Big idea

In this unit you will extend your knowledge of SOH CAH TOA to work with obtuse and reflex angles. This extension will involve the unit circle which will allow you to understand that inverse sine or inverse cosine or inverse tangent actually give more than one solution. You will also learn how to find the exact ratios for special angles without using a calculator.

Feedback & Assessment of Your Success

Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: _____ Questions to ask the teacher:
2days	2-6	New Terminology (MCR) Journal #1,2				
	7-8	Exact Values using Special Angles (MCR) Journal #3				
3days	9-16	Rotational Trigonometry (MCR) Journal #4,5				

Rationalize Denominator

$$\frac{2}{\sqrt{5}}$$

Rationalize Numerator

$$\frac{2\sqrt{5}}{5}$$

Multiplying

$4a^2 \times 9a^3$

$2^2 x^5 y \times 3^2 xy^4$

$5\sqrt{3} \times 2\sqrt{7}$

Adding

$\sqrt{3} - 5\sqrt{3}$

$6\sqrt{2} + \sqrt{3}$

Reducing

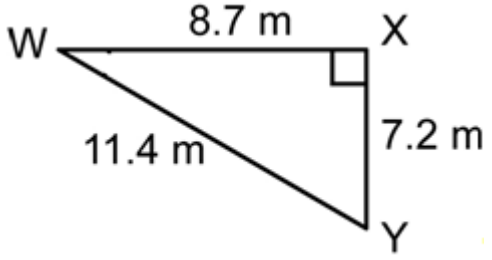
$3\sqrt{8}$

New Terminology

1. Since you will be extending your trigonometry knowledge from grade 10 to be able to solve obtuse and reflex angles, it is a good idea to review the **primary trigonometric ratios,** or SOH CAH TOA, and Pythagorean Theorem. Summarize what you should know:

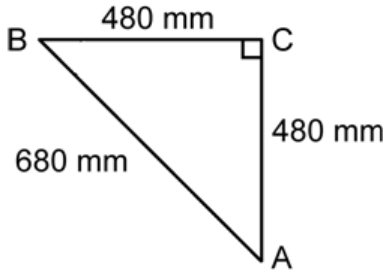
2. Summarize what the **secondary trigonometric ratios** are:

3. Find the following ratios (DON'T find angle unless asked)



a. $\cos Y$ b. $\cot W$

4. Find the following ratios

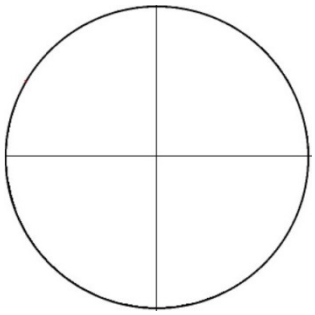


a. $\csc A$ b. $\tan B$

5. If $\sin\theta = \frac{1}{4}$ then show how to find the values of $\csc\theta$ and $\sec\theta$

6. If $\cot\theta = 0.4732$ then what is θ?

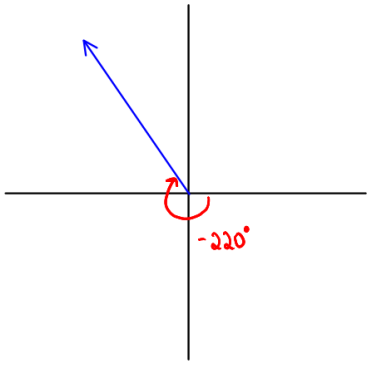
7. To understand the way obtuse and reflex angles relate to the unit circle, you need to learn some new definitions. Define(or show on a diagram) the following terms:



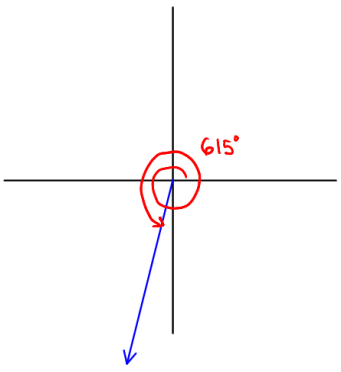
standard position	Initial arm	terminal arm	Positive/Negative angle rotation
Quadrants	quadrantal angles	principle angle	co-terminal angles
related acute angle (or reference angle)	acute angles	obtuse angles	reflex angles

8. State the principal angle and the related acute angle, then state two more co-terminal angles.

a.



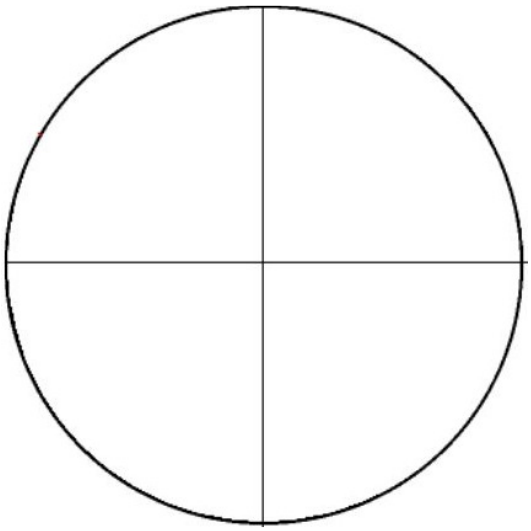
b.



9. You must understand why sometimes ratios are negative and sometimes positive. Answer the following questions to see what the pattern is.

   a. Draw the related acute angle 20^o in each quadrant, and state the principal angles each forms with the initial arm.

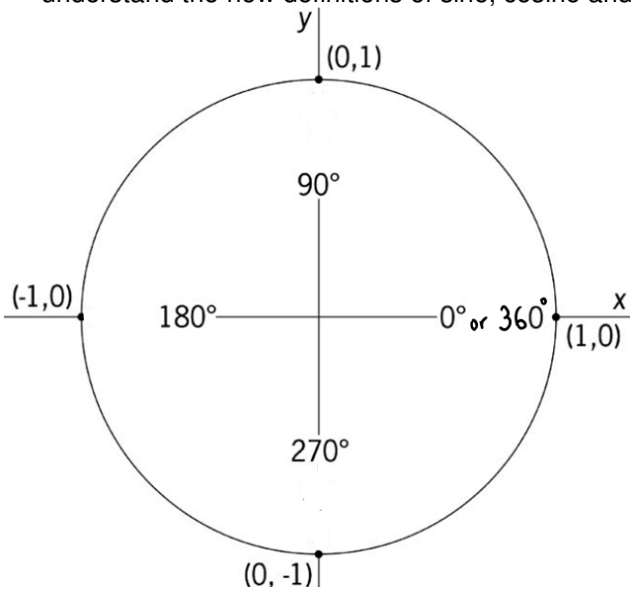
   b. In each quadrant, use the calculator to find all three primary trig ratios for the principal angle in that quadrant.



   c. What do you notice?

   d. What does the acronym CAST stand for?

10. The acronym CAST is not always useful. Angles can fall onto the x or y axes. Answer the following questions to really understand the new definitions of sine, cosine and tangent.



   a. Use the calculator to find all three primary trig ratios of all the angles that fall onto the axes

   b. Compare your answers to the coordinate points that the answers correspond to, what do you notice?

   c. What are the new definitions of sine, cosine and tangent? (Keep in mind the cicle doesn't have to have a radius of 1.) Also, what does the pythagorean theorem remind you of here?

11. Predict whether each value will be positive or negative. Explain the MEANING of each ratio.

a. $\tan 195^o$ b. $\sin(-115^o)$ c. $\cos 670^o$

12. Find the following ratios without using the calculator.

a. $\cos(-90^o)$ b. $\cos 180^o$ c. $\tan 270^o$ d. $\sin 360^o$

13. Find the angles without using the calculator.

a. $\cos\theta = -1$ b. $\cos\theta = 0$ c. $\sin\theta = 1$ d. $\tan\theta = undefined$

Determine the a) principal angle, b) related acute angle, c) two coterminal angles (one positive, one negative)

14. $1045°$ 15. $-197°$

Find the values of all six trig ratios for θ

16. $\sin\theta = \frac{1}{3}$ in quadrant II

17. $\cot\theta = -\sqrt{3}$
not given a quadrant, so find all possibilities for the ratios

Predict what quadrant the angle is in

18. $\sin\theta < 0 \ \& \ \cos\theta > 0$

19. $\csc\theta > 0 \ \& \ \cot\theta = -\frac{\sqrt{3}}{3}$

20. For all of the following state an equivalent trigonometric expression with same value of the ratio.

a. $\tan 195^o$ b. $\sin(-115^o)$ c. $\cos 670^o$

Exact Values using Special Angles

1. What are special angles?

2. What answer is better to record of the two below and why?

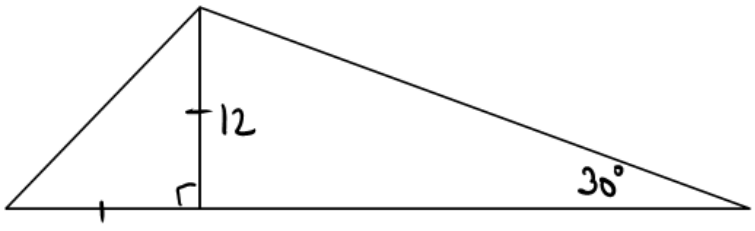
$\cos 30^o = 0.866025403...$ or

$\cos 30^o = \frac{\sqrt{3}}{2}$

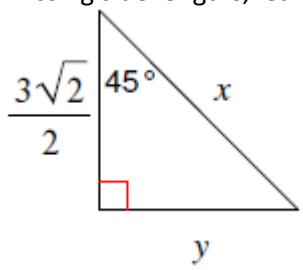
3. Almost everytime trig functions are used there is rounding error. However, it is possible to find exact values for some special angles. Draw two special triangles and explain where the side lengths come from.

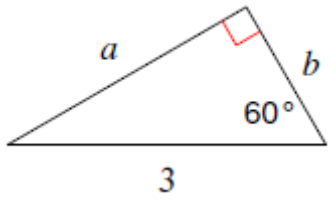
4. Does it matter what is the size of the triangle used when dealing with ratios? Draw different sized triangles and label the dimensions. Show that ratios are still the same.

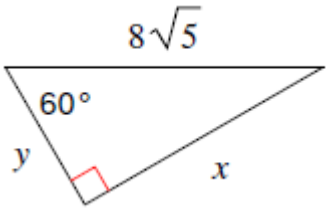
5. Find the exact values for all the dimensions of following diagram

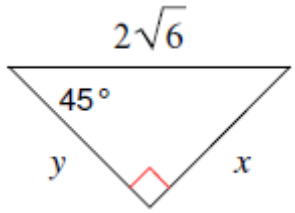


Find the missing side lengths, leave answers as exact and reduced radicals.

6. 

7. 

8. 

9. 

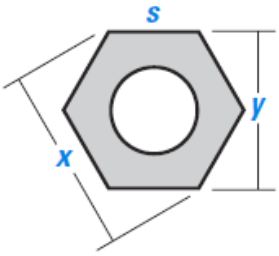
Solve, leave answers exact

10. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude of the triangle.

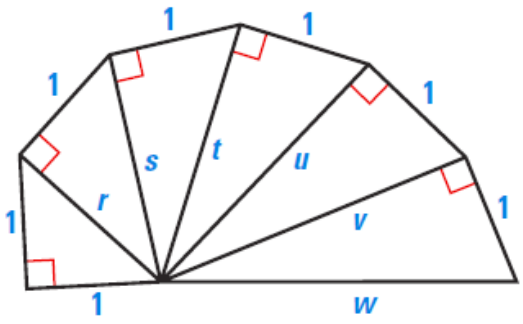
11. The diagonal of a square is 26 inches. Find the length of a side.

Solve, leave answers exact.

12. Find x and y if s=2cm. Hint hexagon is made up of 6 equilateral triangles.

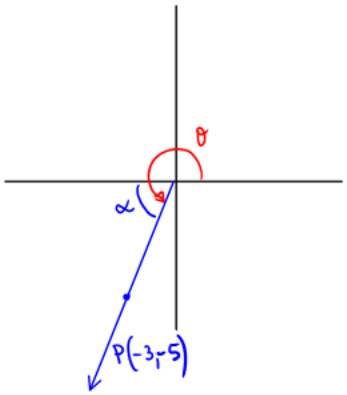


13. Find the missing lengths



Rotational Trigonometry

1. You CANNOT rely on the calculator to give you the answers that you need anymore, to see this, answer the following questions.



a. For the point P(–3, – 5) find the following ratios:

$\sin\theta$

$\cos\theta$

$\tan\theta$

b. Now use the inverse buttons for EACH ratio to find the angle θ

c. What do you notice with answers the calculator gives you for θ?

d. What must be done when you use inverse buttons when dealing with obtuse or reflex angles?

2. For the angle -170^o

a. Find the related acute or the reference angle

b. Predict the signs of all the primary trig ratios. Explain your choice using CAST **or** using the new definitions of the trig ratios.

3. For $\tan(-140^o)$

a. Sketch the given angle

b. State the principal angle and the related acute angle

c. Find a few equivalent expressions to $\tan(-140^o)$ that give the same answer for the ratio.

Find ratios

1. Find the exact values of each of the following, without using the calculator.

a. $\csc 330^o$

b. $\cos 720^o$

c. $\sin 315^o$

d. $\tan(-120^o)$

e. $\sec 225^0$

f. $\cot 120^0$

g. $\sin 270^o \cos 45^o - \cot 60^o \sec 150^o$

h. $2\csc 90^o - 3\tan 135^o \cos 210^o$

Find the values of all six trig functions without using a calculator

2. 600^o

3. $-270°$

4. $225°$

5. $-30°$

Find the ratio values, explain how you can do so without using a calculator

6. $\sin 180°$

7. $\sec 270°$

Find equivalent trig ratio expressions to the given ratio

8. $\cos(-240^\circ)$

a) using principal angle

b) using an angle in quadrant III

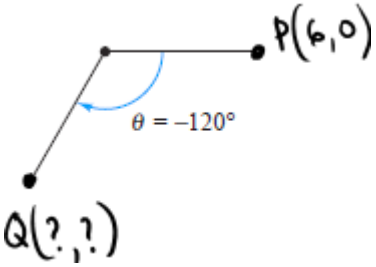
9. $\sin 150^\circ$

a) using an angle in quadrant I

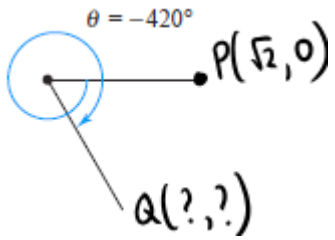
b) using an angle in quadrant IV

Find the new point on the terminal arm given the angle that the point rotated through, round to 2 decimals.

10.



11.



Find several possibilities for point that lies on the terminal arm then find θ.

12. θ lies on the line $7x+5y=0$ in quadrant II

13. $\csc\theta = \frac{6\sqrt{11}}{11}$ & $\cos\theta = -\frac{5}{6}$

Find angles

1. Without using the calculator find the solutions for angles to the following

a. $\cos\theta = -\frac{\sqrt{3}}{2}$

b. $\tan\theta = \frac{\sqrt{3}}{3}$

c. $\sin\theta = -\frac{\sqrt{2}}{2}$

d. $\tan\theta = -1$

e. $\sec\theta = -\sqrt{2}$

f. $\cot\theta = -\sqrt{3}$

g. $\csc\theta = \frac{2\sqrt{3}}{3}$

h. $\sec\theta = \textit{undefined}$

2. Solve for angle if $0^o \le angle \le 360^o$ If possible do not use the calculator.

a. $2 = -5\tan\beta$

b. $-4\sin\theta - 3 = 0$

c. $\sin\beta = 0.5$

d. $2\cos t + 1 = 0$

e. $\tan\theta = 0$

f. $\cos\alpha = 0$

g. $\cot\omega = 5.64$

h. $\sec\lambda = -1$

i. $\csc\varphi = -4$

3. For the ratio $\sin\theta = -\frac{2}{5}$, the angle θ is in standard position i.e. $0^o \le \theta \le 360^o$.
   a. How many answers for θ are there?
   b. Is θ acute, obtuse or reflex in Quadrant III or reflex in Quadrant IV?
   c. Find all possible measures of θ in the given domain.

4. For the point $P(-2,6)$
   a. Sketch the angle, θ, in standard position
   b. Find $\cot\theta$
   c. Find the angle θ.

5. For the ratio $\sec\theta = -\frac{4}{3}$, the angle θ is in standard position $0^o \le \theta \le 360^o$.
   a. Find all 5 other trig ratios for θ
   b. Find all possible measures of θ in the given domain

6. For the point $P(5,-7)$
   a. Find $\csc\theta$
   b. Find the angle θ.

7. For the point $P(-2,-8)$ find the exact values of the three primary trig ratios for the principal angle that is made with the terminal arm with point P on it.

8. The angle θ, is in Quadrant III, and $\sin\theta = -\frac{\sqrt{3}}{2}$. Point P lies on the terminal arm. Determine θ, and state at least two possible coordinates for point P.

9. The terminal arm of θ is in quadrant III and on the line $\sqrt{3}y - 3x = 0$. Determine the angle θ in standard position