

# 6-7 Skills Practice

## Solving Radical Equations and Inequalities

Solve each equation.

1.  $\sqrt{x} = 5$

2.  $\sqrt{x} + 3 = 7$

3.  $5\sqrt{j} = 1$

4.  $v^{\frac{1}{2}} + 1 = 0$

5.  $18 - 3y^{\frac{1}{2}} = 25$

6.  $\sqrt[3]{2w} = 4$

7.  $\sqrt{b-5} = 4$

8.  $\sqrt{3n+1} = 5$

9.  $\sqrt[3]{3r-6} = 3$

10.  $2 + \sqrt{3p+7} = 6$

11.  $\sqrt{k-4} - 1 = 5$

12.  $(2d+3)^{\frac{1}{3}} = 2$

13.  $(t-3)^{\frac{1}{3}} = 2$

14.  $4 - (1-7u)^{\frac{1}{3}} = 0$

15.  $\sqrt{3z-2} = \sqrt{z-4}$

16.  $\sqrt{g+1} = \sqrt{2g-7}$

Solve each inequality.

17.  $4\sqrt{x+1} \geq 12$

18.  $5 + \sqrt{c-3} \leq 6$

19.  $-2 + \sqrt{3x+3} < 7$

20.  $-\sqrt{2a+4} \geq -6$

21.  $2\sqrt{4r-3} > 10$

22.  $4 - \sqrt{3x+1} > 3$

23.  $\sqrt{y+4} - 3 \geq 3$

24.  $-3\sqrt{11r+3} \geq -15$

$$\begin{aligned} 1. \quad \sqrt{x} &= 5 \\ (\sqrt{x})^2 &= 5^2 \\ x &= 25 \end{aligned}$$

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**CHECK**

$$\begin{aligned} \sqrt{x} &= 5 \\ \sqrt{25} &= 5 \\ 5 &= 5 \\ \therefore x &= 25 \end{aligned}$$

$$\begin{aligned} 2. \quad \sqrt{x} + 3 &= 7 \\ \sqrt{x} &= 4 \\ (\sqrt{x})^2 &= 4^2 \\ x &= 16 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt{x} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4 \\ \therefore x &= 16 \end{aligned}$$

$$\begin{aligned} 3. \quad 5\sqrt{j} &= 1 \\ \sqrt{j} &= \frac{1}{5} \\ (\sqrt{j})^2 &= \left(\frac{1}{5}\right)^2 \\ j &= \frac{1}{25} \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt{j} &= \frac{1}{5} \\ \sqrt{\frac{1}{25}} &= \frac{1}{5} \\ \frac{1}{5} &= \frac{1}{5} \\ \therefore j &= \frac{1}{25} \end{aligned}$$

$$\begin{aligned} 4. \quad v^{\frac{1}{2}} + 1 &= 0 \\ v^{\frac{1}{2}} &= -1 \\ (v^{\frac{1}{2}})^2 &= (-1)^2 \\ v &= 1 \end{aligned}$$

**CHECK**

$$\begin{aligned} v^{\frac{1}{2}} &= -1 \\ 1^{\frac{1}{2}} &= -1 \\ 1 &\neq -1 \\ \therefore &\text{no solution} \end{aligned}$$

$$\begin{aligned} 5. \quad 18 - 3y^{\frac{1}{2}} &= 25 \\ -3y^{\frac{1}{2}} &= 7 \\ y^{\frac{1}{2}} &= \frac{7}{-3} \\ (y^{\frac{1}{2}})^2 &= \left(\frac{7}{-3}\right)^2 \\ y &= \frac{49}{9} \end{aligned}$$

**CHECK**

$$\begin{aligned} y^{\frac{1}{2}} &= \frac{7}{-3} \\ \left(\frac{49}{9}\right)^{\frac{1}{2}} &= \frac{7}{-3} \\ \frac{7}{3} &\neq -\frac{7}{3} \\ \therefore &\text{no solution} \end{aligned}$$

$$\begin{aligned} 6. \quad \sqrt[3]{2w} &= 4 \\ (\sqrt[3]{2w})^3 &= 4^3 \\ 2w &= 64 \\ w &= 32 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt[3]{2w} &= 4 \\ \sqrt[3]{2(32)} &= 4 \\ \sqrt[3]{64} &= 4 \\ 4 &= 4 \\ \therefore w &= 32 \end{aligned}$$

$$\begin{aligned} 7. \quad \sqrt{b-5} &= 4 \\ (\sqrt{b-5})^2 &= 4^2 \\ b-5 &= 16 \\ b &= 21 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt{b-5} &= 4 \\ \sqrt{21-5} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4 \\ \therefore b &= 21 \end{aligned}$$

$$\begin{aligned} 8. \quad \sqrt{3n+1} &= 5 \\ (\sqrt{3n+1})^2 &= 5^2 \\ 3n+1 &= 25 \\ 3n &= 24 \\ n &= 8 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt{3n+1} &= 5 \\ \sqrt{3(8)+1} &= 5 \\ \sqrt{24+1} &= 5 \\ \sqrt{25} &= 5 \\ \therefore n &= 8 \end{aligned}$$

$$\begin{aligned} 9. \quad \sqrt[3]{3r-6} &= 3 \\ (\sqrt[3]{3r-6})^3 &= 3^3 \\ 3r-6 &= 27 \\ 3r &= 33 \\ r &= 11 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt[3]{3r-6} &= 3 \\ \sqrt[3]{3(11)-6} &= 3 \\ \sqrt[3]{33-6} &= 3 \\ \sqrt[3]{27} &= 3 \\ 3 &= 3 \\ \therefore r &= 11 \end{aligned}$$

$$\begin{aligned} 10. \quad 2 + \sqrt{3p+7} &= 6 \\ \sqrt{3p+7} &= 4 \\ (\sqrt{3p+7})^2 &= 4^2 \\ 3p+7 &= 16 \\ 3p &= 9 \\ p &= 3 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt{3p+7} &= 4 \\ \sqrt{3(3)+7} &= 4 \\ \sqrt{9+7} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4 \\ \therefore p &= 3 \end{aligned}$$

$$\begin{aligned} 11. \quad \sqrt{k-4} - 1 &= 5 \\ (\sqrt{k-4})^2 &= 6^2 \\ k-4 &= 36 \\ k &= 40 \end{aligned}$$

**CHECK**

$$\begin{aligned} \sqrt{k-4} &= 6 \\ \sqrt{40-4} &= 6 \\ \sqrt{36} &= 6 \\ 6 &= 6 \\ \therefore k &= 40 \end{aligned}$$

$$12. (2d + 3)^{\frac{1}{3}} = 2$$

$$((2d + 3)^{\frac{1}{3}})^3 = 2^3$$

$$2d + 3 = 8$$

$$2d = 5$$

$$d = \frac{5}{2}$$

CHECK

$$(2(\frac{5}{2}) + 3)^{\frac{1}{3}} = 2$$

$$(5 + 3)^{\frac{1}{3}} = 2$$

$$(8)^{\frac{1}{3}} = 2$$

$$2 = 2$$

$$\therefore d = \frac{5}{2}$$

$$13. (t - 3)^{\frac{1}{3}} = 2$$

$$((t - 3)^{\frac{1}{3}})^3 = 2^3$$

$$t - 3 = 8$$

$$t = 11$$

CHECK

$$(t - 3)^{\frac{1}{3}} = 2$$

$$(11 - 3)^{\frac{1}{3}} = 2$$

$$(8)^{\frac{1}{3}} = 2$$

$$2 = 2$$

$$\therefore t = 11$$

$$14. 4 - (1 - 7u)^{\frac{1}{3}} = 0$$

$$4 = (1 - 7u)^{\frac{1}{3}}$$

$$4^3 = ((1 - 7u)^{\frac{1}{3}})^3$$

$$64 = 1 - 7u$$

$$63 = -7u$$

$$-9 = u$$

CHECK

$$4 = (1 - 7u)^{\frac{1}{3}}$$

$$4 = (1 - 7(-9))^{\frac{1}{3}}$$

$$4 = (1 + 63)^{\frac{1}{3}}$$

$$4 = (64)^{\frac{1}{3}}$$

$$4 = 4$$

$$\therefore u = -9$$

$$15. \sqrt{3z - 2} = \sqrt{z - 4}$$

$$(\sqrt{3z - 2})^2 = (\sqrt{z - 4})^2$$

$$3z - 2 = z - 4$$

$$2z = -2$$

$$z = -1$$

CHECK

$$\sqrt{3z - 2} = \sqrt{z - 4}$$

$$\sqrt{3(-1) - 2} = \sqrt{-1 - 4}$$

$$\sqrt{-5} = \sqrt{-5}$$

• negative under radical

$\therefore$  no solution

$$16. \sqrt{g + 1} = \sqrt{2g - 7}$$

$$(\sqrt{g + 1})^2 = (\sqrt{2g - 7})^2 \quad \sqrt{g + 1} = \sqrt{2g - 7}$$

$$g + 1 = 2g - 7$$

$$8 = g$$

$$g = 8$$

CHECK

$$\sqrt{g + 1} = \sqrt{2(8) - 7}$$

$$\sqrt{9} = \sqrt{16 - 7}$$

$$\sqrt{9} = \sqrt{9}$$

$$3 = 3$$

$$\therefore g = 8$$

$$17. 4\sqrt{x + 1} \geq 12$$

RELATED EQUATION:

$$4\sqrt{x + 1} = 12$$

$$\sqrt{x + 1} = 3$$

$$(\sqrt{x + 1})^2 = 3^2$$

$$x + 1 = 9$$

$$x = 8$$

DOMAIN:

$$\overset{\text{(radicand)}}{x + 1} \geq 0$$

$$x \geq -1$$

TEST:

8

-1

$$x = 3$$

$$x = 8$$

$$x = 15$$

$$4\sqrt{3+1} \geq 12$$

$$\sqrt{4} \geq 3$$

$$2 \geq 3$$

**FALSE**

$$4\sqrt{8+1} \geq 12$$

$$\sqrt{9} \geq 3$$

$$3 \geq 3$$

**TRUE**

$$4\sqrt{15+1} \geq 12$$

$$\sqrt{16} \geq 3$$

$$4 \geq 3$$

**TRUE**

• don't need to test  $x < -1$  as domain is  $x \geq -1$

$$\therefore x \geq 8$$

18.  $5 + \sqrt{c-3} \leq 6$

**RELATED EQUATION:**

$$\begin{aligned}(\sqrt{c-3})^2 &= 1^2 \\ c-3 &= 1 \\ c &= 4\end{aligned}$$

**DOMAIN:**

(radicand)  
 $c-3 \geq 0$   
 $c \geq 3$

- don't need to test  $c < 3$  as domain is  $c \geq 3$

3 TEST: 4

$c = 3.5$	$c = 4$	$c = 19$
$5 + \sqrt{3.5-3} \leq 6$	$5 + \sqrt{4-3} \leq 6$	$5 + \sqrt{19-3} \leq 6$
$5 + \sqrt{3.5-3} \leq 6$	$5 + \sqrt{1} \leq 6$	$5 + \sqrt{16} \leq 6$
$\approx 5.7 \leq 6$	$6 \leq 6$	$9 \leq 6$
<b>TRUE</b>	<b>TRUE</b>	<b>FALSE</b>

$\therefore 3 \leq c \leq 4$

19.  $-2 + \sqrt{3x+3} < 7$

**RELATED EQUATION:**

$$\begin{aligned}(\sqrt{3x+3})^2 &= 9^2 \\ 3x+3 &= 81 \\ 3x &= 78 \\ x &= 26\end{aligned}$$

**DOMAIN:**

(radicand)  
 $3x+3 \geq 0$   
 $3x \geq -3$   
 $x \geq -1$

- don't need to test  $x < -1$  as domain is  $x \geq -1$

TEST:

-1 26

$x = 1$	$x = 26$	$x = 50$
$-2 + \sqrt{3(1)+3} < 7$	$-2 + \sqrt{3(26)+3} < 7$	$-2 + \sqrt{3(50)+3} < 7$
$\approx 0.45 < 7$	$7 < 7$	$\approx 10.4 < 7$
<b>TRUE</b>	<b>FALSE</b>	<b>FALSE</b>

$\therefore -1 \leq x < 26$

20.  $-\sqrt{2a+4} \geq -6$

**RELATED EQUATION:**

$$\begin{aligned}(\sqrt{2a+4})^2 &= 6^2 \\ 2a+4 &= 36 \\ 2a &= 32 \\ a &= 16\end{aligned}$$

**DOMAIN:**

(radicand)  
 $2a+4 \geq 0$   
 $2a \geq -4$   
 $a \geq -2$

- don't need to test  $a < -2$  as domain is  $a \geq -2$

TEST:

-2 16

$a = 6$	$a = 16$	$a = 30$
$-\sqrt{2(6)+4} \geq -6$	$-\sqrt{2(16)+4} \geq -6$	$-\sqrt{2(30)+4} \geq -6$
$-\sqrt{16} \geq -6$	$-\sqrt{36} \geq -6$	$-\sqrt{64} \geq -6$
$-4 \geq -6$	$-6 \geq -6$	$-8 \geq -6$
<b>TRUE</b>	<b>TRUE</b>	<b>FALSE</b>

$\therefore -2 \leq a \leq 16$

21.  $2\sqrt{4r-3} > 10$

**RELATED EQUATION:**

$$\begin{aligned}(\sqrt{4r-3})^2 &= 5^2 \\ 4r-3 &= 25 \\ 4r &= 28 \\ r &= 7\end{aligned}$$

**DOMAIN:**

(radicand)  
 $4r-3 \geq 0$   
 $4r \geq 3$   
 $r \geq \frac{3}{4}$

- don't need to test  $r < \frac{3}{4}$  as domain is  $r \geq \frac{3}{4}$

$\frac{3}{4}$  TEST: 7

$r = 1$	$r = 7$	$r = 10$
$2\sqrt{4(1)-3} > 10$	$2\sqrt{4(7)-3} > 10$	$2\sqrt{4(10)-3} > 10$
$2\sqrt{1} > 10$	$2\sqrt{25} > 10$	$\approx 12.2 > 10$
$2 > 10$	$10 > 10$	
<b>FALSE</b>	<b>FALSE</b>	<b>TRUE</b>

$\therefore r > 7$

$$22. 4 - \sqrt{3x + 1} > 3$$

**RELATED EQUATION:**

$$\begin{aligned} -\sqrt{3x + 1} &= -1 \\ (\sqrt{3x + 1})^2 &= 1^2 \\ 3x + 1 &= 1 \\ 3x &= 0 \\ x &= 0 \end{aligned}$$

**DOMAIN:**

(radicand)

$$\begin{aligned} 3x + 1 &\geq 0 \\ 3x &\geq -1 \\ x &\geq -\frac{1}{3} \end{aligned}$$

**TEST:**

$$\frac{-1}{3} \qquad 0$$

$x = -\frac{1}{4}$	$x = 0$	$x = 10$
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$$\begin{aligned} 4 - \sqrt{3(-\frac{1}{4}) + 1} &> 3 & 4 - \sqrt{3(0) + 1} &> 3 & 4 - \sqrt{3(10) + 1} &> 3 \\ 3.5 > 3 & & 4 - \sqrt{1} > 3 & & \approx -1.6 > 3 \\ \text{TRUE} & & 3 > 3 & & \\ & & \text{FALSE} & & \text{FALSE} \end{aligned}$$

$$\therefore -\frac{1}{3} \leq x < 0$$

$$23. \sqrt{y + 4} - 3 \geq 3$$

**RELATED EQUATION:**

$$\begin{aligned} (\sqrt{y + 4})^2 &= 6^2 \\ y + 4 &= 36 \\ y &= 32 \end{aligned}$$

**DOMAIN:**

(radicand)

$$\begin{aligned} y + 4 &\geq 0 \\ y &\geq -4 \end{aligned}$$

**TEST:**

$$-4 \qquad 32$$

$y = 0$	$y = 32$	$y = 60$
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$$\begin{aligned} \sqrt{0 + 4} - 3 &\geq 3 & \sqrt{32 + 4} - 3 &\geq 3 & \sqrt{60 + 4} - 3 &\geq 3 \\ 2 - 3 &\geq 3 & 6 - 3 &\geq 3 & 8 - 3 &\geq 3 \\ -1 &\geq 3 & 3 &\geq 3 & 5 &\geq 3 \\ \text{FALSE} & & \text{TRUE} & & \text{TRUE} \end{aligned}$$

$$\therefore y \geq 32$$

$$24. -3\sqrt{11r + 3} \geq -15$$

**RELATED EQUATION:**

$$\begin{aligned} (\sqrt{11r + 3})^2 &= 5^2 \\ 11r + 3 &= 25 \\ 11r &= 22 \\ r &= 2 \end{aligned}$$

**DOMAIN:**

(radicand)

$$\begin{aligned} 11r + 3 &\geq 0 \\ 11r &\geq -3 \\ r &\geq -\frac{3}{11} \end{aligned}$$

**TEST:**

$$\frac{-3}{11} \qquad 2$$

$r = 1$	$r = 2$	$r = 3$
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$$\begin{aligned} -3\sqrt{11(1) + 3} &\geq -15 & -3\sqrt{11(2) + 3} &\geq -15 & -3\sqrt{11(3) + 3} &\geq -15 \\ \approx -11.2 &\geq -15 & 15 &\geq -15 & -18 &\geq -15 \\ \text{TRUE} & & \text{TRUE} & & \text{FALSE} \end{aligned}$$

$$\therefore -\frac{3}{11} \leq r \leq 2$$