Review

December-09-13 5:42 PM

FORMULAS

Rotatation formulas

$\theta = \frac{a}{r}$ $\omega = \frac{\theta}{t}$ $v = \frac{a}{t}$ $\omega = \frac{v}{r}$ $A = \frac{1}{2}r^2\theta$

Conversion Ratios = all equal ONE

$\frac{\pi}{180^\circ}$ $\frac{360^\circ}{1rev}$ $\frac{1rev}{2\pi}$ $\frac{100cm}{1m}$ $\frac{1000m}{1km}$ $\frac{60\,\text{sec}}{1\,\text{min}}$ $\frac{60\,\text{min}}{1\,\text{hr}}$ $\frac{12\,\text{in}}{1\,\text{ft}}$ $\frac{5280\,\text{ft}}{1\,\text{mi}}$ $\frac{3.28084\,\text{ft}}{1\,\text{m}}$ $\frac{1.600344\,\text{km}}{1\,\text{mi}}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Double Angle

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Sum/Difference (Compound)

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

don't forget there are also many variations of the following:

- ODD + EVEN Symmetry
- COFUNCTION id
- similar in shape
- shift by period
- CAST symmetry

Half-Angle Formulas

$\sin^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$

(1.) State equivalent trig expressions for

$\tan\frac{15\pi}{9}$

a) using odd/even symmetry

b) using cofunction id.

c) using similar shape

d) using CAST

e) using shift of period

f) repeat a)-e) for $\sec\left(\frac{6\pi}{5}\right)$

2. Find the exact values of each of the following. Justify what you use for your answers.

a) $\sin\frac{5\pi}{3}$

b) $\csc\frac{3\pi}{4}$

c) $\sec\frac{7\pi}{2}$

d) $\cos\frac{5\pi}{12}$

e) $\tan\frac{5\pi}{8}$

3. Determine the solutions for each equation where $0 \le x \le 2\pi$.

a) $1 + \frac{17}{3}\tan x = -\frac{14}{3}$

b) $\sin x + 2 + \frac{\csc x}{10} = \frac{9}{5} + \sin x$

4. Determine the solutions for each equation where $0 \le x \le 2\pi$ by using a double-angle formula to transform the equation into a linear trigonometric equation, if necessary.

a) $2\sin x\cos x + \frac{1}{2} = 1$

b) $\cos^2 x - \sin^2 x + \tan^2 x - 3 = \tan^2 x - 2$

5. Use factoring to solve each of the following equations where $0 \le x \le 2\pi$.

a) $\tan^2 x - \sqrt{3}\tan x = 0$

b) $\sin^2 x - \frac{1}{4} = 0$

c) $\cos^2 x + 2\cos x + 1 = 0$

d) $\sin 2\theta = \tan 2\theta$

e) $\sec^2\theta + \tan\theta = 1$

f) $2\sin^2\theta = 2 + \cos\theta$

Solve for $x \in (-\infty, \infty)$ in Radians

6. Write each of the following expressions as a single trigonometric function.

a) $\sin 16^\circ \cos 99^\circ - \cos 16^\circ \sin 99^\circ$

b) $\dfrac{\tan\frac{\pi}{18} + \tan\frac{2\pi}{9}}{1 - \tan\frac{\pi}{18}\tan\frac{2\pi}{9}}$

c) $\sin\frac{13\pi}{20}\cos\frac{\pi}{5} + \cos\frac{13\pi}{20}\sin\frac{\pi}{5}$

d) $\cos 88^\circ \cos 9^\circ - \sin 88^\circ \sin 9^\circ$

7. Prove

a) $\sin^3 x + \cos^3 x = (1 - \sin x \cos x)(\sin x + \cos x)$

b) $\dfrac{\cos 2x}{1 + \sin 2x} = \tan\left(\dfrac{\pi}{4} - x\right)$

c) $\sin 8x = 8 \sin x \cos x \cos 2x \cos 4x$

d) $\sin x = 1 - 2\sin^2\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

e) $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

f) $\cos\left(\dfrac{\pi}{12} - x\right)\sec\dfrac{\pi}{12} - \sin\left(\dfrac{\pi}{12} - x\right)\csc\dfrac{\pi}{12} = 4\sin x$

8. The angle α lies in quadrant II, and angle β lies in quadrant III, if $\tan\alpha = \dfrac{-2}{7}$ and $\cos\beta = \dfrac{-1}{3}$, find

a) $\sin\alpha$, $\sin\beta$ and $\cos\alpha$

b) $\sin(\alpha - \beta)$

c) $\tan 2\alpha$

d) $\cos\dfrac{\beta}{2}$

9. The cable lifting a garage door turns around a pulley at a rate of 20 cm per second. How long will it take to lift the door 2.2 meters?

10. A wind turbine has three blades, each measuring 3 m from centre to tip. At a particular time, the turbine is rotating four times a minute.
   a) Determine the angular velocity of the turbine in radians/second.
   b) How far has the tip of a blade travelled after 5 min?

11. If the sector formed by a central angle of 15° has an area of $\frac{\pi}{3}\text{cm}^2$, find the radius of a circle.

12. When Lance Armstrong blazed up Mount Ventoux in the 2002 tour, he was equipped with a 150-millimeter-diameter chainring and a 95-millimeter-diameter sprocket. Lance is known for maintaining a very high cadence, or pedal rate. The sprocket and rear wheel rotate at the same rate, and the diameter of the rear wheel is 700 mm. If he was pedaling at a rate of 90 revolutions per minute, find his speed in kilometers per hour. (1 km = 1,000,000 mm or 10^6 mm)