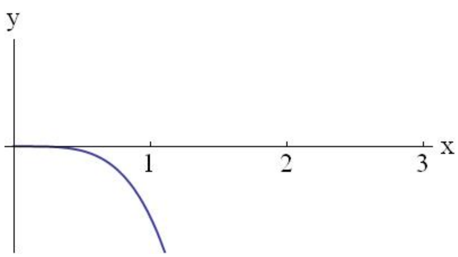
Questions

1. State the power and constant of variation for the function $f(x) = -2x^{-3}$. Graph the function and analyze it (domain, range, asymptotes, extrema, etc.).

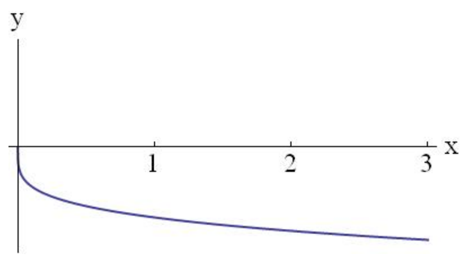
2. The power P (in watts) produced by a windmill is proportional to the cube of the wind speed v (in mph). If a wind of 10 mph generates 15 watts of power, how much power is generated by winds of 20, 40, and 80 mph? Make a table and explain the pattern.

3. Match the function to its graph

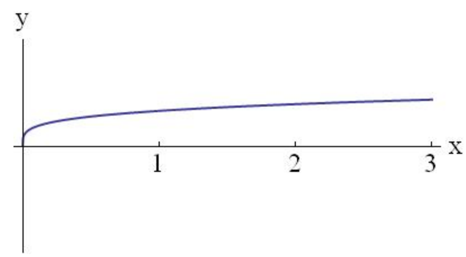
(i) $y = -2x^4$ (ii) $y = -2x^{1/4}$ (iii) $y = x^{1/4}$ (iv) $y = x^{-2}$ (v) $y = -\dfrac{1}{4}x^{-2}$ (vi) $y = x^3$



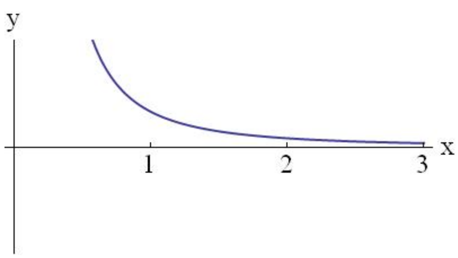
(a)



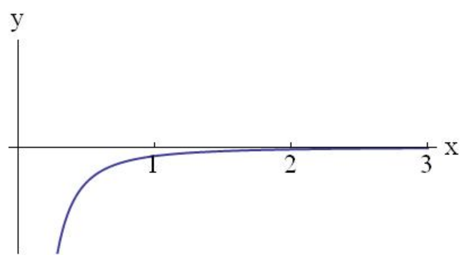
(b)



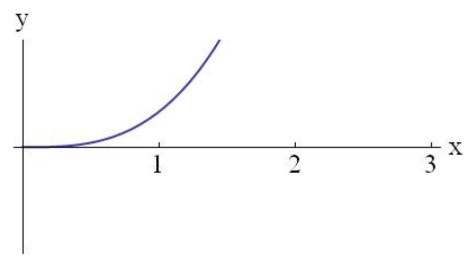
(c)



(d)



(e)



(f)

4. Is the function $f(x) = \frac{1}{3}x^{-3}$ even, odd, or neither? Is it continuous?

5. Is the function $f(x) = x^{1/3}$ even, odd, or neither?

6. Evaluate the following limits by thinking of what the sketch of the function looks like.

(a) $\lim_{x\to-\infty} x^4 =$

(b) $\lim_{x\to-\infty} x^3 =$

(c) $\lim_{x\to-\infty} x^{1/2} =$

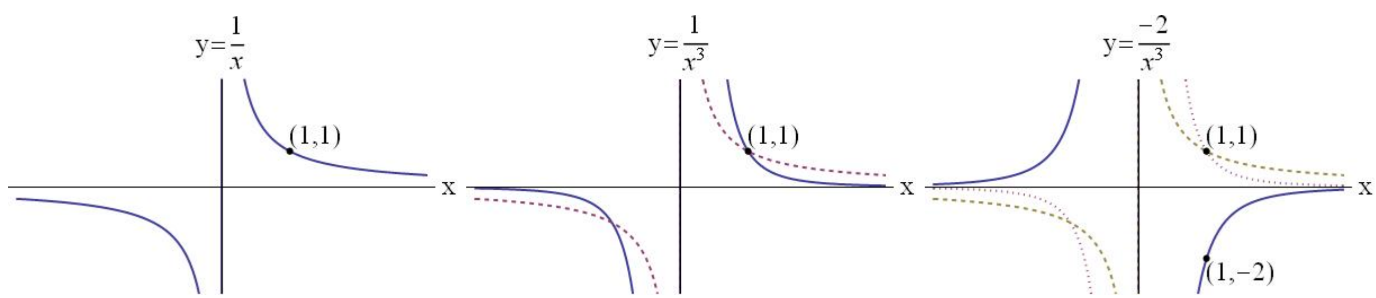
(d) $\lim_{x\to\infty} x^3 =$

Solutions

1. State the power and constant of variation for the function $f(x) = -2x^{-3}$. Graph the function and analyze it.

The function has power -3. Since the power is less than zero, this is an inverse variation function. The constant of variation is -2.

Sketch (from the sketch of the basic function $y = x^{-1}$, so sketched by hand, not with a computer, although I used a computer to draw the sketches).



Domain: $x \in (-\infty, 0) \cup (0, \infty)$.
Range: $y \in (-\infty, 0) \cup (0, \infty)$.
Continuous: the function is discontinuous at $x = 0$.
Increasing/Decreasing: The function is increasing for $x \in (-\infty, 0)$, and increasing for $x \in (0, \infty)$.
Symmetric: The function is odd ($f(-x) = f(x)$).
Boundedness: The function is not bounded above or below.
Extrema: none.
Asymptotes: The function has a horizontal asymptote of $y = 0$, and a vertical asymptote at $x = 0$.
Vertical Asymptotes: $\lim_{x\to 0^+} (-2x^{-3}) = -\infty$, $\lim_{x\to 0^-} (-2x^{-3}) = \infty$.
End Behaviour: $\lim_{x\to -\infty} (-2x^{-3}) = 0$ and $\lim_{x\to \infty} (-2x^{-3}) = 0$

2. The power P (in watts) produced by a windmill is proportional to the cube of the wind speed v (in mph). If a wind of 10 mph generates 15 watts of power, how much power is generated by winds of 20, 40, and 80 mph? Make a table and explain the pattern.

We are told the power is proportional to the cube of the wind speed. Converting that to mathematics leads us to write

$$P \propto v^3$$

where P is the power in watts and v is the wind speed in mph. The symbol between them indicates that they are proportional to each other.

We need to know a relation for when they are equal, and we can get that by inserting a *proportionality constant* k:

$$P = kv^3$$

You could have started your solution with this relation. We don't know the value of k yet, but we can find it using some of the information given.

We are told that when $v = 10$ mph, $P = 15$ watts, and we can use this do determine the constant k:

$$\begin{aligned} P &= kv^3 \\ 15 &= k10^3 \\ \frac{15}{1000} &= k \\ k &= \frac{3}{200} \end{aligned}$$

and we can write the relation between wind speed in mph and power in watts as:

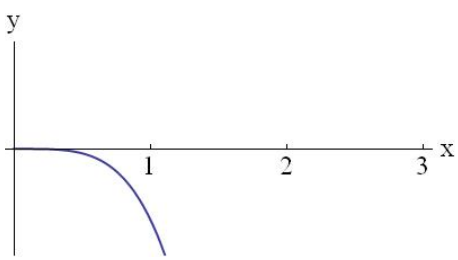
$$P = \frac{3}{200}v^3.$$

Now we can construct the table that is asked for:

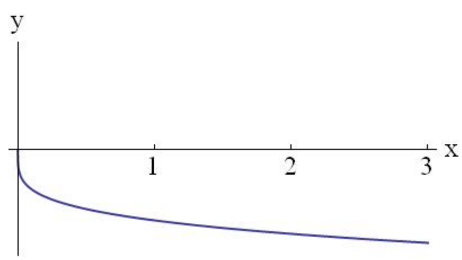
v (mph)	$P = \frac{3}{200}v^3$ (watts)
10	15 (this data was given)
20	120
40	960
80	7680

The values for P at wind speeds of 20, 40, 80 mph were calculated using the relation. As the wind speed increases, the power grows rapidly, by a cubic relation. The proportionality constant $k = 3/200$ scales the cubic growth.

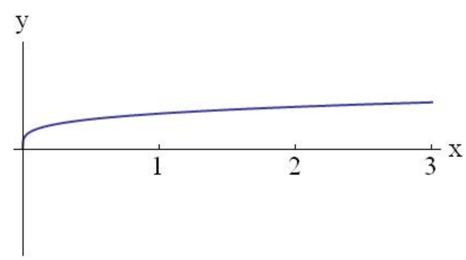
3. Match the function to its graph



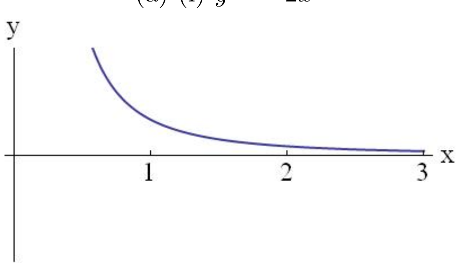
(a) (i) $y = -2x^4$



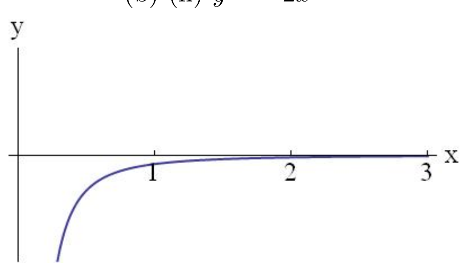
(b) (ii) $y = -2x^{1/4}$



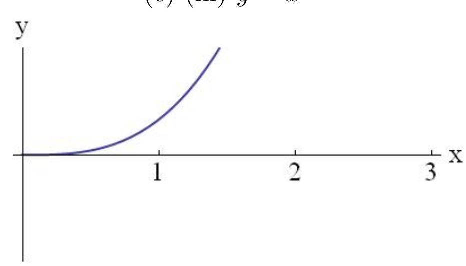
(c) (iii) $y = x^{1/4}$



(d) (iv) $y = x^{-2}$



(e) (v) $y = -\frac{1}{4}x^{-2}$



(f) (vi) $y = x^3$

4. Is the function $f(x) = \frac{1}{3}x^{-3}$ even, odd, or neither? Is it continuous?

To check even/odd/neither, we evaluate $f(-x)$ and see what it simplifies to.

$$f(-x) = \frac{1}{3}(-x)^{-3} = \frac{1}{3} \cdot \frac{1}{(-x)^3} = -\frac{1}{3} \cdot \frac{1}{x^3} = -\frac{1}{3}x^{-3} = -f(x)$$

So, since $f(-x) = -f(x)$, we have that f is odd. Since it is not defined at $x = 0$, it is not continuous at $x = 0$.

5. Is the function $f(x) = x^{1/3}$ even, odd, or neither?

$$f(-x) = (-x)^{1/3} = -x^{1/3} = -f(x)$$

So, since $f(-x) = -f(x)$, we have that f is odd.

6. Evaluate the following limits by thinking of what the sketch of the function looks like.

(a) $\lim\limits_{x \to -\infty} x^4 = \infty$

(b) $\lim\limits_{x \to -\infty} x^3 = -\infty$

(c) $\lim\limits_{x \to -\infty} x^{1/2}$ does not exist, since the domain of $x^{1/2} = \sqrt{x}$ is $x > 0$

(d) $\lim\limits_{x \to \infty} x^3 = \infty$