

POLYNOMIALS – journal questions (MHF)

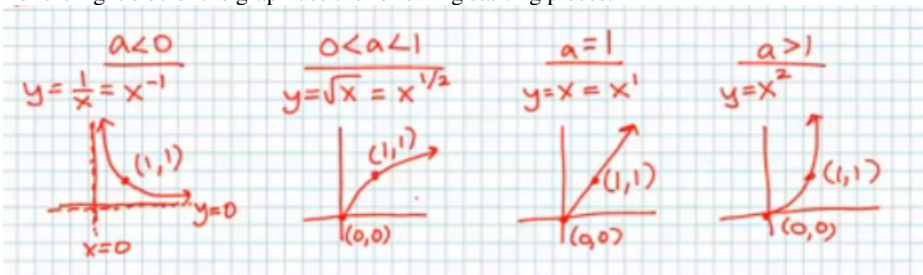
Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. POWER FUNCTIONS, POLYNOMIAL INTRO

a. Copy the following

SKETCHING POWER functions of the form $y = x^a$:

- For the right side of the graph use the following starting pieces:



- Decide what the left side would look like – think: Will there be a left side? Will it be odd symmetry? Will it be even symmetry?
- Adjust the graph if there is a reflection

b. Do a little sketch beside each of the following, add in any notes for yourself within part a – if you find the above steps are not enough

$y = -2x^{\frac{2}{3}}$	$y = 6x^{\frac{1}{5}}$	$y = -x^{\frac{1}{4}}$	$y = \frac{2}{x^3}$	$y = -5x^{-4}$	$y = x^{-\frac{1}{3}}$
$y = \frac{2}{3}x^{\frac{8}{5}}$	$y = -9x^{\frac{5}{3}}$	$y = 0.5x^{1.5}$	$y = \frac{1}{\sqrt[4]{x}}$	$y = -2x$	$y = 3x^5$

c. Define what is a POLYNOMIAL graph, equation, table. Which of the above power functions are polynomial?

d. DEGREE & LEADING COEFFICIENT, define and explain how to find both for the following questions:

from standard form: $y = 3x - 9x^3 + 10x^6 - x^2 + 8$ and from factored form: $y = -2(x - 4)(3x^3 - x^2)(1 - x^4)$

2. PROPERTIES & GRAPHS of polynomials

a. Take a look at degree 4 and 5 examples of possible polynomial graphs then fill in the following chart with the required information:



		# of possible roots	# of possible t.p.	Domain & Range	End Behaviour	
					pos LC	neg LC
Odd degree polynomials	$n = 7$					
	$n = 9$					
Even degree polynomials	$n = 6$					
	$n = 8$					

- Make a note that the end behaviour can be described in several ways, be comfortable in all of them:
 Informally: “up, up” or “quadrant II to quadrant I”
 Formally: “as $x \rightarrow -\infty$, $y \rightarrow \infty$ and as $x \rightarrow \infty$, $y \rightarrow \infty$ ” or “ $\lim_{x \rightarrow -\infty} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = \infty$ ”

- Make a note that “roots” can be thought of as “solutions” or sometimes “zeros/x-int” (if equation is manipulated to have one side zero). “Turning points” can also be called “local extreme values”

b. Explain how to SKETCH FROM FACTORED FORM. Ensure to talk about “cut” like linear function for multiplicity of 1, “bounce” like quadratic function for even multiplicity, and “bend” like cubic function for odd multiplicity of higher than one.

$y = x^3(2 - x)^2(3 - 10x)$ and $y = -2(x + 2)^4(3 - x)^2(x + 5)^3$

3.

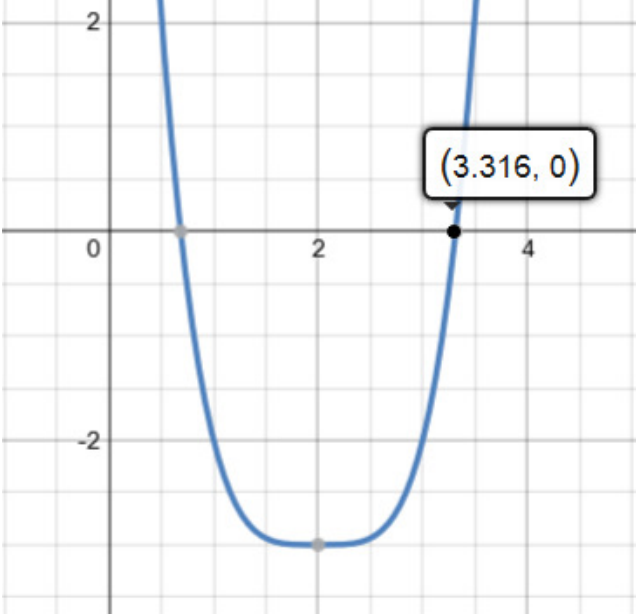
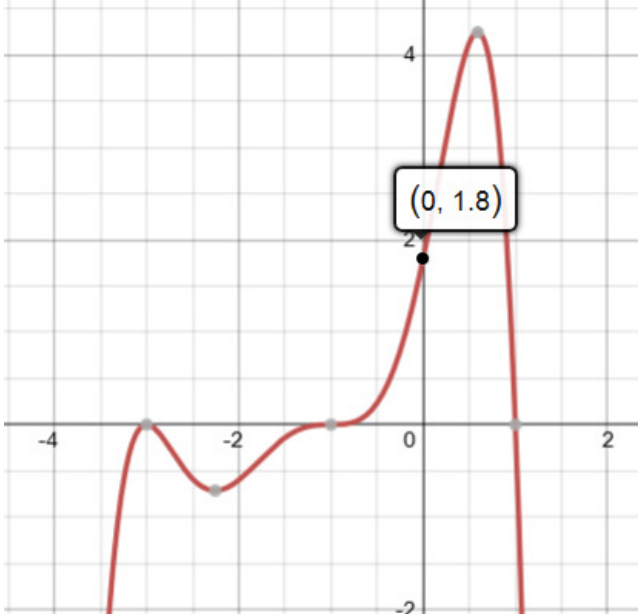
a. TRANSFORMATIONS OF POLYNOMIALS.

Finish the sentence: "Transformations are visible only if _____ and graphs will resemble _____."

Sketch $y = -\frac{1}{3}(3x + 15)^3 + 6$ by showing only shifts and reflections. Make a note on how to find other points.

b. FAMILIES of EQUATIONS. What is a FAMILY of equations?

Write a NOTE: "Look over UNIT 3 Discrete & Finance journal #2 on how to find a cubic/quadratic polynomial equation using differences". Then, provide solutions to:

<p>i. Find equation of degree 8 graph</p> 	<p>ii. Find equation of degree 6 graph</p> 
<p>iii. State the family of degree 4 polynomials that has been reflected in x-axis, vertically stretched by 2 (decide to use 2 or 1/2), with x-intercepts at $4, 1 \pm \sqrt{3}$ (all multiplicity 1).</p>	<p>iv. State the family of degree 5 polynomials, that pass through points (0, 50) and (-2, -18) and have ONLY two real roots, one at -1 (multiplicity 1) and another at -5 (multiplicity 2).</p>

4. DIVIDING POLYNOMIALS

a. Explain the terms divisor, dividend, remainder, quotient and how they all relate to one another. Include numeric example of recording mixed and improper fractions to help recall the 2 result statement formulas.

b. Long division example and key steps. Ensure you end with a final statement of result – rational and polynomial versions.

$$(6x^4 - 5x^2 - 3x + 9) \div (2x^2 - 5)$$

c. Synthetic division example and key steps. Ensure you end with a final statement of result – just polynomial version.

$$(-12x^4 + 2x^3 - 500x + 1) \div (2x + 9)$$

d. Add in IMPORTANT NOTES

- Explain when you must insert zeros as placeholders
- Synthetic division only works with linear divisors so unfortunately you have to know the long division method even if you like the synthetic method better.
- Make a note of how to record the result properly after synthetic division if the original divisor had a coefficient on x.

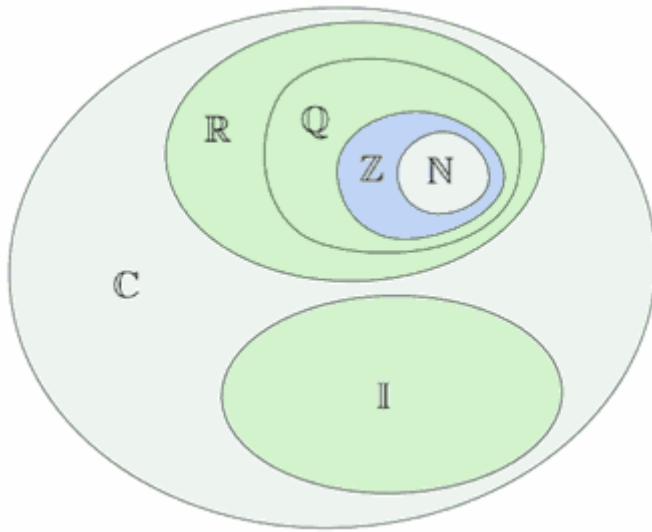
5. THEOREMS

a. Copy/Paste the following Theorems

<p>What is a root? For a polynomial $f(x)$, r is a root if and only if $f(r) = 0$</p>	<p>Remainder Theorem When a polynomial $f(x)$ is divided by a linear divisor $(x - a)$, then the remainder is equal to $f(a)$.</p> <p>Proof: $f(x) = (\text{quotient})(\text{divisor}) + \text{remainder}$ divisor is $x - a$ $f(x) = (\text{quotient})(x - a) + \text{remainder}$ sub in the zero of the divisor $x = a$ $f(a) = (\text{quotient})(0) + \text{remainder}$ $f(a) = \text{remainder}$ (If the divisor is not linear then the remainder might not be just one constant number but a polynomial – and this theorem will not work)</p>	<p>Factor Theorem The polynomial $f(x)$ has a factor $(x - k)$ if and only if (this means the implication goes both ways) $f(k) = 0$ <u>Proof - "if" implication:</u> if $x - k$ is a factor of $f(x)$ then it divides into $f(x)$ evenly This means the remainder is zero. By Remainder theorem $f(k) = \text{remainder}$ Therefore $f(k) = 0$, since remainder is zero. <u>Proof - "only if" implication:</u> if $f(k) = 0$ then by Remainder theorem $\text{remainder} = 0$ but that implies the divisor $x - k$ goes into $f(x)$ evenly which means the divisor $x - k$ is a factor.</p>
<p>Fundamental Theorem of Algebra Any polynomial of degree n will have exactly n roots. Roots can be rational, irrational or complex.</p>	<p>Descarte's Rule of Signs The number of positive roots of $f(x)$ is equal to the # of sign changes in the list of coefficients of $f(x)$, or less than this #, skip count down by 2. The number of negative roots of $f(x)$ is equal to the # of sign changes in the list of coefficients of $f(-x)$, or less than this #, skip count down by 2.</p> <p>example: $f(x) = 2x^3 - x^2 - 6x + 3$ coefficients: 2, -1, -6, 3 #of sign changes = 2 ∴ two, or no pos roots $f(-x) = -2x^3 - x^2 + 6x + 3$ #of sign changes = 1 ∴ one neg root $f(x) = (2x - 1)(x^2 - 3)$ actual roots are $\frac{1}{2}, \pm\sqrt{3}$</p>	<p>Irrational Root Theorem Let a and b be rational numbers and \sqrt{b} an irrational number. If $a + \sqrt{b}$ is a root of a polynomial, then $a - \sqrt{b}$ is also a root.</p> <p>Imaginary(Complex) Root Theorem If $a + bi$ is a root of a polynomial, then $a - bi$ is also a root.</p>
<p>Rational Root Test A polynomial $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ can have rational roots only of the form $\pm \frac{p}{q}$ where p is any factor of a_0, and q is any factor of a_n. These are possible roots, you must test the candidates. example: $f(x) = 6x^2 - 7x - 20$ possible roots are $\pm \frac{1, 2, 4, 5, 10, 20}{1, 2, 3, 6}$ and all its combinations or $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{20}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$ $f(x) = (2x - 5)(3x + 4)$ actual roots are $\frac{5}{2}$ and $-\frac{4}{3}$</p>	<p>b. REMAINDER THEOREM example Find the remainder (without doing the division) of $f(x) = 2x^5 + 4x^4 - 5x^3 - 6x^2 + 2x - 4$ when it is divided by $x + 3$</p> <p>c. FACTOR THEOREM example to factor a polynomial fully. (Show solution using the synthetic division method) $f(x) = 6x^4 - 19x^3 - 2x^2 + 44x - 24$</p> <p>d. COMPARING COEFFICIENTS method If the dividend is $-2x^4 - 5x^3 + 2x^2 + 3x - 1$, quotient is $-2x^3 - 7x^2 - 5x - 2$ and remainder is -3, find the divisor</p>	<p>6. SOLVE WORD PROBLEM A box that holds an expensive pen has square ends, and its length is 13 cm longer than its width. The volume of the box is 60 cm^3. Determine the dimensions of the box.</p> <p>7. Solve INEQUALITIES a. Use a sketch of the related polynomial $6x^3 - 41x > 11x^2 - 60$</p> <p>b. Use a +/- table $16x^2 + 36x + 18 \leq 2x^4 + 4x^3$</p>

8. IMAGINARY #s and COMPLEX Roots of Polynomials (AP)

a. Copy/paste the following into your journal, name each set of numbers, insert at least one numerical example into each circle.



b. Define the imaginary number, i , complex numbers $z = a + bi$, their conjugate z^* OR \bar{z} and illustrate how to sketch complex numbers on the Argand-Gauss coordinate plane.

c. OPERATIONS WITH COMPLEX NUMBERS: Simplify into $a + bi$ form

i. $3\sqrt{-98}$

ii. $i^{345} + 2i^{-23}$

iii. $2(3 - i) - 5(-2 + 3i)$

iv. $(2 - 5i)(7 + 2i)$

v. $\frac{6}{2-i}$

vi. $|\sqrt{17} - i\sqrt{8}|$

d. Find the unique polynomial of degree 4 that has zeros $1 - 2i$ and $1 + i$ that passes through the point $(0, 20)$.

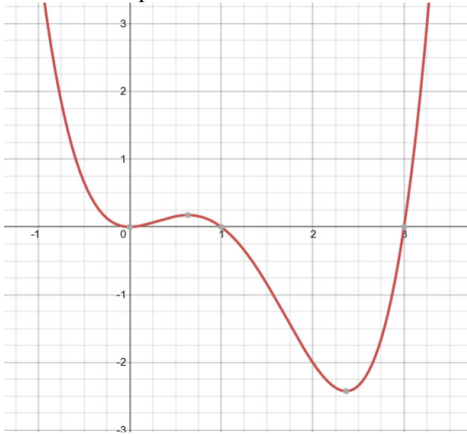
e. Solve

$$5x^3 - 15x^2 + 20x = 60; \quad x \in \mathbb{C}$$

9. Summarize, using your own examples, the key information on how to sketch:

a. ABSOLUTE VALUE of Polynomials (AP)

Sketch overtop

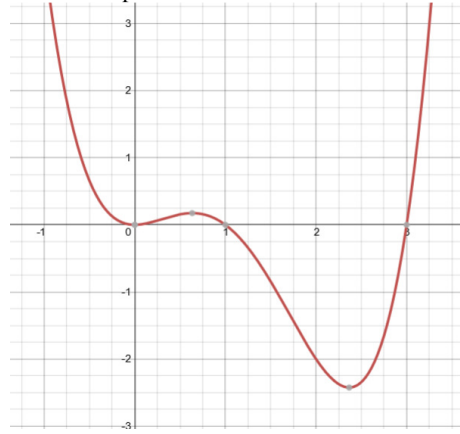


d. Sketch, label each turning point

$$f(x) = -2|(x+1)(x-5)| + 1$$

b. RECIPROCALs of Polynomials (MHF)

Sketch overtop

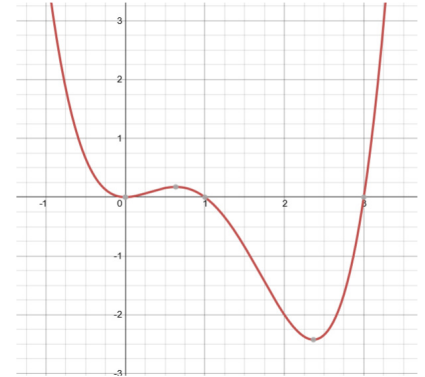


e.

$$f(x) = \frac{1}{x^2 + 2}$$

c. ROOTS of Polynomials (AP)

(do a different colour for even root and odd root)



f.

$$y = \sqrt{(x-3)^2 - 4}$$