Big idea

You have already been exposed to radians. Recall that just like length can be measured in meters or yards, angles can be measured using different units of measure (degrees, radians, revolutions). Radian measure, if you remember, actually has no units at all. Radian measure is used in calculation of arc-length, area sector of a circle, and angular speed. In this unit you will review the major topics you learned last semester, and then learn about some new identities. These identities will allow you to do more complicated proofs and you will also be able to find exact values of angles that are NOT directly related to the special triangles. Use of proper notation is key to minimize errors as well as to get full marks. Finally you will learn how to solve more complicated trig equations that involve factoring.

Feedback & Assessment of Your Success

Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: ____________________ Questions to ask the teacher:
0.5day	2-3	Arc Length and Sector Area (MHF) Journal #1				
1.5days	4-7	Linear and Angular Speed (MHF) Journal #2				
	8-11	Review Algebra, Special Triangles and Rotational Trig (MHF)				
	12-15	Review Sketching Trig, Word Problems (MHF) & Inverse Trig (AP) Journal #3				
2days	16-21	Identities and Exact Values (MHF) Journal #4				
	22-23	Proofs (MHF) Journal #5				
2days	24-29	Solve Trig Equations (MHF) Journal #6				

ASSIGNMENT Arc Length & Sector Area (MHF)

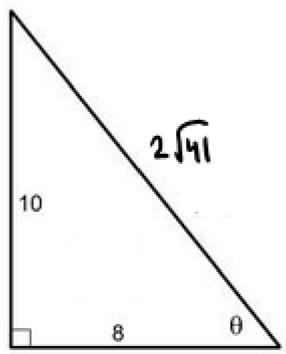
Convert radians to degrees or vice versa

1. $\frac{\pi}{24}$

2. $240°$

Find the exact values of the 6 trig ratios

3.

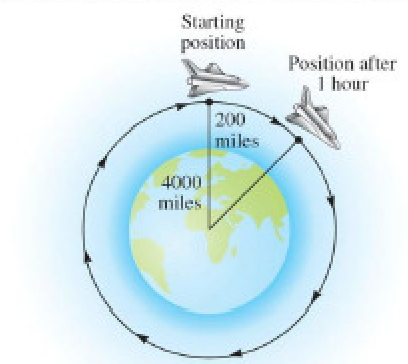


4. $\sin\theta = -\frac{1}{3}$

5. The minute hand of a clock is 1.2 cm long. How far does the tip of the minute hand travel in 40 minutes?

6. Find the radian measure if angle θ, if θ is a central angle in a circle of radius $r = 4$ inches, and θ cuts off an arc of length $s = 12\pi$ inches.

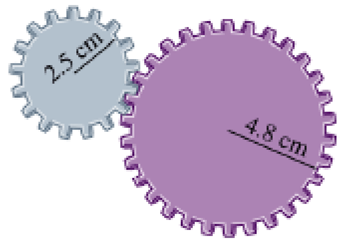
7. A space shuttle 200 miles above the earth is orbiting the earth once every 6 hours. How long, in hours, does it take the space shuttle to travel 8,400 miles? (Assume the radius of the earth is 4,000 miles.) Give both the exact value and an approximate value for your answer.



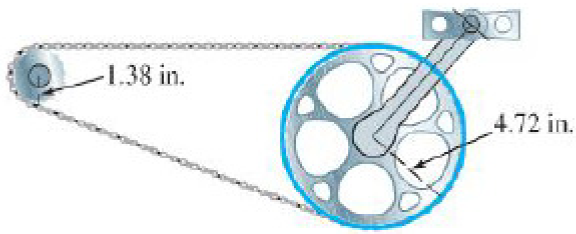
8. The pendulum on a grandfather clock swings from side to side once every second. If the length of the pendulum is 4 feet and the angle through which it swings is 20°. Find the total distance traveled in 1 minute by the tip of the pendulum on the grandfather clock.

one arclength in 1 sec

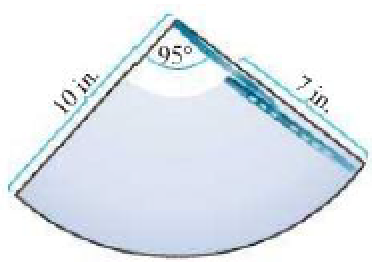
9. Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of 225°, through how many degrees will the larger gear rotate?



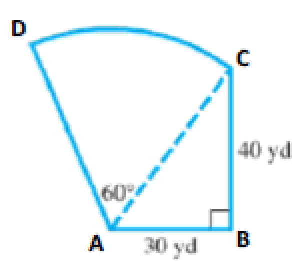
10. The figure shows the chain drive of a bicycle. How far will the bicycle move if the pedals are rotated through 180°? Assume the radius of the bicycle wheel is 13.6 in.



11. The total arm and blade of a single windshield wiper was 10 in. long and rotated back and forth through an angle of 95°. The shaded region in the figure is the portion of the windshield cleaned by the 7-in. wiper blade. What is the area of the region cleaned?



12. A frequent problem in surveying city lots and rural lands adjacent to curves of highways and railways is that of finding the area when one or more of the boundary lines is the arc of the circle. Find the area of the lot.



ASSIGNMENT Linear and Angular Speed (MHF)

1. A wheel with a 15-inch diameter rotates at a rate of 6 radians per second. What is the linear speed of a point on its rim in feet per minute?

2. An earth satellite in circular orbit 1200 km high makes one complete revolution every 90 minutes. What is its linear speed in km/min, given that the earth's radius is 6400 km?

3. Through how many radians does the minute hand of a clock rotate from 12:45pm to 1:25pm?

4. A car travels at 60 miles per hour. Its wheels have a 24-inch diameter. What is the angular speed of a point on the rim of a wheel in revolutions per minute?

5. A car travels 3 miles. Its tires make 2640 revolutions. What is the radius of a tire in inches?

6. A satellite 290 miles above Mars' surface makes one revolution every 2 hours. What is its linear speed in miles per hour, given that the radius of Mars is 2110 miles?

7. A cylinder with a 2.5 ft radius is rotating at 120 rpm.
(a) Give the angular velocity in rad/sec and in degrees per second.
(b) Find the linear velocity of a point on its rim in mph.

8. A tire with a 9 inch radius is rotating at 30 mph. Find the angular velocity of a point on its rim. Express the result in radians per minute and revolutions per minute.

9. The radius of Earth is approximately 6400 km. It takes 23h 56m 4.1s for the Earth to rotate once on its axis. Find:
a) angular speed of Earth in radians per day and radians per hour.
b) linear speed at the North or South Pole.
c) linear speed at Quito, Ecuador, a city on the equator.
d) linear speed at Salem, Oregon (halfway from the equator to the North Pole)

10. Suppose that a point P is on a circle with center at the origin, O, with radius $r = 20cm$, and ray $\overrightarrow{OP}$ is rotating with angular speed $\omega = \frac{\pi}{12}$ radians per sec. Find
a) the angle generated by P in 6 seconds.
b) the distance traveled by P along the circle in 6 seconds.
c) the linear speed of P.

11. Find the angular velocity for each of the following, all in radians:
a) the hour hand of a clock.
b) the minute hand of a clock.
c) a line from the center to the edge of a CD revolving 300 times per minute.

12. A cylinder with a 2 ft radius is spinning at 450 rpm.
(a) Find its angular velocity in degrees per sec.
(b) Find the linear speed on the rim in mph.

13. Jupiter rotates in approximately 9h 50m. Its radius is 11.194173 times that of Earth's. Find the linear velocity at the equator of Jupiter.

14. A pulley has a 48-inch diameter, and moves a belt at a rate of 8 miles per hour. What is the angular speed of a point on the edge of the pulley in revolutions per minute?

15. If a cylinder with a 6 in. radius is spinning at 24 mph, find the angular velocity in rpm of a point on its rim.

16. The angle of depression to the bottom of a slide is 60 degrees. If a child slides down at a rate of 5 feet per second and it takes 2 seconds for the child to reach the bottom, what is the vertical height of the slide, in feet?

17. A circle has a radius of 3 feet. What is the measure, in degrees, of an angle that subtends an arc of 4 inches?

18. A tire has a radius of $\frac{\pi}{5}$ meters. The tire is rolled and travels a total distance of 28π meters. By the time the tire stops what is the angle that it rolled through?

19. A tire rolls $\frac{8}{9}\pi$ meters while turning 240°. Determine the area of the wheel.

ASSIGNMENT Review Algebra, Special Triangles & Rotational Trig (MHF)

Rewrite in simpler form

1. $\sqrt{1200}$

2. $\sqrt{18}+\sqrt{108}+\sqrt{50}+\sqrt{48}$

3. $\sqrt{x^7b^9}$

4. $\dfrac{3\sqrt{6}}{4\sqrt{10}}$

5. $\sqrt[4]{\dfrac{t^{1/2}\sqrt{st}}{s^{2/3}}}$

6. $\dfrac{a^2b^4}{\sqrt{a^5b^9}}$

7. $\dfrac{2\sqrt{6}}{2\sqrt{6}+1}$

8. $\dfrac{2\sqrt{7}+\sqrt{5}}{3\sqrt{7}-2\sqrt{5}}$

9. $\left(\dfrac{3x^{-3}y^{-2}}{x^{-2}y^{-4}}\right)^2$

10. $(x^{-5}y^3z^{10})^{-3/5}$

Rewrite in simpler form (note factored form is better than expanded form for solving or sketching, so factor if possible)

11. $\left(\frac{32r^2}{2s^4}\right)^{\frac{1}{4}}$

12. $\dfrac{\left(x^{-2/3}y\right)^{-2}}{\left(x^{1/2}y^{-1}\right)\left(x^2y^{3/2}\right)}$

13. $((\frac{1}{5})^{-1}+(\frac{1}{5})^{-3})^{-1}$

14. $(\frac{1}{a}+\frac{1}{b})^{-1}$

15. $3x^{1/3}-2x^{-2/3}$

16. $\dfrac{x^5+x^{-2}}{x^{-3}}$

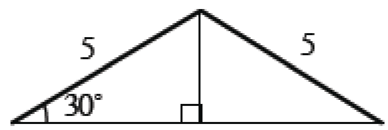
17. $-3(x-1)^{-1}+4(x-1)^{-2}$

18. $3(x+1)^{-1/2}+x(x+1)^{-3/2}$

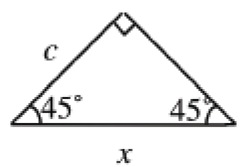
19. Given a 30-60-90 triangle with hypotenuse length 8, find the exact length of the long leg.

20. If a 45-45-90 triangle's legs are 17 mm long, exactly how long is its hypotenuse?

21. Find the exact area of the following isosceles triangle.



22. Solve for length x in terms of c.



Find the ratios stated after the semi-colon

23. $\tan\theta = -\frac{4}{7}, 270^\circ < \theta < 360^\circ; \sec\theta$

24. $\csc\theta = \frac{\sqrt{11}}{3}, \frac{\pi}{2} < \theta < \pi; \cot\theta$

Evaluate the following without a calculator

25. $\sin\frac{5\pi}{3}$

26. $\csc\frac{3\pi}{2}$

27. $\cos\frac{5\pi}{4}$

28. $\tan\frac{14\pi}{3}$

29. $\csc(-330°)$

Find all angles that satisfy the following equations (do left column in degrees, and right column in radians)

30. $\sin x = -\frac{\sqrt{2}}{2}$

31. $\cos x = 0$

32. $\cot\theta = -\sqrt{3}$

33. $\sec\theta = -2$

34. $\tan x = -3.284$ approximate to one decimal place

35. $\csc\theta = \frac{7}{5}$ approximate to one decimal place

ASSIGNMENT Review Sketching Trig & Word Problems(MHF) & Inverse Trig (AP)

A For each of the following equation state their domain, range and period. then sketch each function for one cycle

1. $y = 2\cos(3x - 4\pi) + 1$

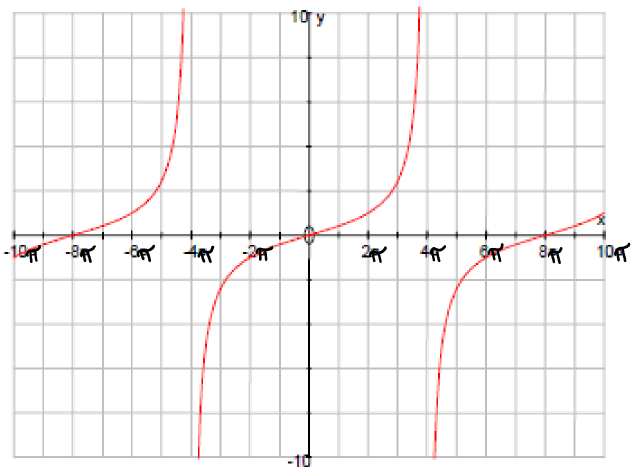
2. $y = 5 + 2\sec(\frac{1}{2}x)$

3. $y = -\csc(\frac{\pi}{3}x + \pi) - 4$

4. $y = 3\tan(\frac{1}{3}x - 2\pi) - 2$

State equations for each of the following graphs: Do two different trig types

5.



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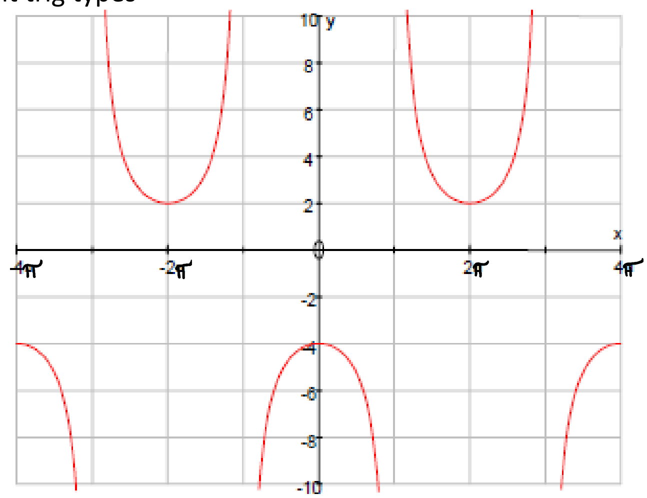

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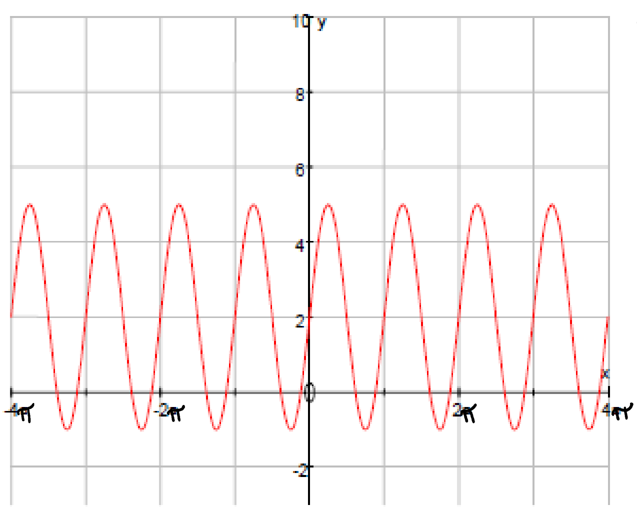

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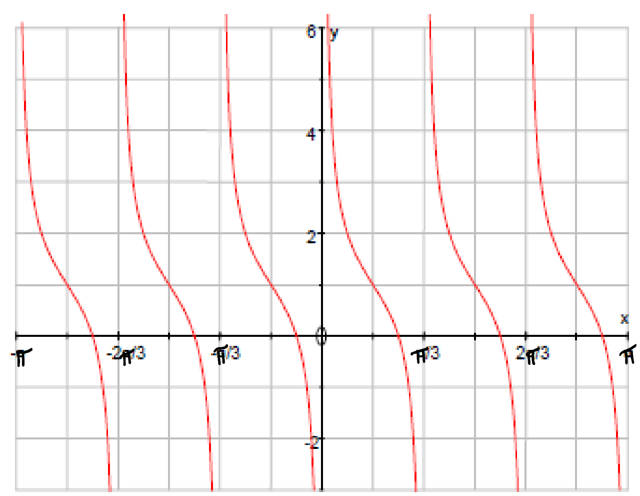


9. The swimming motion of an orca whale can be modeled using the sinusoidal function $h(t) = 5\cos(\frac{\pi}{5}(t-1)) - 3$, where h(t) is the height of the whale, in metres, above the water and t is the time, in seconds, that has elapsed since the Ralph first sighted the whale.

(a) Is the whale visible above the water as it moves through the water. Explain.
(b) If you are on a boat with Ralph and you sighted the whale at the same time that he did, for what length of time would the whale be visible to you?
(c) How long would you have to wait before you would see the whale again?

10. When you board a Ferris wheel your feet are 1 foot off the ground. At the highest point of the ride, your feet are 99 feet above the ground. Your feet move at 10feet/sec

a) Write a trigonometric equation for your height above the ground at t seconds after the ride starts.
b) Find at what two times within one cycle you are exactly at 90 feet off the ground.

B

Determine exact values for each of the following.

11. $\sin(Arc\tan\frac{4}{5})$

12. $\sec(Sin^{-1}(\frac{-3}{5}))$

13. $\csc(Arc\sin(\frac{12}{13}))$

14. $\arccos\left(\cos\frac{-\pi}{3}\right)$

15. $\sin(\arctan x)$

16. $\sec\left(\sin^{-1}\frac{x}{\sqrt{x^2+4}}\right)$

17. Sketch the graph of the equation: $y=\sin^{-1}(x-2)+\frac{\pi}{2}$

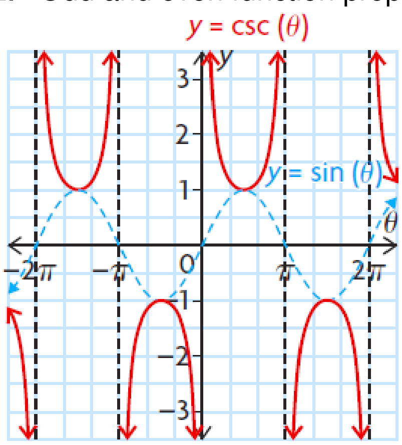
ASSIGNMENT Identities & Exact Values (MHF)

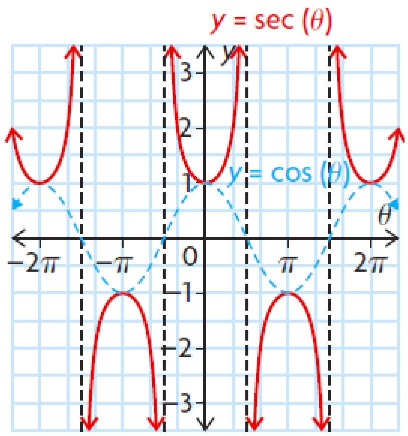
1. Clarify for yourself the difference between the following three words, use examples in your explanations.

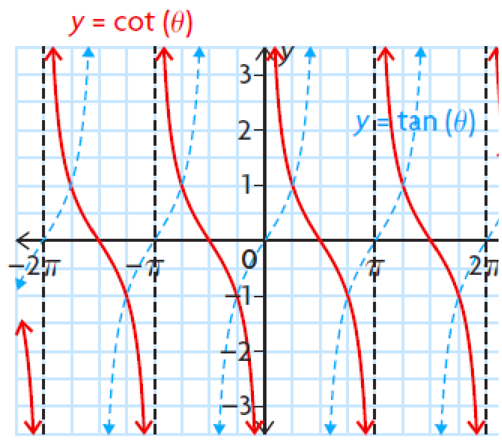
Expression Equation Identity

Understand how to use all the identities (gouped by topic) ALL formulas are found in journal

2. Odd and even function properties



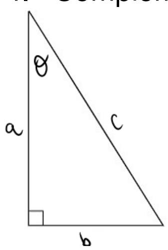




3. Practice writing the equivalent expressions using "symmetry" for the following

i) $\sin\left(-\frac{\pi}{6}\right)$ ii) $\sec\left(\frac{\pi}{2}\right)$ iii) $\cos(-A-B)$

4. Complementary angles property = Cofunction identities



5. Practice writing the equivalent expressions using "cofunction identities" for the following

i) $\sin\left(\frac{\pi}{6}\right)$ ii) $\sec\left(\frac{\pi}{8}\right)$ iii) $\cot\left(2x+\frac{\pi}{4}\right)$

6. Horizontal translations of period multiples

7. Practice writing the equivalent expressions using “translation of period multiple” for the following

i) $\cos\left(\frac{7\pi}{3}\right)$ ii) $\csc\left(\frac{19\pi}{8}\right)$ iii) $\tan\left(x-4\pi\right)$

d. Similar shape of the graph just with a shift

8. Practice writing the equivalent expressions using “shift of a similar type of graph” for the following

i) $\sin\left(\frac{7\pi}{6}\right)$ ii) $\sec\left(\frac{4\pi}{5}\right)$ iii) $\cot\left(2x\right)$

9. Principal and related acute angle characteristics

10. Practice writing the equivalent expressions using “same related acute angle” for the following

i) $\sin\left(\frac{11\pi}{6}\right)$ ii) $\cos\left(\frac{5\pi}{7}\right)$ iii) $\tan\left(-\frac{\pi}{4}\right)$

11. Write down several equivalent expressions for the following, identify what property was used for each

a) $\sin\frac{5\pi}{6}$ b) $\sec\frac{3\pi}{4}$

Write each of the following as a single trigonometric function.

12. $\cos a \cos 30° - \sin a \sin 30°$

13. $\sin(4t)\cos(2t) - \cos(4t)\sin(2t)$

14. $\dfrac{\tan(3d) + \tan(5d)}{1 - \tan(3d)\tan(5d)}$

15. $2\cos^2(5x) - 1$

16. $2\sin\frac{x}{2}\cos\frac{x}{2}$

17. $\sin 5p \cos 3p + \cos 5p \sin 3p$

Evaluate the following without your calculator

18. $1 - 2\sin^2(\frac{\pi}{12})$

19. $2\sin(\frac{\pi}{12})\cos(\frac{\pi}{12})$

20. $\dfrac{2\tan\frac{\pi}{8}}{1 - \tan^2\frac{\pi}{8}}$

21. $\dfrac{\tan\frac{\pi}{8} + \tan\frac{7\pi}{8}}{1 - \tan\frac{\pi}{8}\tan\frac{7\pi}{8}}$

22. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} + \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

23. $\sin 11^o - \cos 79^o$

Determine the exact value for each of the following:

24. $\sin(165^\circ)$

25. $\cos(\frac{13\pi}{12})$

26. $\tan(\frac{7\pi}{12})$

27. $\csc(\frac{11\pi}{12})$

28. If $\tan\theta = -\frac{\sqrt{7}}{3}$ and θ is a second-quadrant angle, find:

a. $\sec\theta$ **b.** $\cos\theta$ **c.** $\sin\theta$

d. $\sin 2\theta$ **e.** $\cos 2\theta$ **f.** $\tan 2\theta$

g. In what quadrant does 2θ lie?

find **a.** $\sin 2\theta$, **b.** $\cos 2\theta$, **c.** $\tan 2\theta$, **d.** the quadrant in which 2θ lies. Show all work.

29. $\sin\theta = -\frac{\sqrt{2}}{3}$ in the third quadrant

30. $\csc\theta = \sqrt{5}$ in the second quadrant

for each value of $\cos A$, find **a.** $\sin\frac{1}{2}A$ **b.** $\cos\frac{1}{2}A$ **c.** $\tan\frac{1}{2}A$. Show all work.

31. $\cos A = -\frac{5}{12}, 90° < A < 180°$

32. $\cos A = \frac{7}{9}, 360° < A < 450°$

33. If $180° < A < 270°$ and $\sin A = -\frac{\sqrt{5}}{3}$, find:

a. $\sin \frac{1}{2}A$ **b.** $\cos \frac{1}{2}A$ **c.** $\tan \frac{1}{2}A$

Determine the exact values for each of the angles:

34. $\tan\frac{5\pi}{6}$

35. $\cos(270°)$

36. $\sin(\frac{7\pi}{12})$

37. $\cos 22.5°$

38. $\sin\frac{9\pi}{8}$

39. $\tan\frac{-\pi}{8}$

ASSIGNMENT Proofs (MHF)

Prove. Do not change the original by moving terms around or cross multiplying. Explain your steps

1. $\frac{1}{\cos^2\theta} + 1 + \frac{\sin^2\theta}{\cos^2\theta} = 2\sec^2\theta$

2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

3. $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

4. $\sin 4x = 2\cot(2x)\sin^2(2x)$

5. $\cot^2 x - \cos^2 x = \cot^2 x\cos^2 x$

Prove. Do not change the original by moving terms around or cross multiplying. Explain your steps

6. $\frac{\sin\theta}{\csc\theta - \cot\theta} = 1 + \cos\theta$

7. $\frac{1+\sin\theta}{\cos\theta} = \frac{\cos\theta}{1-\sin\theta}$

8. $\tan x = \frac{\sin 2x}{1+\cos 2x}$

9. $\sqrt{2}\cos(x - \frac{\pi}{4}) = \cos x + \sin x$

10. $\sin 3x\cos x - \cos 3x\sin x = 2\sin x \sin\left(\frac{\pi}{2} - x\right)$

ASSIGNMENT Solve Trig Equations (MHF)

Solve exactly if possible, if not possible - round to one decimal.

left column solutions in $[0, 2\pi]$ and right column solutions in $[0^o, 360^o]$

1. $\sin\theta = 0.8134$

2. $\cot\theta = 6.6173$

3. $6\cos t - 4\sqrt{3} = 0$

4. $\tan(x) - \sqrt{3} = -2\tan(x)$

5. $4\sin^2(x) + 5 = 8$

6. $\sec(x) + 4 = 6$

Factor the following

1. $2\cos x \sin^2 x + 3\cos x \sin x - 5\cos x$

2. $\cos^4 x - 4$

3. $3\sin^2 x + 5\sin x - 2$

4. $\cos^2 x - \sin^2 y$

5. $\cos^3 x - e^{3x}$

6. $6\cos^2 x + 5\cos x - 4$

7. $125\sin^3 x + 1$

8. $216\tan^3 x - 125$

Solve the following for ALL values of x in $(-\infty, \infty)$, record as a sequence of solutions.
Still do left column in radians, and right column in degrees.

7. $2\cos(x) + \sqrt{3} = 0$

8. $\cos(-2x) + 2 = 3\cos(2x)$

9. $\cot(3x) - 4 = 2\cot(3x) + 7$

10. $\sin(x) = 3 + 2\sin(-x)$

11. $\cos x \csc x + \cos x\sqrt{2} = 0$

12. $3 \sec x \sin x - 2\sqrt{3} \sin x = 0$

Solve for x.

left column solutions in $[0, 2\pi]$ and right column solutions in $[0^o, 360^o]$

13. $\sin 2x - \sqrt{3}\sin x = 0$

14. $\sin 2x + \sqrt{2}\cos x = 0$

15. $2\tan x = \tan 2x$

16. $\cos 2x - 3\sin x = 2$

Solve the following equations for the indicated domain:

9. $6\cos(x+30^\circ)\sin(x+30^\circ)+5=2; \quad x\in[-270^\circ,180^\circ]$

10. $\sin(x-\frac{\pi}{4})=\sin(2x-\frac{\pi}{2}); \quad x\in[0,3\pi]$

Solve the following equations for the indicated domain:

11. $\tan(10x) + \tan 50^\circ = \sqrt{3} - \sqrt{3}\tan(10x)\tan 30^\circ; \quad x \in (0, 90^\circ)$

12. $\sin(6x) = \cos(3x); \quad x \in [\frac{\pi}{4}, \frac{3\pi}{4}]$

Solve the following equations for the indicated domain:

13. $3-3\sin(4x-12^{\circ})=2\cos^{2}(4x-12^{\circ});\quad x\in[0^{\circ},270^{\circ}]$

14. $\sin(2x)=\sqrt{2}\cos x;\quad x\in(-\pi,2\pi)$