Additional Practice Problems for Function Transformations

In Problems 1-6, explain how the graph of g is created from the given function f.

Example 1 The basic function is $f(x) = x^{-2}$ and the function g is defined by $g(x) = -\frac{3}{(4-2x)^2}$.

Solution. First, note that the standard form for the function g is

$$g(x) = (-3) \frac{1}{\left(-2\left[x-2\right]\right)^2} = -3f\left(-2\left[x-2\right]\right) + 0$$

Now we can see that the transformation constants are h = 2, b = -2, a = -3, and v = 0. Working from the inside out, we know the graph of g is created by

- 1. First taking the graph of f and shifting it to the right by two units.
- 2. Taking the result of Step 1 and scaling it horizontally by a factor of 2, then reflecting the result about the line x = 2.
- 3. Taking the result of Step 2 and scaling it vertically by a factor of 3, then reflecting the result about the x-axis.

- 1. The basic function is f(x) = |x| and the function g is defined by g(x) = 2|x| + 1.
- 2. The basic function is f(x) = |x| and the function g is defined by g(x) = 2|x+1|.
- 3. The basic function is f(x) = |x| and the function g is defined by g(x) = |2x + 1|.
- 4. The basic function is $f(x) = \sqrt{x}$ and the function g is defined by $g(x) = -\sqrt{x-3}$.
- 5. The basic function is $f(x) = \sqrt{x}$ and the function g is defined by $g(x) = \sqrt{3-x}$.
- 6. The basic function is $f(x) = \sqrt{x}$ and the function g is defined by $g(x) = 4\sqrt{2x-3} + 7$.

In Prolems 7-10, construct the formula for the function g representing the given transformation of the function f.

Example 2 Find a formula g representing the graph of $f(x) = x^3$ after it has been shifted left by two units, reflected about the x-axis, then shifted down by three units.

Solution. We want to construct g(x) = af(b[x - h]) + v. We are told that h = -2, b = 1 (there is no mention of horizontal scaling), a = -1, and v = -3. Therefore, we know

$$g(x) = (-1)f((1)[x - (-2)]) + (-3)$$

= $-f(x + 2) - 3$
= $-(x + 2)^3 - 3$

- 7. Find a formula g representing the graph of $f(x) = x^2$ after it has been shifted three units right and four units down.
- 8. Find a formula g representing the graph of $f(x) = x^{-1}$ after it has been shifted three units right, reflected about the line x = 3, then shifted one unit up.
- 9. Find a formula g representing the graph of $f(x) = x^3$ after it has been scaled vertically by a factor of 4 then shifted three units up.
- 10. Find a formula g representing the graph of $f(x) = \sqrt{x}$ after the following steps have been performed.
 - (a) The graph of f is shifted two units to the right.
 - (b) The result of Step (a) is scaled horizontally by a factor of three, then reflected about the line x = -2.
 - (c) The result of Step (b) is reflected about the x-axis.
 - (d) The result of Step (c) is shifted five units up.

In Problems 11-13, the transformed graph of a basic function is given. Construct a formula g(x) = af(x - h) + v that represents this graph. (Assume b = 1.)

Example 3 Construct a formula g for the graph shown.



Solution. The basic graph is a parabola, and the basic parabola has a turning point (vertex) at (0,0). The turning point for the given graph lies at (-1,-1), so we know the graph has been shifted one unit left and one unit down. This tells us that

$$g(x) = af(x+1) - 1 = a(x+1)^2 - 1$$

To find the constant a, pick a point on the graph other than the vertex and plug this point into the formula. For example, the point (1, -3) lies on the graph. This tells us

$$-3 = a(1+1)^2 - 1 \Longrightarrow -2 = 4a \Longrightarrow -\frac{1}{2} = a$$

Thus, we know that the desired formula is $g(x) = -\frac{1}{2}(x+1)^2 - 1$.

11. Construct a formula g for the graph shown.



12. Construct a formula g for the graph shown.



13. Construct a formula g for the graph shown.



The graph of a function f is given below. In problems 14-16, sketch the graph represented by the function g.



14. On the grid provided, sketch the graph of g(x) = 1 + f(x - 2).



Graph of g

15. On the grid provided, sketch the graph of g(x) = f(1 - x).



Graph of g

16. On the grid provided, sketch the graph of g(x) = f(2x).



Answers.

- 1. The function f is first scaled vertically by a factor of 2, then shifted one unit up.
- 2. The function f is first shifted one unit to the left, then scaled vertically by a factor of 2.
- 3. The function f is first shifted one-half unit to the left, then scaled horizontally by a factor of 2.
- 4. The function f is first shifted three units to the right, then reflected about the x-axis.
- 5. The function f is first shifted three units to the right, then reflected about the line x = 3.
- 6. The function f is first shifted 3/2 units to the right, scaled horizontally by a factor of 2, scaled vertically by a factor of 4, and finally shifted seven units up.
- 7. We have $g(x) = (x-3)^2 4$.
- 8. We have $g(x) = 1 + \frac{1}{3-x}$.
- 9. We have $g(x) = 4x^3 + 3$.
- 10. We have $g(x) = 5 \sqrt{6 3x}$.
- 11. The desired formula is $g(x) = (x-2)^2 1$.
- 12. The desired formula is g(x) = 2|x+1| 3.
- 13. The desired formula is $g(x) = -3\sqrt{x} + 2$.
- 14. The desired graph will be



Graph of g(x) = 1 + f(x - 2)



16. The desired graph will be

