Big idea

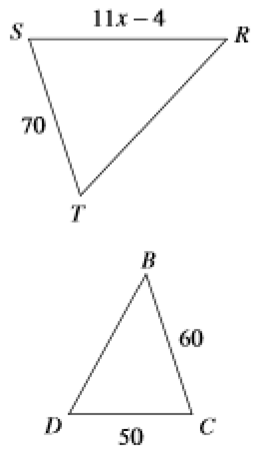
In this unit you will work review how to work in degrees and also begin learning about radian measure. You will then learn the differences between a trigonometric equation and a trigonometric identity. You will practice proving different trigonometric identities. Finally you will review Sine and Cosine laws that you've studied in grade 10 and see that there exist ambiguous solutions. You will learn how to interpret the results from the calculations and decide on the proper answer.

Feedback & Assessment of Your Success

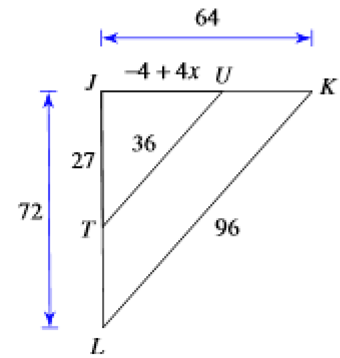
Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: ____________________ Questions to ask the teacher:
2days	2-8	Review Trigonometry (MCR) Journal #1, 2				
	9-11	Radians (MHF) Journal #3				
	12-14	Working in Radians (MHF) Journal #4				
1.5days	15-17	Proving Trig Identities (MCR) Journal #5				
1.5days	18-21	Ambiguous Case (MCR) Journal #6				
	22-23	Bearings (AP) Journal #7				

ASSIGNMENT Review of Trig Topics (MCR)

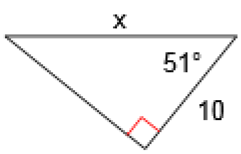
Solve for x, the triangles are similar

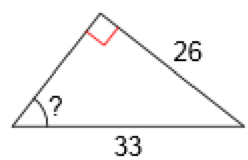
1. 

ΔSRT ~ ΔCBD

2. 

Find the missing measure

3. 

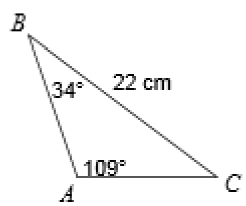
4. 

5. An airplane is flying at an altitude of 6000 m over the ocean directly toward a coastline. At a certain time, the angle of depression to the coastline from the airplane is 14°. How much farther (to the nearest kilometer) does the airplane have to fly before it is directly above the coastline?

6. From a horizontal distance of 80.0 m, the angle of elevation to the top of a flagpole is 18°. Calculate the height of the flagpole to the nearest tenth of a metre.

Solve each triangle (find all angles and all sides). Round to nearest tenth

7.



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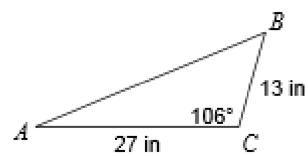
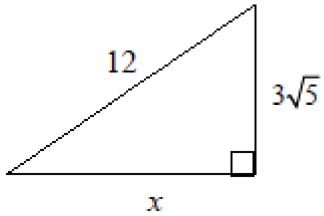


9. Find x.



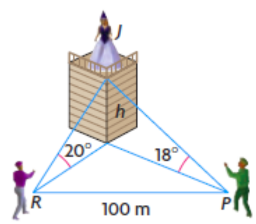
10. Given the points A(-1, 7) and B(-4, -2)

Use the Pythagorean Theorem to find the distance from A to B. Write your answer in simplest radical form.

Solve, round to nearest tenth.

11. The helicopter flies parallel to the ground. The angle of elevation from you to the helicopter is 28°. Twenty seconds later, the helicopter has moved 200ft. At that point, the angle of elevation to the helicopter is 48°. Find the height of the helicopter.

12. Suppose Romeo is serenading Juliet while she is on her balcony. Romeo is facing north and sees the balcony at an angle of elevation of 20°. Paris, Juliet's other suitor, is observing the situation and is facing west. Paris sees the balcony at an angle of elevation of 18°. Romeo and Paris are 100 m apart as shown. Determine the height of Juliet's balcony above the ground, to the nearest metre.



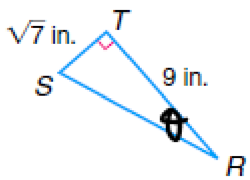
Review how to rationalize the denominators

13. $\dfrac{\sqrt{5}}{\sqrt{3}}$

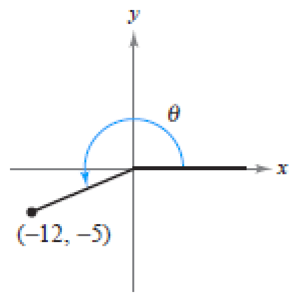
14. $\dfrac{3}{4+4\sqrt{5}}$

Find the values of all six trig ratios for θ

15.



16.



17. Point P(θ) is $(4,0)$ what is P(θ+240°)?

18. Point P(θ) is $\left(\frac{1}{2},\frac{-\sqrt{3}}{2}\right)$ what is P(θ+180°)?

Find a) the six trig ratios, b) find the angle, c) state all the coterminal angles (as a sequence formula)

19. $\csc\theta = -\frac{13}{5}$ and $\cos\theta < 0$

20. $P(\sqrt{3},-1)$

ASSIGNMENT Working in Degrees (MCR)

Find the exact output ratio values

1. $\cos 135°$

2. $\sin 750°$

3. $-\csc 1380°$

4. $\sec^2(-420°)$

5. $\sec 0$

6. $\cot(-10\pi)$

7. $3\sin 135^o - 2\cot(-60^o)$

8. $\dfrac{3\cot 270^o - \sec 210^o \sin 120^o}{\sin 450^o - 2\cos 315^o}$

Find all input angles within $0^o \le angle \le 360^o$ do without calculator, if possible.
Record a general sol as a sequence of all angles in $(-\infty, \infty)$

9. $\sec\theta$ = undefined

10. $\sin\theta = 0$

11. $\sin\theta = -\frac{\sqrt{3}}{2}$

12. $\cot\theta = -1$

13. $\cot\theta = -\sqrt{3}$

14. $2\cos\theta + \sqrt{2} = 0$

15. $\sec x = -1.3342$

16. $2\csc x = 1$

INVESTIGATION: What are radians?

Materials needed:
Compass Protractor Measuring tape (bendy type) – or string and ruler will do the trick

1. Use the compass to draw a circle, any size you wish, but not too small. Draw in the centre you used as a point.
2. Label the circle with angles $0^o, 90^o, 180^0, 270^0, 360^o$
3. Measure your radius

   my radius is r = _______________

4. Measure the different arc lengths corresponding to given angles. Finish the table.
5. Conclude the definition of radians is:

θ	arclength, a	Ratio $\frac{a}{r}$
80°		
115°		
380°		

6. Angles to memorize:

ASSIGNMENT **Radians (MHF)**

Convert to radians

1. $300°$

2. $-100°$

Convert to degrees

3. $\frac{13\pi}{11}$

4. $\frac{-5\pi}{12}$

Find equivalent expressions to the given ratio

5. $\csc\left(\frac{-5\pi}{4}\right)$
   a) using principal angle
   b) using an angle in quadrant III

6. $\tan\frac{10\pi}{9}$
   a) using an angle in quadrant I
   b) using an angle in quadrant IV

Find the ratio values, explain how you can do so without using a calculator

7. $\sec\left(\frac{-11\pi}{2}\right)$

8. $\cos 10\pi$

9. Determine the number of radians between the hour hand and the minute hand at 7:00.

Draw radian angles

10. $\frac{7\pi}{6}$

11. $\frac{13\pi}{4}$

12. $-\frac{4\pi}{3}$

13. 7.58

Using questions #10-13 answer the following

14. For #10 find all secondary trig ratios without a calculator

15. For #11 find all primary trig ratios without a calculator

16. For #12 find the principal, related acute and all coterminal angles

17. For #13 find the tangent value accurate to 2 decimals and explain what this value can represent

ASSIGNMENT Working in Radians (MHF)

Find the exact output ratio values, if possible. No calculator, if possible

1. $\cot\left(\frac{20\pi}{3}\right)$

2. $\sin^2 \frac{5\pi}{6}$

3. $\tan\left(-\frac{23\pi}{6}\right)$

4. $\csc\left(\frac{-3\pi}{4}\right)$

5. $\sec\left(\frac{3\pi}{2}\right)$

6. $-\sin\left(-\frac{9\pi}{2}\right)$

7. $\tan \frac{10\pi}{9}$

8. $\cot(1.35)$

Find all input angles within $0 \le angle \le 2\pi$ do without calculator, if possible.

Record a general sol as a sequence of all angles in $(-\infty, \infty)$

9. $\cos\theta = -1$

10. $\csc\theta = undefined$

11. $\tan\theta + 1 = 0$

12. $\sqrt{3}\cot\theta + 3 = 0$

13. $\sin\theta = \dfrac{\sqrt{2}}{2}$

14. $\cos\theta = \dfrac{1}{2}$

15. $\tan\theta = -\dfrac{\sqrt{3}}{2}$

16. $\csc x = 3.4219$

Each of the following is written incorrectly; fix the mistake(s)

17. $\sin\frac{10\pi}{3} = 600^o$

18. $\sin = \frac{1}{2}$

19. $\sec\frac{5\pi}{6} = \cos\frac{6}{5\pi}$

20. $2\sec\frac{\pi}{8} = \frac{1}{2\cos\frac{\pi}{8}}$

21. $\tan\frac{5\pi}{3} = \tan\frac{-\sqrt{3}}{1}$

22. These mistakes are easily avoided if you understand function notation and the difference of input (angle) and output (ratio value) of trig functions. What is a good tip/reminder that you can tell someone or yourself to avoid these types of mistakes? (Look over the past pages of this assignment and ensure you didn't record things improperly)

ASSIGNMENT Proving Trig Identities (MCR)

Simplify into a single ratio

1. $\frac{1}{\sec t - \tan t} - \frac{1}{\sec t + \tan t}$

2. $\frac{2 \sin t \cos t + (\sin t - \cos t)^2}{\sec t}$

Prove the following, record beside each step what was done

3. $\sec^2\theta\cot^2\theta - \cos^2\theta\csc^2\theta = 1$

4. $\sin^2 x + 2\cos^2 x - 1 = \cos^2 x$

Prove the following, record beside each step what was done

5. $\tan x + \dfrac{1}{\tan x} = \dfrac{1}{\sin x \cos x}$

6. $\sec^2 x - 2\sec x \cos x + \cos^2 x = \tan^2 x - \sin^2 x$

7. $\dfrac{\sec^2 \theta - \tan^2 \theta}{\sec^2 \theta} = \cos^2 \theta$

8. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$

Prove the following, record beside each step what was done, **state restrictions on the variables**

9. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

10. $\dfrac{\sin^2 x}{1 - \cos x} = 1 + \cos x$

11. $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$

12. $\sin\theta + \dfrac{\cos\theta}{\tan\theta} = \dfrac{1}{\cos\theta \tan\theta}$

ASSIGNMENT Ambiguous Case (MCR)

1. View an applet http://www.mathopenref.com/congruentsss.html and then circle which of the following conditions implies the triangles are congruent (same) ie only one unique triangle is possible.

a) AAA

b) ASA

c) SSS

d) SAA

e) SSA

f) SAS

2. **(a)** *Determine whether* SAA, ASA, SSA, SAS, *or* SSS *is given.*
(b) *Decide whether the law of sines or the law of cosines should be used to begin solving the triangle.*

1. a, b, and C

2. A, C, and c

3. a, b, and A

4. a, b, and c

5. A, B, and c

6. a, c, and A

7. a, B, and C

8. b, c, and A

SSA – ambiguous case:

For questions #3-8, decide how many, if any, triangles are possible to be drawn from the given information

OBTUSE type:

3. $\angle B = 110°,\ a = 10,\ b = 12$

4. $\angle B = 110°,\ a = 10,\ b = 9$

ACUTE type:

5. $\angle B = 32°,\ a = 10,\ b = 5$

6. $\angle B = 32°,\ a = 10,\ b = 12$

7. $\angle B = 32°,\ a = 10,\ b = 8$

8. $\angle B = 32°,\ a = 10,\ b = 10$

Solve the triangle, if possible, using only Sine Law - watch for ambiguous case!

9. In ΔDEF $\angle E = 28^o, e = 24, f = 20$

10. In ΔABC $\angle$A$=96^o, a = 13, b = 20$

11. In ΔABC $\angle$A$=35^o, a = 3, b = 4$

12. $a = 1.3$ cm, $b = 2.8$ cm, $\angle A = 33^\circ$

Solve the triangle using only Sine Law - watch for ambiguous case!

13. $\angle B = 42°,\ a = 20,\ b = 14$

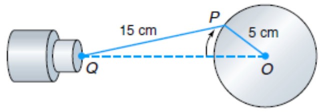
14. $A = 30°,\ b = 16,\ a = 6.3$

Solve the triangle using only Cosine Law – no need to watch for ambiguous case – it will become apparent why

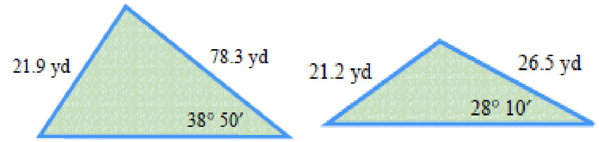
15. $A = 46°,\ b = 16,\ a = 18.4$

16. A = 63° , a = 18, and b =25

17. **Mechanics** The blades of a power lawn mower are rotated by a two-stroke engine with a piston sliding back and forth in the engine cylinder. As the piston moves back and forth, the connecting rod rotates the circular crankshaft. Suppose the crankshaft is 5 centimeters long and the connecting rod is 15 centimeters. If the crankshaft rotates 20 revolutions per second and P is at the horizontal position when it begins to rotate, how far is the piston from the rim of the crankshaft after 0.01 second?



18. A surveyor reported on two triangular pieces of properties: The properties are triangular in shape with dimensions as shown in the figures". What is wrong with both diagrams?



Hint: Convert the 'minutes' portion of the degree into decimal version of the degree, example:

$25^0\ 15'$

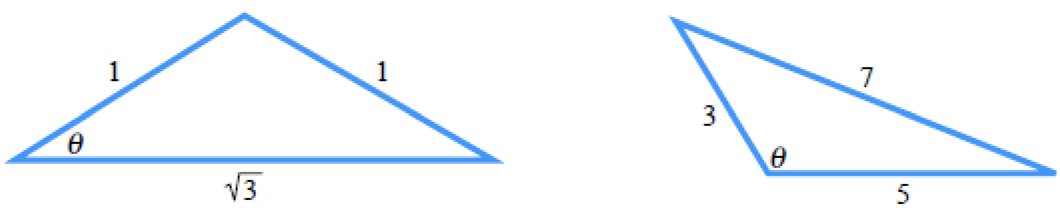
since there are 60 minutes in a degree (and 60 seconds in a minute)

$\frac{15}{60} = 0.25^0$ so

$25^0 + 0.25^0 = 25.25^0$

Find the value of θ in each triangle, use cosine law and explain how you can do these without a calculator.

19.



ASSIGNMENT Bearings (AP)

1. Convert each true bearing to its equivalent quadrant bearing.

a) 070° **b)** 180° **c)** 300°

d) 140° **e)** 210° **f)** 024°

2. Convert each quadrant bearing to its equivalent true bearing.

a) N35°E **b)** N70°W **c)** S10°W

d) S52°E **e)** S18°E **f)** N87°W

Practice drawing diagrams only, DO NOT SOLVE, but explain what law(s) would work

3. Two fire-lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N35°E, and the bearing of the fire from station B is N49°W. How far, to the nearest mile, is the fire from station B?

4. ***Path Traveled by a Plane*** The aircraft carrier *Tallahassee* is traveling at sea on a steady course with a bearing of 30° at 32 mph. Patrol planes on the carrier have enough fuel for 2.6 hr of flight when traveling at a speed of 520 mph. One of the pilots takes off on a bearing of 338° and then turns and heads in a straight line, so as to be able to catch the carrier and land on the deck at the exact instant that his fuel runs out. If the pilot left at 2 P.M., at what time did he turn to head for the carrier?

5. Airport B is 320 miles from airport A on a bearing of S40°E. A pilot wishes to fly from A to B, but to avoid a storm must first fly due East at a speed of 210 mph for an hour, and then from this point (call it C) turns to fly to B. Find the distance, to the nearest mile, and the bearing, to the nearest degree, that the pilot must fly to airport B

6. A boat is anchored off a long straight shoreline which runs due east and west. The bearing of the boat from two cottages on the shore 12 miles apart are S41.2°E and S58.8°W. How far is the boat from each cottage, to the nearest tenth of a mile?

Solve

7. *Distance and Direction of a Motorboat* A motorboat sets out in the direction N 80° E. The speed of the boat in still water is 20.0 mph. If the current is flowing directly south, and the actual direction of the motorboat is due east, find the speed of the current and the actual speed of the motorboat.

8. *Distance of a Ship from Its Starting Point* Starting at point X, a ship sails 15.5 km on a bearing of 200°, then turns and sails 2.4 km on a bearing of 320°. Find the distance of the ship from point X.

9. *Distance and Bearing of a Luxury Liner* A luxury liner leaves port on a bearing of 110.0° and travels 8.8 mi. It then turns due west and travels 2.4 mi. How far is the liner from port, and what is its bearing from port?

10. Airplane takes off at a heading of N58°E from a boat travelling directly east. After travelling for 10 km, it turns and heads back toward the boat and travels 8 km back and lands on the boat. How far did the boat travel in that time?