PreCalculus UNIT 8

TRIG IDENTITES (MHF) – journal questions

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

Read over your old journals before you answer the following:

- *UNIT A part 3 about radicals and UNIT A part 2 about factoring smallest exponent out ,*
- *UNIT G Rotational Trig journal about special triangles*
- *UNIT 4 Trig Ratios about radians and trig identities*
- *UNIT 5 Graphing Trig journal about transformations of trig functions, sinusoidal word problems, and about inverse trig graphs and simplifications using triangles*

1.

a. ARC LENGTH $a = \theta r$

A pendulum with length of 1.4 meters swings through an angle of 30º. How far does the bob at the end of the pendulum travel as it goes from left to right?

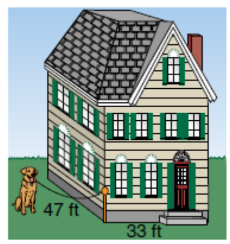
b. SECTOR AREA $A = \frac{1}{2} r^2 \theta$ explain where formula comes from. Make a note that in both equations above θ must be in radians, which is not a unit but a ratio

c.

A rectangular house is 33 feet by 47 feet.
A dog is placed on a leash that is connected to a pole at the corner of the house.

i) If the leash is 15 feet long, find the area the dog has to play.

ii) If the owner wants the dog to have 750 square feet to play, how long should the owner make the leash?



2. LINEAR and ANGULAR Speed Word Problems

a. Copy down the following information into your journal

$t =$ time (sec, min, hr)
$a =$ arc length (mm, cm, m, km)
$\theta =$ angle (degrees, revolutions, radians)
$r =$ radius or distance from the centre of rotation (cm, in, m, ...)
$v =$ linear velocity/speed (cm/sec, km/hr, ...)
$\omega =$ angular velocity/speed (°/sec, rev/sec, rad/sec, rpm, ...)

$$\theta = \frac{a}{r} \qquad \omega = \frac{\theta}{t} \qquad v = \frac{a}{t} \qquad \omega = \frac{v}{r}$$

$$\frac{2\pi}{360^o} \quad \frac{360^o}{1\text{ rev}} \quad \frac{1\text{ rev}}{2\pi} \quad \frac{100\text{ cm}}{1\text{ m}} \quad \frac{1000\text{ m}}{1\text{ km}} \quad \frac{60\text{ sec}}{1\text{ min}} \quad \frac{60\text{ min}}{1\text{ hr}} \quad \frac{12\text{ in}}{1\text{ ft}} \quad \frac{1\text{ mi}}{5280\text{ ft}} \quad \frac{3.28\text{ ft}}{1\text{ m}} \quad \frac{1.6\text{ km}}{1\text{ mi}}$$

b. The tires on a race car have a diameter of 30 inches. If the tires are turning at a rate of 2000 revolutions per minute, determine the race car's speed in miles per hour (mph).

c. The Kit Carson County Carousel makes 3 revolutions per minute.

i. Find the linear velocity in feet per second of someone riding a horse that is 22.5 feet from the center.

ii. The linear velocity of the person on the inside of the carousel is 3.1 feet per second. How far is the person from the center of the carousel?

3. Review and apply past trig topics:

a. Find the sine of the angle formed by two rays that start at the origin of the Cartesian plane if one ray passes through the point $(3\sqrt{3}, 3)$ and the other ray passes through the point $(-4, 4\sqrt{3})$. Round your answer to the nearest hundredth, if necessary.	b. The terminal arm of θ is in the fourth quadrant. If $\cot\theta = -\sqrt{3}$, then calculate $\sin\theta\cot\theta - \cos^2\theta$.
c. A clock is hanging on a wall, with the centre of the clock 3 m above the floor. Both the minute hand and the second hand are 15 cm long. The hour hand is 8 cm long. For each hand, determine the equation of the cosine function that describes the distance of the tip of the hand above the floor as a function time. Assume that the time, t, is in minutes and that the distance, D(t), is in centimetres. Also assume that t = 0 is midnight.	d. SKETCH INVERSE TRIG with transformations, explain (AP) $y = \arccos(0.5x) - \pi$

4. IDENTITIES

a. Copy the following into your journal.

COMPOUND angle identities (ADD/SUBTRACTION id):

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

DOUBLE or HALF angle identities:

$$\sin(2A) = 2\sin A \cos A$$
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$
$$\cos(2A) = \cos^2 A - \sin^2 A$$
$$\cos(2A) = 1 - 2\sin^2 A$$
→ **same as** $\boxed{\sin^2 A = \frac{1-\cos(2A)}{2}}$ or $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$
$$\cos(2A) = 2\cos^2 A - 1$$
→ **same as** $\boxed{\cos^2 A = \frac{1+\cos(2A)}{2}}$

b. OTHER identities - add in the following from class notes + include examples that use these

- Odd/even symmetry identities
- Cofunction identities
- Translation of period multiples
- Shift of a similar type of graph
- Same related acute angle

c. α and β are acute angles in quadrant I, with $\sin\alpha = \frac{7}{25}$ and $\cos\beta = \frac{5}{13}$. Without using a calculator, determine the values of i) $\sin(\alpha+\beta)$ ii) $\tan(\alpha+\beta)$.	d. The angle x lies in the interval $\frac{\pi}{2} \le x \le \pi$, and $\sin^2 x = \frac{8}{9}$. Without using a calculator, determine the value of i) $\sin 2x$ ii) $\cos\frac{x}{2}$

5. PROOFS

a. List strategies of things to try when doing proofs

Prove, explain steps

b. $\dfrac{1-2\sin^2 x}{\cos x + \sin x} + 2\sin\dfrac{x}{2}\cos\dfrac{x}{2} = \cos x.$	c. $\sec 2t = \dfrac{\csc t}{\csc t - 2\sin t}$

6. SOLVE TRIG EQUATIONS. Explain and give general solutions written as sequences.

a. $\sin 3x = -\dfrac{\sqrt{3}}{2}$	b. $\tan^2 x \sin x - \dfrac{\sin x}{3} = 0$

c. The quadratic trigonometric equation $a\cos^2 x + b\cos x - 1 = 0$ has the solutions $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$ in the interval $0 \le x \le 2\pi$. What are the values of a and b?