing of 61°. Station B simultaneously detects the same plane, on a bearing of 331°. Find the distance from A to C.



Solution

$$\underline{|A = 90° - 61° = 29°|}$$

$$\cos 29° = \frac{b}{3.7}$$

$$b = 3.7\cos 29° \approx 3.2\ km$$

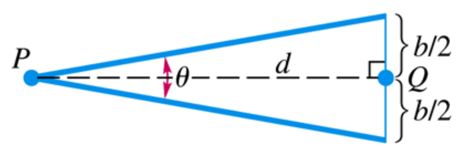
Exercise

A method that surveyors use to determine a small distance d between two points P and Q is called the ***subtense bar method***. The subtense bar with length b is centered at Q and situated perpendicular to the line of sight between P and Q. Angle θ is measured, then the distance d can be determined.

a) Find d with $\theta = 1°\ 23'\ 12''$ and $b = 2.000\ cm$

b) Angle θ usually cannot be measured more accurately than to the nearest 1″. How much change would there be in the value of d if θ were measured 1″ larger?

Solution



a) $\cot\frac{\theta}{2} = \frac{d}{b/2}$

$$d = \frac{b}{2}\cot\frac{\theta}{2}$$

$$\begin{aligned}\theta &= 1°\ 23'\ 12'' \\ &= 1° + \frac{23°}{60} + \frac{12°}{3600} \\ &\approx 1.38667°\end{aligned}$$

$$\begin{aligned}d &= \frac{2}{2}\cot\frac{1.38667°}{2} \\ &\approx 82.6341\ cm\end{aligned}$$

b) $\theta = 1°\ 23'\ 12'' + 1'' = 1°\ 23'\ 13'' \approx 1.386944°$

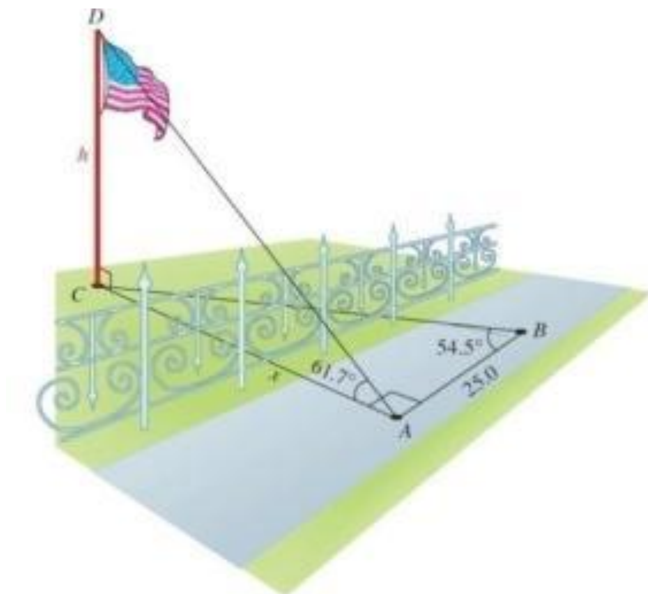
$$d = \frac{2}{2}\cot\frac{1.386944°}{2} \approx 82.617558\ cm$$

The change is: $82.6341 - 82.6175 \approx 0.0166\ cm$

Exercise

A diagram that shows how Diane estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way. So she stands at point *A* facing the pole and finds the angle of elevation from point *A* to the top of the pole to be 61.7°. Then she turns 90° and walks 25.0 ft to point *B*, where she measures the angle between her path and a line from *B* to the base of the pole. She finds that angle is 54.5°. Use this information to find the height of the pole.

Solution



$$\tan 54.5° = \frac{x}{25.0}$$

$$x = 25.0 \tan 54.5°$$

$$= 35.0487\ ft$$

$$\tan 61.7° = \frac{h}{35.0487}$$

$$h = 35.0487 \tan 61.7°$$

$$\boxed{= 65.1\ ft}$$