Big idea

In the first unit you practiced how to work with quadratic expressions by expanding and factoring. In this unit you will learn different versions of quadratic equations:

Standard form:

Factored form:

Vertex form:

You will also learn how these forms tell you different things about the u-shape curve (called _____), how to graph quadratics from different forms and how to interpret these graphs This unit is very important because of where quadratics are used in real life. Here are some examples: If you were to open up a business selling an item you produce, how do you maximize profit and minimize cost? If you are an engineer, how do you find dimensions of a shape that will minimize cost of material yet maximize space inside; or how to model a flight path of a launched rocket?

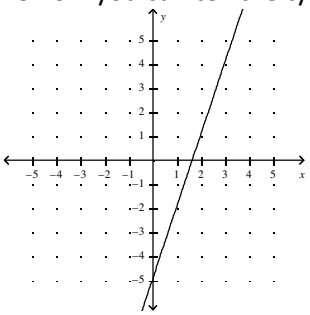
Feedback & Assessment of Your Success

Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: _____ Questions to ask the teacher:
	2-4	Non Linear Relations (MPM) Journal #1				
	5-7	Transformations of Quadratics (MPM) Journal #2				
	8-10	Different Forms of Quadratics (MPM) Journal #3				
	11-13	Find Equations (MPM) Journal #4				
	14-16	Graph Quadratics (MPM) Journal #5				
1.5days	17-20	Solve Quadratics & Quadratic Formula (MPM) Journal #6				
0.5days	21	# of Zeros (MPM) Journal #7				
1.5days	22-25	Word Problems WITH Equations (MPM) Journal #8				
1.5days	26-30	Word Problems WITHOUT Equations (MPM) Journal #9				

ASSIGNMENT Non Linear Relationships (MPM)

Summarize how you can tell the type of relationship from a graph/equation/table:

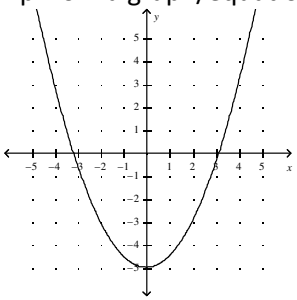
1.



$y = 2x - 5$

x	y
0	-5
1	-3
2	-1
3	1

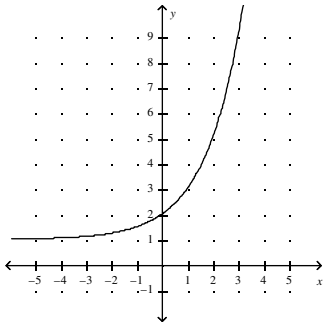
2.



$y = 0.5x^2 - 5$

x	y
0	-5
2	-3
4	3
6	13

3.

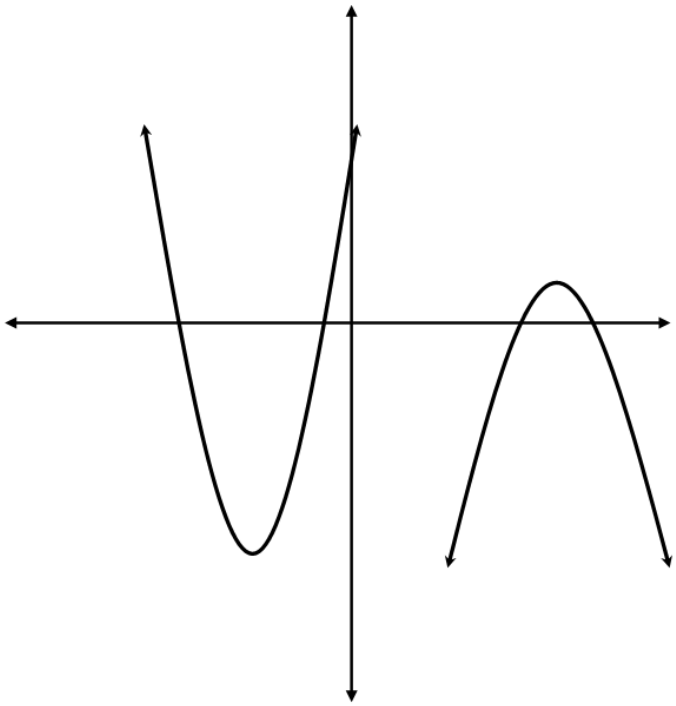


$y = 2^x + 1$

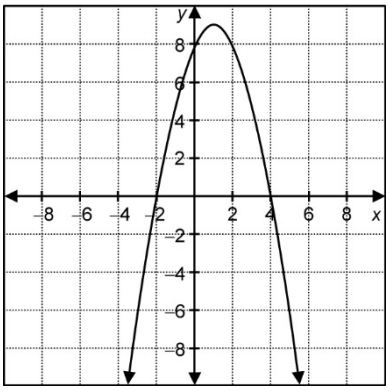
x	y
0	1
1	2
2	4
3	8

Vocabulary of quadratics:

4.

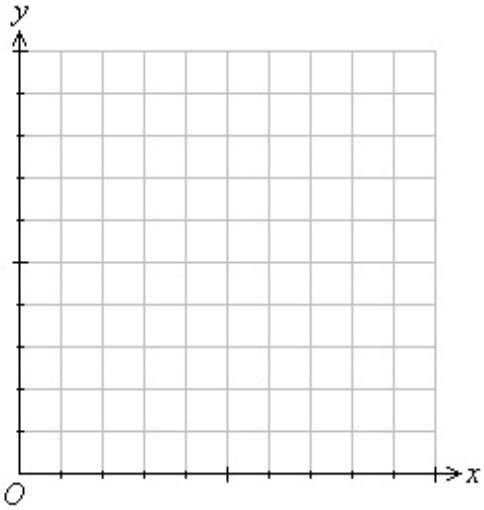


5. For the graph, identify
   a) the coordinates of the vertex
   b) the equation of the axis of symmetry
   c) the *y*-intercept
   d) the maximum or minimum value
   e) the *x*-intercepts

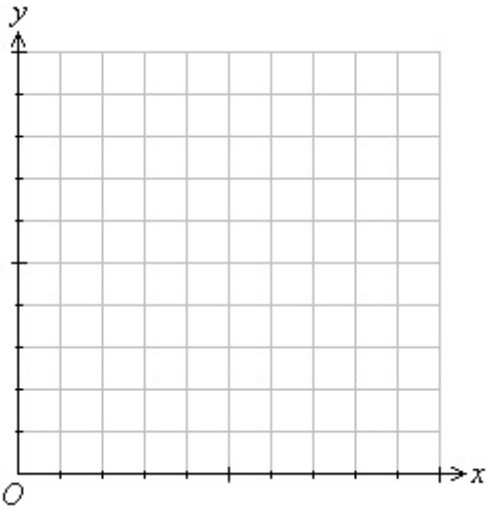


6. **a)**Complete the table of values for the relations between the area of a circle and its radius and between the circumference of a circle and its radius. Then, make two scatter plots of the data.

Radius(cm)	$A = \pi r^2$ (cm^2)
1	
2	
3	
4	
5	
6	



Radius(cm)	$C=2\pi r$ (cm)
1	
2	
3	
4	
5	
6	



b) Describe the two relations.

c) Draw lines or curves of best fit for the data.

d) Use your models to predict the area and circumference for a radius of 2.5 cm.

e) Use your models to predict the area and circumference for a radius of 8 cm.

7. Use finite differences to determine whether each relation is linear, quadratic, or neither.

a)

x	y
0	3
1	6
2	9
3	12
4	15

b)

x	y
–5	–125
–3	–27
–1	–1
1	1
3	27

8. A ball is thrown upward with an initial velocity of 10 m/s. Its approximate height, h, in metres, above the ground after t seconds is given by the relation $h = -5t^2 + 10t + 35$.

a) Sketch a graph of the quadratic relation.

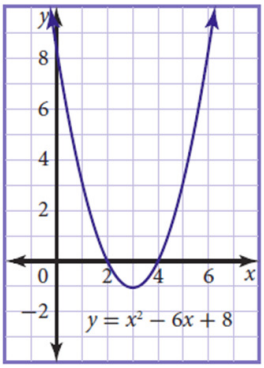
b) Find the maximum height of the ball.

c) How long does it take the ball to reach this maximum height?

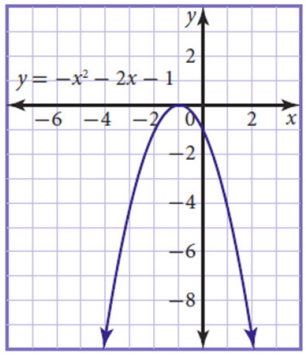
d) Find when the ball is at the ground level

9. Identify all the key features of these graphs

a.



b.



Max or Min ?	Max or Min ?
Optimal Value	Optimal Value
Axis of symm	Axis of symm
Vertex	Vertex
Zeros/x-int	Zeros/x-int
Y-intercept	Y-intercept

ASSIGNMENT Transformations of Quadratics (MPM)

In grade 9 LINES: **Now in grade 10 for QUADRATICS:**

In this investigation you will graph different parabolas and determine the link between the equation in "vertex form" $y=a(x-h)^2+k$ and the transformations from the basic parabola $y = x^2$

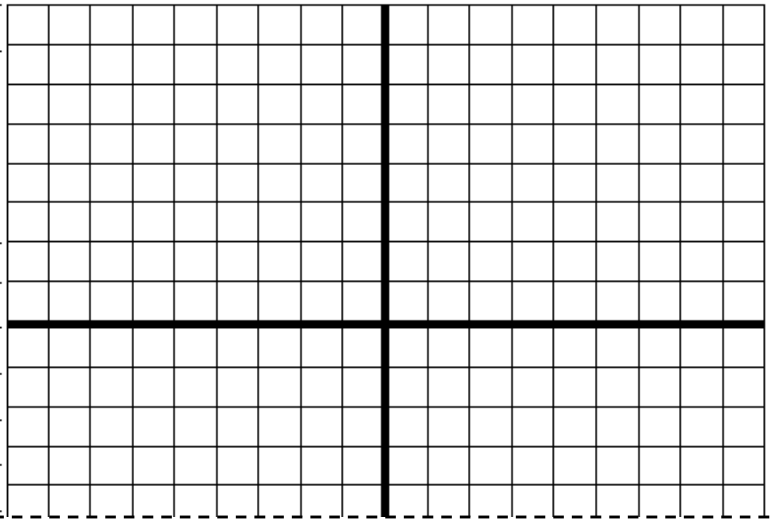
Parabola Investigation

Basic Equation	$y = x^2$

Vertex Form $y = a(x – h)^2+k$

Change values for a
keep h=0 and k=0 for now

values	Equations	Colour
a=2		
a=0.2		
a=-1		
a=-2		
a=-0.2		



What effect does changing *"a"* have on the graph of $y = x^2$

1. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = 1/3x^2$ b. $y = – 5x^2$ c. $y = – 0.001x^2$

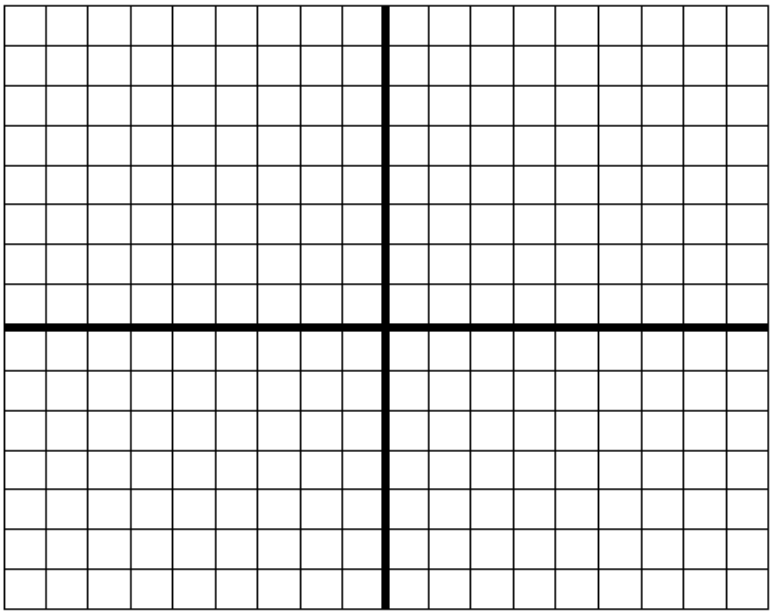
Basic Equation	$y = x^2$

Vertex Form $y = a(x - h)^2 + k$

Change values for **k**
keep a=1 and h=0 for now

values	Equations	Colour
k=4		
k=-6		

What effect does changing *k* have on the graph of $y = x^2$?



2. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = 2x^2 - 9$ b. $y = -0.5x^2 - 16$ c. $y = -3x^2 + 9$

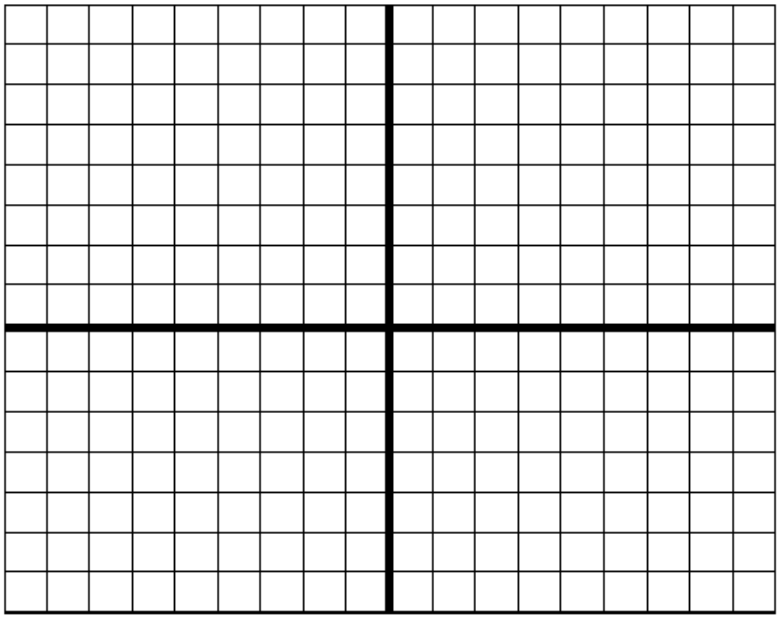
Basic Equation	$y = x^2$

Vertex Form $y = a(x - h)^2 + k$

Change values for **h**
keep a=1 and k=0 for now

values	Equations	Colour
h=3		
h=-5		

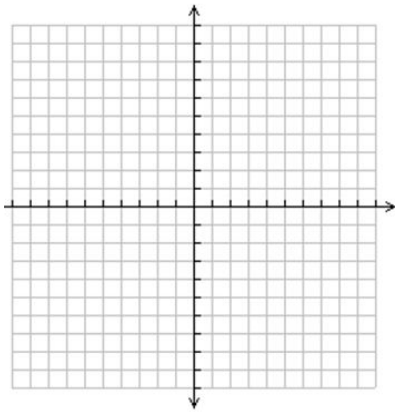
What effect does changing *h* have on the graph of $y = x^2$



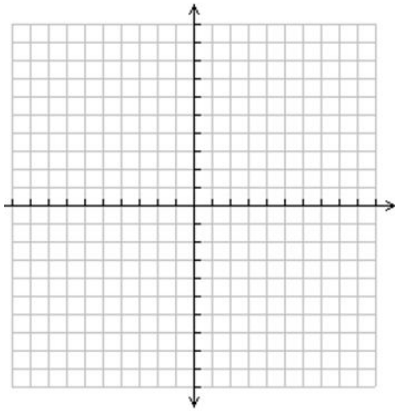
3. State the transformations performed on $y = x^2$ in each of the following quadratics
   a. $y = (x+2)^2$ b. $y = (x - 4)^2 - 7$ c. $y = -(x+4)^2 + 3$ d. $y = 2(x - 1)^2$

State the transformations performed on $y = x^2$, then do a little sketch showing the image points in tables

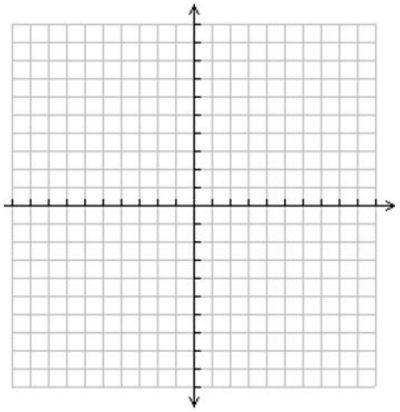
4. $y = (x+2)^2$



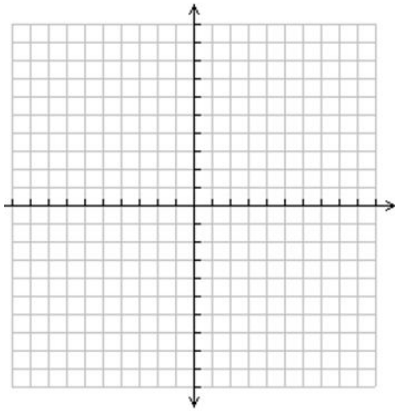
5. $y = (x-4)^2 - 7$



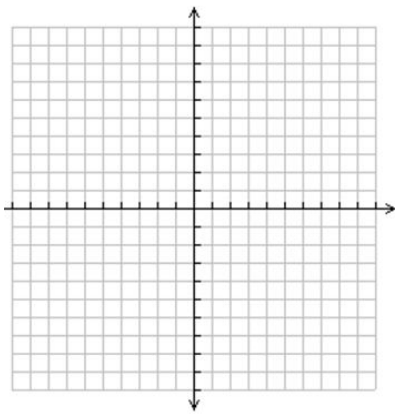
6. $y = -(x+4)^2 + 3$



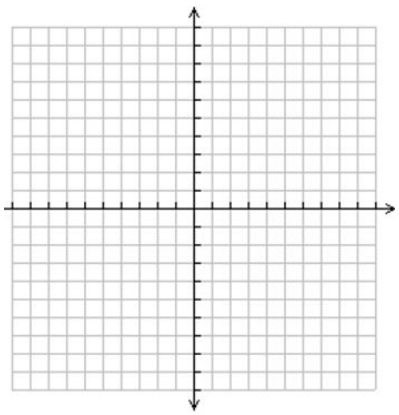
7. $y = 2(x-1)^2$



8. $y = -\frac{1}{2}x^2 + 9$



9. $y = \frac{1}{4}(x+4)^2 - 1$

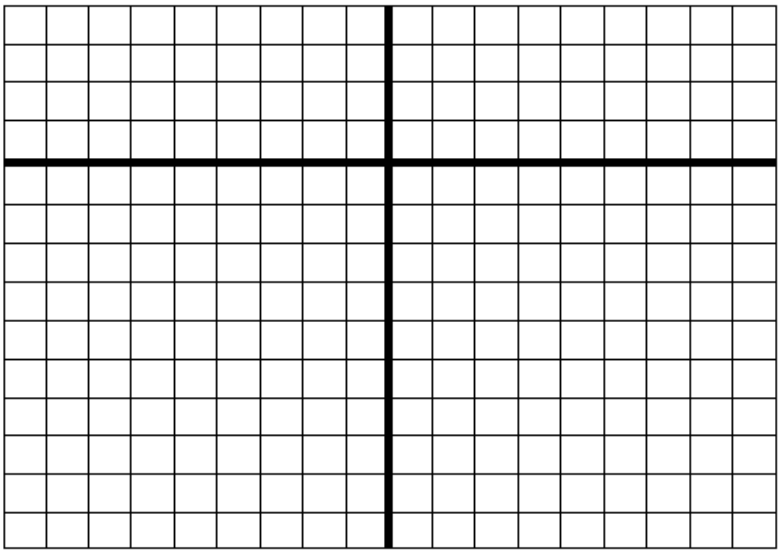


ASSIGNMENT Different Forms of Quadratics (MPM)

1. Table of Values

x	y
-2	0
-1	-4
0	-6
0.5	-6.25
1	-6
2	-4
3	0

a. Plot the points and draw a curve of best fit



b. What is the y-intercept?

c. What is the direction of opening?

d. Is the vertex at max/min?

e. What are the zeros/x-int?

f. What is the axis of symmetry?

g. What is the Optimal Value?

h. What is the vertex?

2. Check that BOTH of the following equations model this parabola

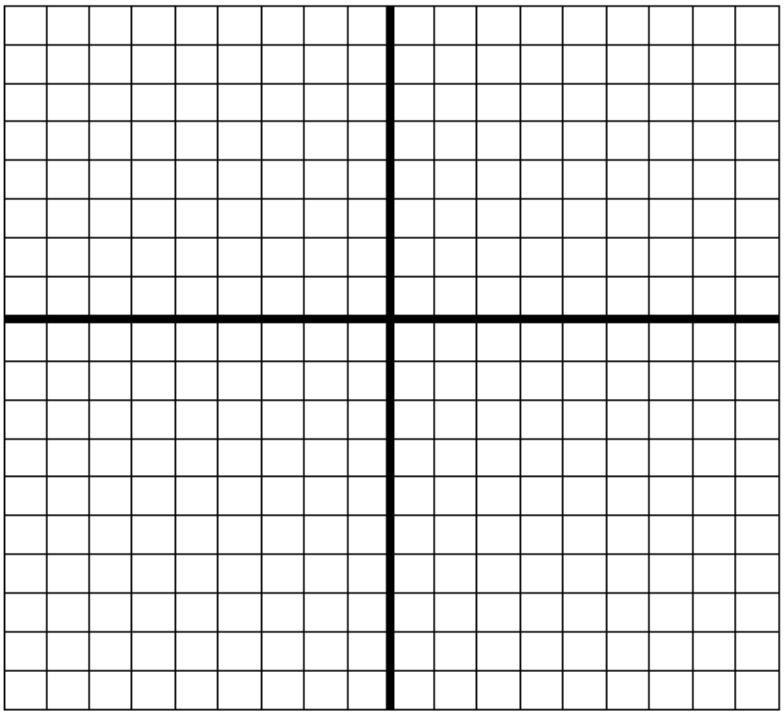
Standard form $y = x^2 - x - 6$

Factored form $y = (x - 3)(x + 2)$

3. Table of Values

x	y
-4	-10
-3	0
-2	6
-1	8
0	6
1	0
2	-10

a. Plot the points and draw a curve of best fit



b. What is the y-intercept?

c. What is the direction of opening?

d. Is the vertex at max/min?

e. What are the zeros/x-int?

f. What is the axis of symmetry?

g. What is the Optimal Value?

h. What is the vertex?

4. Check that BOTH of the following equations model this parabola

Standard form $y = -2x^2 - 4x + 6$

Factored form $y = -2(x - 1)(x + 3)$

Summarize:

5. **Standard Form** **Factored Form** **Vertex Form**

How to get axis of symmetry from just zeros?

How to get Max/Min optimal value?

6. i) Fix and identify what form(s) each parabola is in ii) state the characteristic(s) that can easily be seen from the equation given

a. $y = -2x - 35 + x^2$ b. $y = (x-7)(10x+5)$ c. $y = -(4x+2)^2 + 6$ d. $y = -(2x-4)(x+6)$

e. $y = (3x-6)^2 - 36$ f. $y = 12 - x^2 - 4x$ g. $y = 0.5(x+5)^2$ h. $y = 2x^2 - 8$

i. $y = 2(x-2)(4x+8)$ j. $y = 4x(x-6)$ k. $y = 2(x-4)^2$ l. $y = 2x^2 + 5$

7. $y = -2x^2 + 5x - 1$ Try to factor. Then talk about how to use a table of values to sketch. Show that the graph will not look symmetrical if vertex is not part of the table. Discuss the need to completing the square to see vertex.

8. The path of a soccer ball can be defined by the relation $h = -0.025d^2 + d$, where h represents the height, in metres, and d represents the horizontal distance, in metres, that the ball travels before it hits the ground.
   a) Find the d-intercepts.
   b) Sketch a graph of the relation.
   c) For what values of d is the relation invalid? Explain.
   d) What is the maximum height?
   e) How far will the ball have travelled horizontally at its maximum height?

9. Write a quadratic equation in factored form, using integers, the roots of the equation are $\frac{1}{2}$ and -5.

10. Write a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers and the roots are $\frac{1}{5}$ and $-\frac{2}{3}$.

ASSIGNMENT Find Equations (MPM)

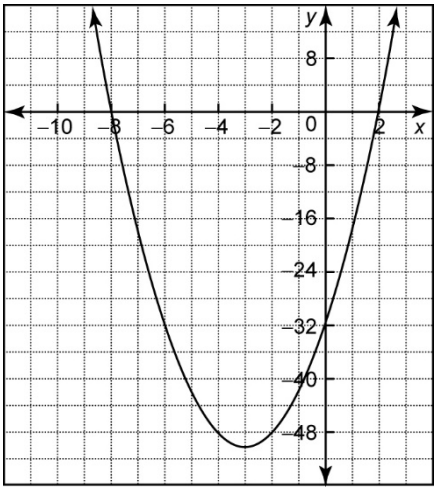
1. Write an equation for the parabola that satisfies each set of conditions.

a) vertex (–2, – 4), opening downward with a vertical stretch

b) The graph of $y = x^2$ is translated 9 units downward, translated 10 units to the left, reflected in the *x*-axis and compressed vertically

2. Determine an equation in factored form

a)



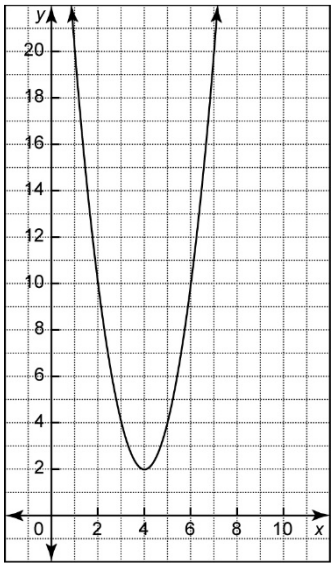
b) the parabola has one of the zeros at 2, axis of symmetry at -0.5 and goes through the point (4, -28)

c) the quadratic has only one zero at 3 and is reflected in the x-axis and has a vertical compression factor.

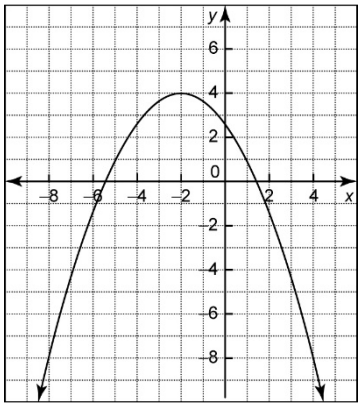
3. A parabola $y = ax^2 + k$ passes through the points (1, 5) and (3, 29). Find the values of *a* and *k*.

4. Write an equation for each parabola.

a)

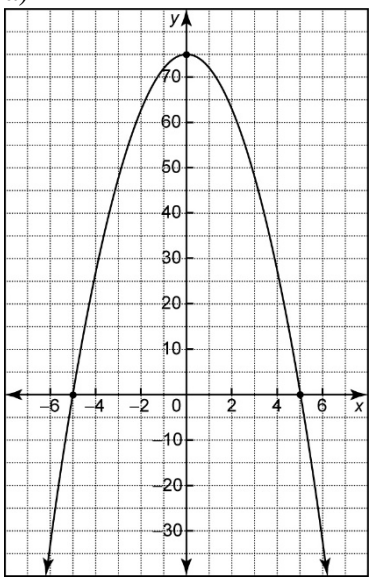


b)



5. Write an equation in the form $y = ax^2 + bx + c$ to represent each parabola.

a)



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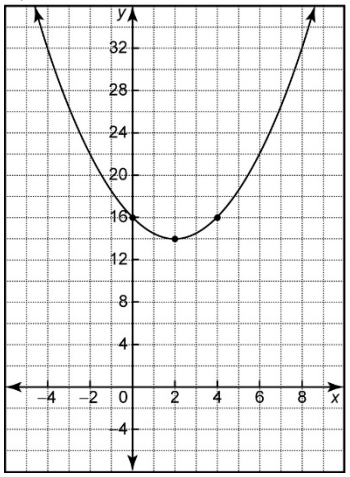


Use differences to check if tables are quadratic, then find the equations.

6.

x	y
0	-90
2	-30
4	6
6	18
8	6
10	-30

7.

X	-4	-3	-2	0	2	4
Y	12	6	2	0	6	20

8. $i: \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \rightarrow \begin{pmatrix} 5 \\ 10 \\ 17 \\ 26 \end{pmatrix}$

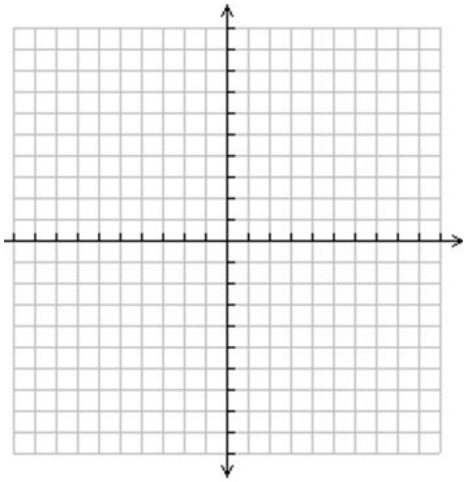
9. $l: \begin{pmatrix} -1 \\ 2 \\ 5 \\ 8 \\ 11 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ -54 \\ -162 \\ -324 \end{pmatrix}$

ASSIGNMENT Graph Quadratics (MPM)

i) state the coordinates of the vertex, **ii)** state all the transformations **iii)** sketch the graph on grid (show step by step transformations using different colours– this will be important for future types of non quadratic graphs)

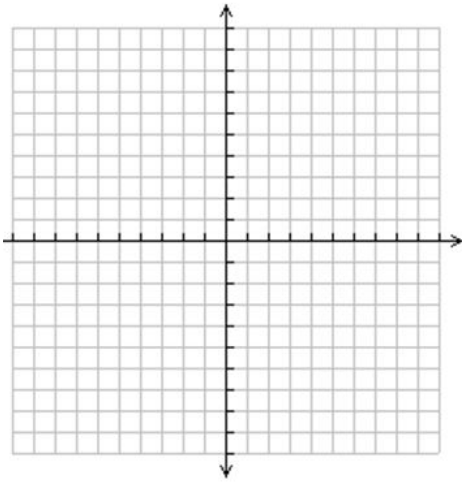
1.

$y = -2(x+4)^2 + 8$



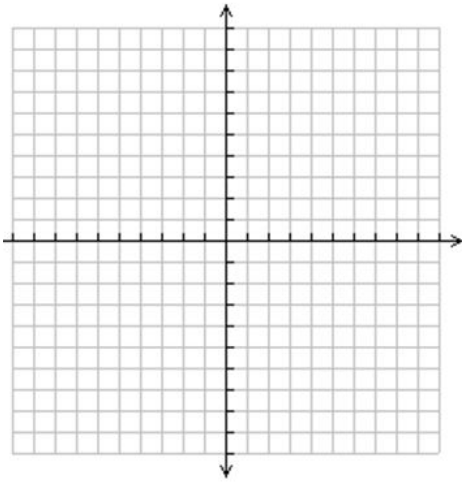
2.

$y = 0.5(x-2)^2 - 6$

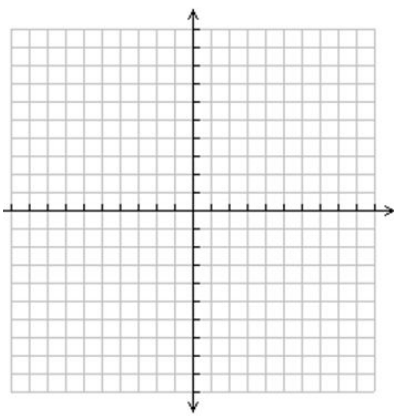
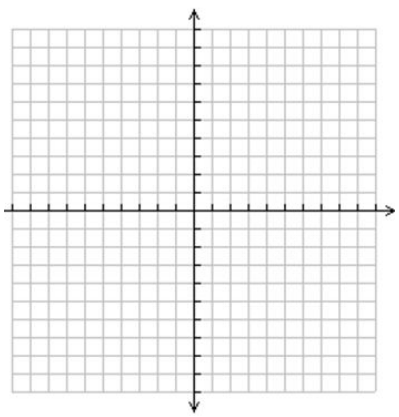
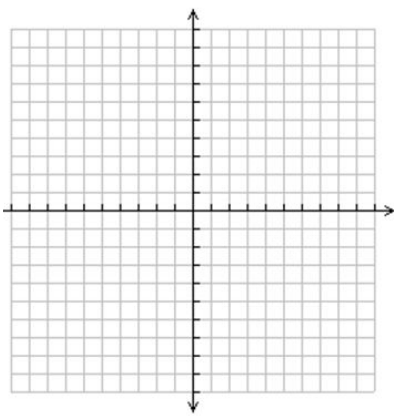


3.

$y = -(x+5)^2 + 7$



4. Complete the table for each parabola

Property	$y = (x-3)^2 + 2$	$y = -2(x+4)^2 + 3$	$y = \frac{1}{3}(x+1)^2 - 4$
Vertex			
step pattern (SHORTCUT for transformations)			
sketch			

5. Sketch each parabola. Label the vertex and the x-intercepts

a) $y = -\frac{1}{2}(x-3)(x-7)$

b) $y = 2(x-8)(2x+3)$

c) $y = -0.5(4-2x)(x+5)$

d) $y = 3(2+3x)(2-x)$

6. A rocket travels according to the equation $h = -4.9(t - 6)^2 + 182$, where h is the height, in metres, above the ground and t is the time, in seconds.

a) Sketch a graph of the rocket's motion.

b) Find the maximum height of the rocket.

c) How long does it take the rocket to reach its maximum height?

d) How high was the rocket above the ground when it was fired?

7. The path of a kicked football can be modelled by the relation $h = -0.02x(x - 45)$, where h represents the height, in metres, above the ground and x represents the horizontal distance, in metres, measured from the kicker.

a) Sketch the path of the ball.

b) When the ball hits the ground, how far has it travelled?

c) What is the maximum height of the ball?

d) What is the horizontal distance when this occurs?

e) If the goal post is 40 m away, will the kick clear the 3-m-high crossbar for a field goal?

8. Find an equation for the parabola with vertex $(-3, 1)$ that passes through the point $(-2, -1)$

ASSIGNMENT Solve Quadratic Equations (MPM)

Solve notes:

1. $6x = 3x^2$

2. $9c^2 = 49$

3. $\frac{3y^2 + 7}{2} = 5$

4. $3x^2 + 2x = 0$

5. $(5x + 6)(4x + 3) = 0$

6. $(3x - 1)(10x - 3) = 0$

7. $-x^2 + 5x + 6 = 0$

8. $4a^2 + 12a = -9$

9. $\frac{x^2}{6} + 2x + \frac{10}{3} = 0$

10. $\frac{x^2}{4} - \frac{x}{3} = \frac{1}{3}$

Solve using the LONG way, to understand the proof for the shortcut FORMULA on the next page,

11. $0 = x^2 + x - 7$

12. $2c^2 + 7c = 4$

13. $-2x^2 + 32x + 72 = 0$

ASSIGNMENT Quadratic Formula (MPM)

1. Show a proof of the quadratic formula Hint: complete the square using the standard form without actual numbers for a, b, c

Solve, can you the shortcut - formula

2. $x^2 - x = 5$

3. Find the x-intercepts, to the nearest hundredth; the vertex; and the equation in factored form for each quadratic relation. Sketch the parabola that each relation defines.

a) $y = -2x^2 + 4x + 7$

4. $x(2x - 3) = 7$

5. $3y^2 - (5y + 1)(2y - 3) = 3$

b) $y = 3x^2 + 6x + 4$

ASSIGNMENT # of Zeros (MPM)

For each quadratic relation, state the coordinates of the vertex, the direction of opening, and the number of x-intercepts.

1. $y = (x - 2)^2 + 3$
2. $y = -2(x + 5)^2 + 4$
3. $y = -(x + 1)^2$

NOTES:

Find the x-intercepts, to the nearest hundredth; the vertex; and the equation in factored form.

4. $y = 3x^2 + 6x + 4$
5. $y = -2x^2 + 4x + 7$
6. $y = -x^2 + 8x - 16$

NOTES:

ASSIGNMENT Solve Word Problems WITH Equations Given (MPM)

NOTES:
If you see "initial":

If you see "maximum" or "minimum":

OTHERWISE

1. The flight of a baseball is modelled by $y = -4.9x^2 + 9.8x + 14.7$ where x is the time, in sec, and y is the height, in m, above the ground.

a) What is the initial height of the ball?
b) What is the height of the ball 0.5 seconds after it was hit?
c) How long does it take for the ball to reach the ground?
d) Find the maximum height.

2. A regular polygon with n sides has $\frac{n(n-3)}{2}$ diagonals. Find the number of sides of a regular polygon that has 44 diagonals.

3. The path of a soccer ball can be defined by the relation $h = -0.025d^2 + d$, where h represents the height, in metres, and d represents the horizontal distance, in metres, that the ball travels before it hits the ground.
   a) Find the d-intercepts.
   b) Sketch a graph of the relation.
   c) For what values of d is the relation invalid? Explain.
   d) What is the maximum height?
   e) How far will the ball have travelled horizontally at its maximum height?

4. Sipapu Natural Bridge is in Utah. Find the horizontal distance, x, in metres, across this natural arch at the base by solving the equation $-0.04x^2 - 1.56x + 3.28 = 0$.

5. The path of a skydiver can be modelled by the relation $h = -40t^2 + 6000$, where h represents the height of the skydiver in metres, and t represents time in seconds.

a) From what height does the skydiver jump out of the plane?

b) How long does the skydiver take to reach the ground?

6. A textbook falls from the top shelf of a shaky bookcase. The path of the book can be modelled by the relation $h = -9t^2 + 90$, where h represents the height of the book above the floor, in centimetres, and t represents time in milliseconds.

a) What is the height of the top shelf?

b) How long does it take the book to reach the floor?

7. A supporting arch of a bridge can be represented by the quadratic function $y = -0.0625x^2 + 9$, where x is the horizontal distance (in metres) and y is the height of the arch (in metres).

a) What is the vertex of this parabola?

b) What is the maximum height of the arch?

c) If the x-intercepts represent the beginning and the end of the arch, how wide is the base of the arch

8. A rectangle has dimensions $x + 11$ and $2x + 5$, both measured in centimetres. Determine the value of x so that the area is 117 cm^2.

9. Michael owns a trampoline. He wants to see how high he can jump. The path of one jump can be modelled by the relation $h = -4t^2 + 80t + 12$, where h represents Michael's height above the ground in centimetres and t represents time in seconds.

a) What is the height of the trampoline?

b) What is the maximum height Michael reaches?

c) How long does it take Michael to reach this height?

d) What is the height at 2 seconds?

e) How long would it take for Michael to reach a height of 348 cm?

10. A family restaurant has daily expenses that can be modelled by the quadratic relation $C = 4t^2 - 28t + 40$, where C represents the total cost in dollars, and t represents the time in hours the restaurant is open.

a) What is the number of hours the restaurant is open for minimum cost?

b) What is the cost per day when the restaurant is not open for business?

c) How many hours was the restaurant open if the total cost per day was $160?

d) What is the cost per day if the restaurant is open for 8 hours?

ASSIGNMENT Solve Word Problems WITHOUT Equations Given (MPM)

Revenue Problem

1. Angie sold 1200 tickets for the holiday concert at $20 per ticket. Her committee is planning to increase the prices this year. Their research shows that for each $2 increase in the price of a ticket, 60 fewer tickets will be sold.
   a) Determine the revenue relation that describes the ticket sales.
   b) What should the selling price per ticket be to maximize revenue?
   c) How many tickets will be sold at the maximum revenue?
   d) What is the maximum revenue?

Fence/Rope off an Area Problem

2. For a park swimming area, 840 m of line is used to mark off the permissible area in a shape of a rectangle. One side not roped off is next to the beach. Find the dimensions of the swimming area that will make it a maximum.

3. Suppose that half of a piece of 40 cm wire is bent to construct a rectangle. Use a quadratic model to determine the dimensions that will give an area of 24 cm^2

Geometry Problem

4. The hypotenuse of a right triangle measures 20 cm. The sum of the lengths of the legs is 28 cm. Find the length of each leg.

5. A rectangular skating rink measures 30 m by 20 m. It is doubled in area by extending each side of the rink by the same amount. Determine by how much each side was extended.

Frame-Border Problem

6. A picture that measures 10 cm by 5 cm is to be surrounded by a mat. The mat is to be the same width on all sides of the picture. The area of the mat is to be twice the area of the picture. What is the width of the mat?

Volume Problem

7. A rectangular piece of tin 50 cm by 40 cm is made into a lidless box of base area 875 cm^2 by cutting squares of equal sizes from the corners and bending up the sides.

a) Find the side length of each removed square.

b) Find the volume of the box.

Translate English to Math Problems

8. A triangle has an area of 308 cm^2. If the base is 2 cm more than three times the height of the triangle, find the base and height of the triangle

Falling object Problem

9. A model rocket is launched from the deck that is 15 meters high, with an initial speed of 100 m/sec.
   a) What is the equation that would model this?
   b) What is the height of the model rocket after 2 s?
   c) What is the maximum height reached by the model rocket?
   d) How long did the model rocket take to reach this height?

10. The sum of the squares of four consecutive integers is 630. Find the integers.

Revenue Problem

11. A harbour ferry service has about 240 000 riders per day for a fare of $2. The port authority wants to increase the fare to help with increasing operational costs. Research has shown that for every $0.10 increase in the fare the number of riders will drop by 10 000.
    a) What is the revenue equation that will represent this?
    b) How many times should the fare be increased to maximize the revenue? (show two methods)
    c) What is the new fare that maximizes the revenue?
    d) How many riders are needed for the maximum revenue?
    e) What is the maximum revenue?