Big idea

In this unit will you extend your knowledge from linear, quadratic and cubic functions to general polynomial functions. Recall that the quadratic function can be written in 3 forms – factored, expanded/standard and vertex/transformed forms. When you study polynomials you will also use all three of these forms. The vertex form will be called the transformed form since a polynomial function may have saddle point, or many turning points, not just one vertex as a parabola did. You will learn that polynomial functions can be described by their end-behaviour, symmetry, number of zeros and number of turning points. You will be introduced to how to sketch the polynomial function from factored form as well as how to find the equation from a given graph. The main part of this unit will be to learn how to factor polynomials, which can be a long process involving long division or synthetic division. You will not find where exactly the turning points occur just yet – that is part of calculus.

Feedback & Assessment of Your Success

Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: _____ Questions to ask the teacher:
	2-3	Power functions Polynomials Intro Journal #1				
	4-5	Properties & Graphs of Polynomials Journal #2				
2days	6-9	Transformations of Polynomials and Families of Equations Journal #3				
0.5day	10-11	Long & Synthetic Division Journal #4				
1.5days	12-16	Theorems Journal #5				
	17-19	Solve Polyn Word Problems Journal #6				
0.5day	20	Regression with Technology				
0.5day	21-22	Recap of Factoring and Inequalities Journal #7				
	23-24	Imaginary numbers & Complex roots. (AP) Journal #8				
	25-28	Absolute Values (AP) Reciprocals (MHF) Roots of Polynomials (AP) Journal #9				

ASSIGNMENT Power Functions & Polynomials Intro

Sketch the following power functions. State if the given is odd/even/neither symmetry – that should help you draw the sketch

1. $y = 2x^7$

2. $y = -4x^8$

3. $y = -x^{\frac{1}{6}}$

4. $y = \frac{1}{4}x^{-2}$

5. $y = -\frac{2}{x^5}$

6. $y = 7x^{\frac{2}{3}}$

7. $y = 2\sqrt[4]{5x}$

8. $y = 6x^{\frac{8}{3}}$

9. $y = 4x^{\frac{-1}{3}}$

10. $y = -\frac{2}{3}x^{\frac{5}{7}}$

Explain how to classify from each of the following "Is it a POLYNOMIAL?"

- Graphs
- Tables
- Equations

Are all power functions polynomial functions?

Define degree and leading coefficient.

Determine if the following are polynomials or not. Justify why or why not.

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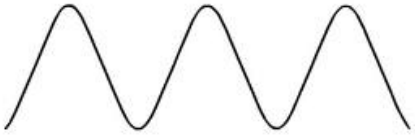

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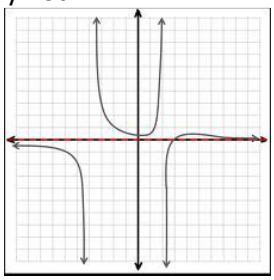

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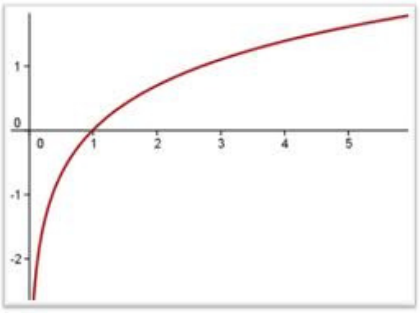

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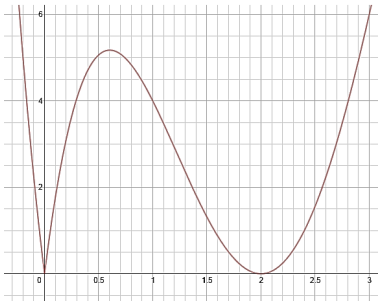

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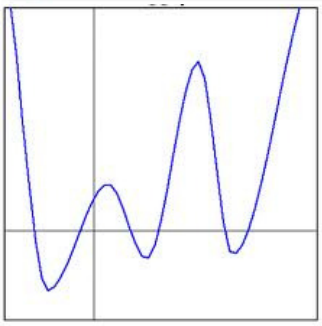

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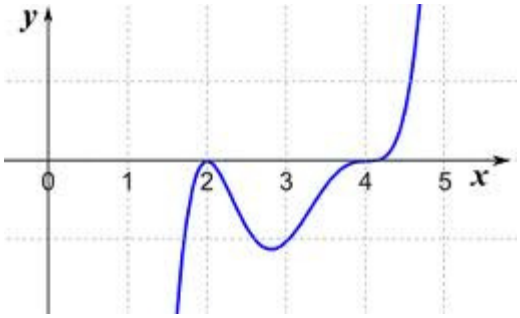


17.

x	0	1	2	3	4	5
y	3	5	19	57	131	253

18.

x	0	1	2	3	4	5
y	4	5	7	11	19	35

19. $y = -\frac{2}{x^5} + 4x - 3$

20. $y = \sqrt{3}x^2 - 7^{-1}x^4 + 6x$

21. $y = \sqrt{x} - 5x^4$

22. $y = \frac{5x^4 - x^3}{x}$

ASSIGNMENT Properties and Graphs of Polynomials

Determine the a) degree, b) leading coefficient, c) end behaviour using limit notation $\lim_{x\to-\infty} f(x) = \ldots$

d) # of possible zeros and # of possible turning points, e) sketch possible graphs

1. $y = -9x^4 + 11x^2 + 2x^6 + x - 13$

2. $y = 8x^2 - 4x - x^3 + 1$

Determine the a) degree, b) leading coefficient, c) end behaviour using notation: $x \to -\infty$, $y \to \ldots$, d) sketch

3. $y = -x^2(2-x)^4(3-2x)^3$

4. $y = -x^3(4-2x)(3x+5)^2$

Determine the a) degree, b) leading coefficient, c) end behaviour using notation: $x \to -\infty,\ y \to \ldots$,
d) # of possible zeros and # of possible turning points e) sketch possible graphs

5. $y = x^3 - 2x^8 - 3$

6. $y = -9x - 5x^3 + x^5 + 18$

Determine the a) degree, b) leading coefficient, c) end behaviour using limit notation $\lim_{x \to -\infty} f(x) = \ldots$ d) sketch

7. $y = (4-2x)^3(6+3x)^3(x-1)$

8. $y = x^5(-2x-10)^4(4x+9)$

9. $y = 7 - (4-x)^3(x^2-1)$

10. $y = (-x)^3(2+x)^2(x-1) - 5$

ASSIGNMENT Transformations of Polynomials & Families of Equations

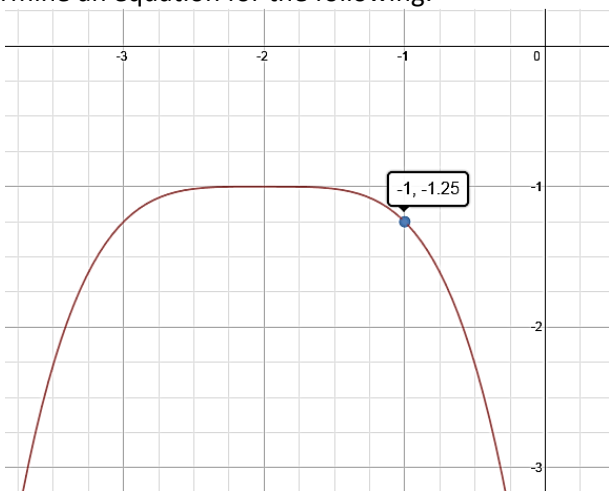
Determine the a) parent power function, b) transformations, c) sketch

1. $y = \frac{1}{5}(-2x)^5 + 1$

2. $y = -2(3(x-4))^6 - 1$

Determine an equation for the following:

3.



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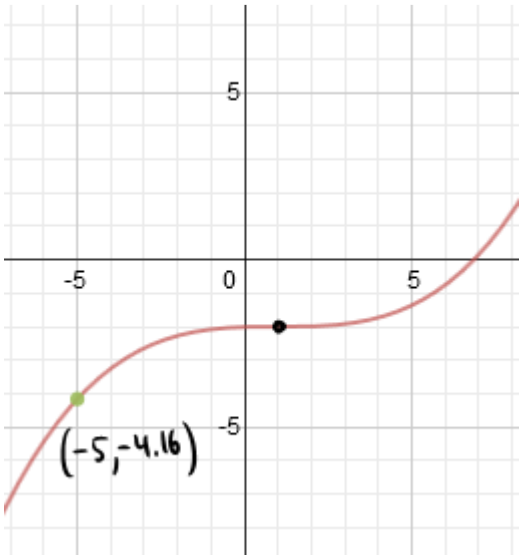
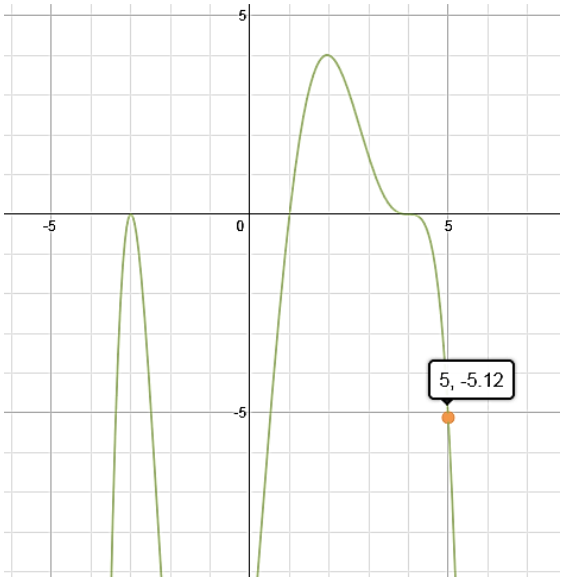


Determine an equation for the following:

5.

6.



Determine a family of equations that satisfies the following conditions. Record answers with rational coefficients only.

7. State the family of degree 3 polynomials that has only one real root at 2, a y-intercept at -10, and going through point (-1,-15).

8. Write the equation of the family of quartic functions that has roots at $1 \pm \sqrt{5}$ and two other real roots. The family must pass through points (0, -8) and (1, -10).

Determine a family of equations that satisfies the following conditions. Record answers with rational coefficients only.

9. State the family of quadratics in ***standard form***, given that the parabola has zeros at $6 \pm \sqrt{12}$, and $\lim_{x \to \infty} f(x) = -\infty$

10. State the family of quartics with two distinct zeros, both of multiplicity 2, one of them at -6 and as $x \to -\infty, y \to -\infty$

11. State the family of degree 5 polynomials that has a y-intercept at 4, x-intercepts at 5(multiplicity 1), and at -1(multiplicity 3)

12. Write the equation of the family of quartic functions that has a double root at -3 and no other real roots. The family must pass through points (0, 18) and (-1, 12).

Determine the unique equation for the following. Record answers with rational coefficients only.

13. The parabola has the axis of symmetry at 3 and x-intercept at 7 and y-intercept at -2.

14.

x	2	3	4	5	6	7
y	3	−1	−9	−21	−37	−57

15. A quartic polynomial has a y-intercept at -7, x-intercepts at 2(multiplicity 2), and $-3 \pm \sqrt{2}$.

16.

x	−3	−2	−1	0	1	2	3
y	−105	−41	−11	−3	−5	−5	9

ASSIGNMENT Dividing Polynomials

Result statement after division can be recorded in two ways:

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$$ or $$dividend = (quotient)(divisor) + remainder$$

Divide using long division, and state the result statement

1. $\left(10x^4 - 15x^3 + 12x^2 + 7x + 5\right) \div \left(5x^2 + 1\right)$

2. $\dfrac{-2x^4 + 12x^2 - 3x + 4}{x^2 - x + 3}$

Divide using synthetic division, and state the result statement

3. $\dfrac{2x^4 - 5x^2 + 3}{x + 4}$

4. $(4x^5 - 2x^4 + 3x - 9) \div (x - 2)$

5. $\left(9x^3 + 7x - 12x^2 - 10\right) \div (3x - 5)$

6. $\dfrac{8x^3 - 12x^2 - 11}{2x + 1}$

ASSIGNMENT Theorems

Factor

1. $81x^3 + 192$

2. $2x^3 - x^2 - 6x + 3$

3. $3x^3 + 8x^2 - 21x + 6$

4. Use Remainder Theorem to find out the remainder if $2x+1$ is divided into $2x^3 - x^2 - 6x + 3$.

5. Use Remainder Theorem to find out the remainder if $x-3$ is divided into $f(x) = x^3 - 12x^2 - 42$.

6. Use Rational Root Theorem to make a list of all possible zeros for polynomial $3x^3 + 8x^2 - 21x + 6$. Then use Descartes's Rule of signs to predict the number of positive and negative zeros. Then use Factor Theorem to decide if any of the factors listed are actual roots

7. Use Rational Root Theorem to make a list of all possible zeros for polynomial $-8x^3 - 3x^2 + 2x + 2$. Then use Descartes's Rule of signs to predict the number of positive and negative zeros. Then use Factor Theorem to decide if any of the factors listed are actual roots

8. If the dividend is $x^4 + x^3 - 7x^2 + 6x - 2$, quotient is $x^3 - 3x^2 + 5x - 14$ and remainder is 54, find the divisor by using comparing coefficients method.

9. If the dividend is $-6x^4 - 2x^3 - 5x^2 + 2x + 8$, quotient is $3x^2 + x + 4$ and remainder is $x+4$, find the divisor by using comparing coefficients method.

10. Determine the remainder, r, to make this multiplication statement true
$(x^2 - 4)(3x^2 + 1) + r = 3x^4 - 10x^2 + 5$

11. If the divisor is $x^2 + 5x + 8$, the dividend is $x^4 - 5x^3 + 3x^2 - 7x + 11$. Find the remainder.

12. Use Factor Theorem to verify which of the following are factors of $2x^3 - 5x^2 - x + 6$
$2x-3$, $x-1$, $x-2$. Then use comparing coefficients to factor all the way.

13. Use Factor Theorem to verify which of the following are factors of $2x^3 + 3x^2 - 18x + 8$
$2x-1$, $x-1$, $x+2$. Then use comparing coefficients to factor all the way.

Factor using synthetic division

14. $2x^3 - 9x^2 + 10x - 3$

15. $12x^3 + 8x^2 - 3x - 2$

Factor using any method

16. $18x^3 - 21x^2 - 10x + 8$

17. $8x^5 - 402x^3 + 490x$

18. $3x^3 + 8x^2 - 21x + 6$

19. $x^5 - 45x^3 + 324x$

20. Find k if the polynomial has $x-3$ as a factor $4x^4-3x^3-2x^2+kx-9$

21. For the function $f(x)=x^3-5x^2+kx-16$, the remainder from $f(x)\div(x+1)$ is twice the remainder from $f(x)\div(x-1)$. Determine the value of k.

22. When $ax^3+5x^2+bx+10$ is divided by $x+1$, the remainder is 18. When it is divided by $x-2$, the remainder is 42. Find the quantity $a+b$.

23. For the function $f(x)=4x^3+x^2+kx+5$, the remainder from $f(x)\div(x-1)$ is two more than the remainder from $f(x)\div(x+1)$. Determine the value of k.

ASSIGNMENT Solve Polynomial Word Problems

Solve

1. $6x^3 + 49x^2 + 8x - 12 = 2x + 4$

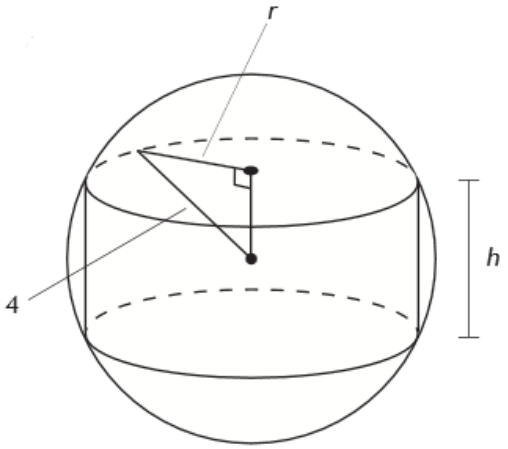
2. $113x - 30 = 8x^3 - 30x^2$

3. The distance of a ship from its harbour is modelled by the function $d(t) = -3t^3 + 3t^2 + 18t$ where t is the time elapsed in hours since departure from the harbour.
   a) When does the ship return to the harbour?
   b) Draw a sketch of the function where $0 \le t \le 3$
   c) What do you need to find to determine the time that the ship begins its return trip back to the harbour. Use technology to verify that this point is NOT halfway between zeros. Thus the need to learn derivatives in calculus

4. The width of a rectangular prism is w centimeters. The height is 2 cm less than the width. The length is 4 cm more than the width. If the magnitude of the volume of the prism is 8 times the measure of the length, what are the dimensions of the prism?

5. A rectangular piece of cardboard measuring 12 in by 8 in is made into an open box by cutting squares from the corners and turning up the sides. If the volume of the box is 60 in^3, what are its dimensions?

6. A cylinder with a radius of r and a height of h is inscribed (see pic) within a sphere that has a radius of 4 units. Derive a polynomial function V(h) in standard form, that expresses the volume of the cylinder as a function of its height.



$V_{cylinder} = \pi r^2 h$

7. Three students share a birthday on the same day. Quinn and Ralph are the same age, but Audrey is two years older. The product of their ages is 11548 greater than the sum of their ages. How old is each person.

ASSIGNMENT Regression With Technology

1. Data: average claims paid per policy for automobile insurance in New Brunswick in the years 1971-1980:

Create a scatter plot of the data set using technology. Find the best regression model for your data (with regression coefficient as close to 1 as you can get it)

Record:

Technology used	Equation	R^2 value
TI-89		
desmos		
excel		

Year	Cost
1971	45.13
1972	51.71
1973	60.17
1974	64.83
1975	65.24
1976	65.17
1977	67.65
1978	79.80
1979	96.13
1980	115.19

2. A certain business company spends money of advertising to get specific revenues back. The data is provided below.

Advertising (in $10 000's)	Revenue (in $100 000's)
50	34.375
100	125.000
150	253.125
200	400.000
250	546.875
300	675.000
350	765.625
400	800.000
450	759.375
500	625.000

a. Use techology to determine the equation that best fits the data.
b. What is the domain of this function for this real life situation?
c. For what amount of advertising does the company get an absolute maximum in revenue?

ASSIGNMENT Recap Factoring & Inequalities

A

Factor each expression completely

1. $\cos^2 x - \sin^2 y$

2. $(t-1)^2 - 49$

3. $2y^3 - 7y^2 - 15y$

4. $(x^2+x)^2 - 8(x^2+x) + 12$

5. $104x^5 + 1625x^2$

6. $x^4 - 16x^2 + 60$

7. $\cos^3 x - e^{3x}$

8. $\frac{1}{81}x^2 + \frac{2}{9}x - 8$

9. $12a^2x - 4a^2 - 9bx + 3b$

10. $(2^x)^2 - 10(2^x) + 16$

11. $x^5 - 5x^{-3}$

12. $2x(x-5)^4 - x^2(4)(x-5)^3$

13. $x^3 - x^2 - 5x + 2$

14. $-\frac{1}{2}x^2(4-x)^{-1/2} + 2x(4-x)^{1/2}$

15. $5(x^6+1)^4(6x^5)(3x+2)^3 + 3(3x+2)^2(3)(x^6+1)^5$

B Solve the following inequalities, state the solution in interval notation

1. $3(x + 1) < x + 7$

2. $0 \leq \frac{x + 3}{2} < 5$

3. $|x + 14| + 3 > 17$

4. $|1 - 2x| \leq 5$

5. $x^2 \leq 8x$

6. $x^2 + x > 12$

Solve the following inequalities by sketching polynomials

7. $(x - 1)^2(x + 4)^3(x - 2) \geq 0$

8. $2x^3 + x^2 - 5x + 2 < 0$

ASSIGNMENT Imaginary Numbers and Complex Roots of Polynomials (AP)

1. Simplify the following into a + bi form

a) $\sqrt{-121}$

b) $\sqrt{-36}\times\sqrt{-9}$

c) $\sqrt{-50}-\sqrt{-18}$

d) $\left(3\sqrt{-6}\right)^2$

e) $\dfrac{\sqrt{-144}}{\sqrt{400}}$

f) $-i(2i)^3$

g) $(2 - 5i)(7 + 2i)$

h) $7i^7$

i) $-2(3 - i) + 5(-2 + 3i)$

j) $\left(1+i\sqrt{3}\right)^3$

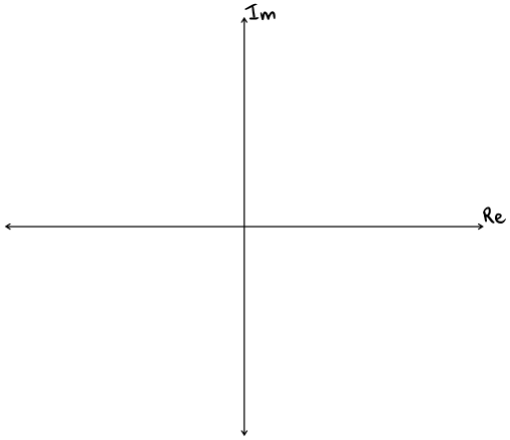
k) $15 \div 6i$

l) $\dfrac{45-25i}{15i}$

m) $\dfrac{2+3i}{2-3i}$

n) $\dfrac{\sqrt{2}}{\sqrt{2}+3i}$

o) Graph the answers to #1 f), g), h), i) on the complex plane (Argand diagram)



p) $|11 - 7i|$

q) $\left|\sqrt{17}-i\sqrt{8}\right|$

2. Find an equation of lowest degree with integral coefficients that has zeros $-2i$ and $2+2\sqrt{2}$.

3. Find the value of b if $1-\sqrt{3}\,i$ is a root of $x^2+bx-2b=0$

4. Find all complex roots.

a $x^3-2x^2+3x-6=0$

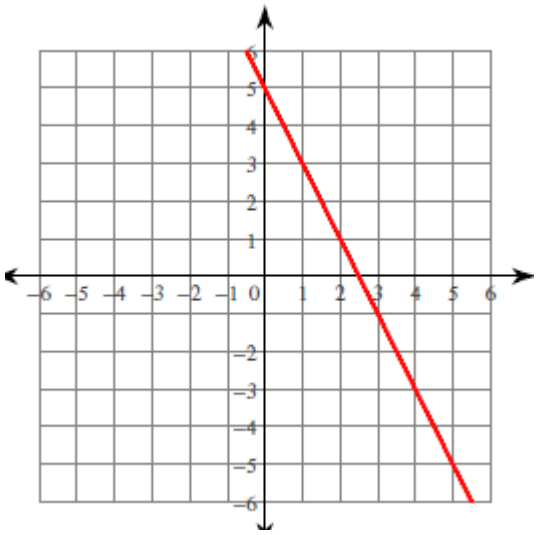
b) $x^4-5x^2-36=0$

5 Given that $1+i$ is a zero of $f(x)=x^4-2x^3-x^2+6x-6$, find all the zeros and write a linear factorization of f.

ASSIGNMENT Absolute Values/Reciprocals/Roots with Polynomials (MHF+AP)

A Sketch Absolute Value of these graphs overtop of the given sketches

1.



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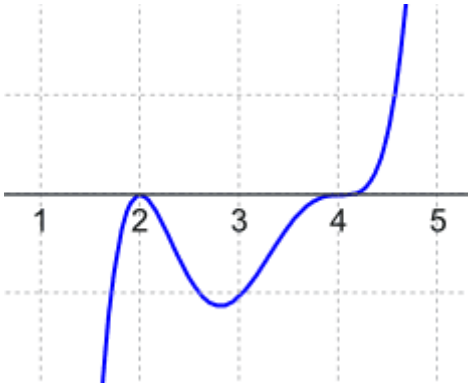

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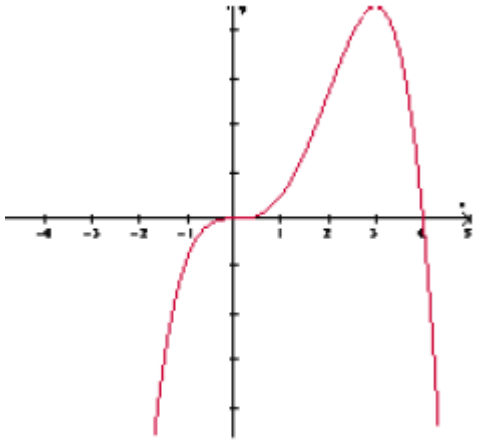

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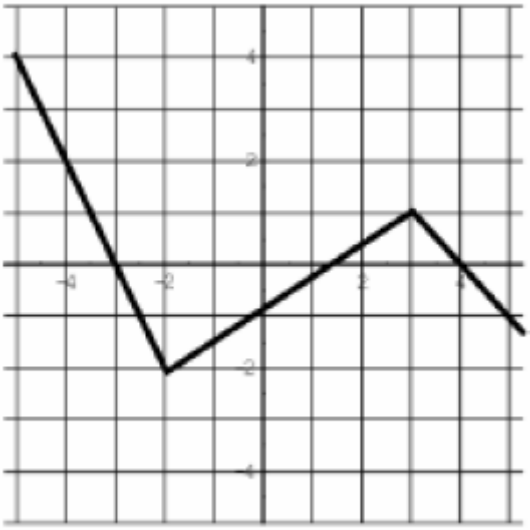
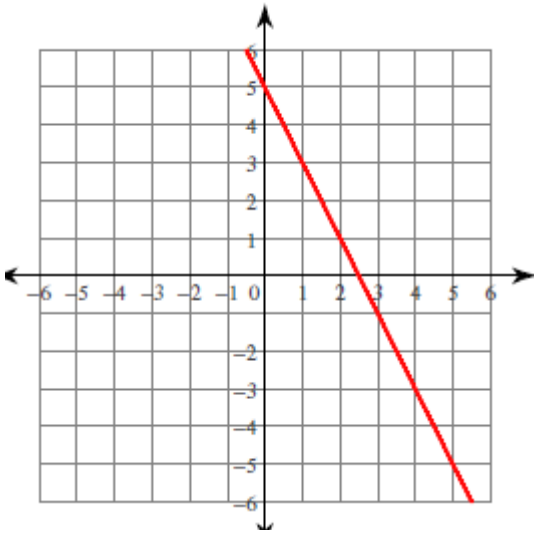


5. State the equations of your sketch for questions # 1-3 (questions 2 and 3 don't show scale of y-axis so you can assume any vertical stretch factor or none at all)

6. State the equation for original graph #4

B Sketch Reciprocals of these graphs overtop of the given sketches

7.



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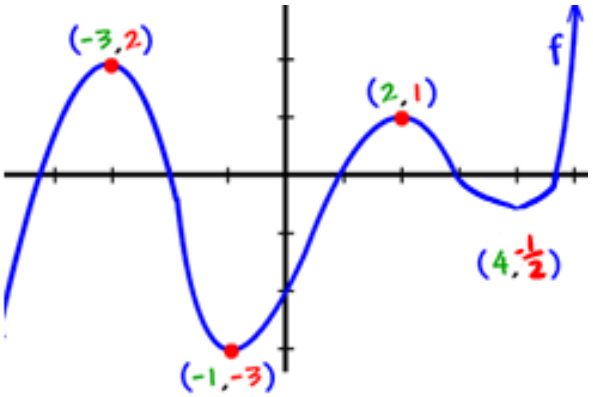

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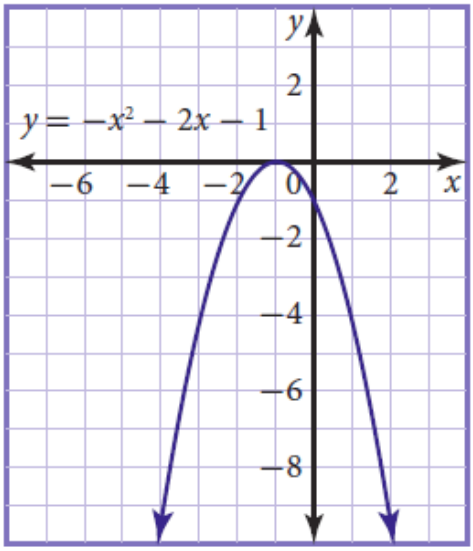

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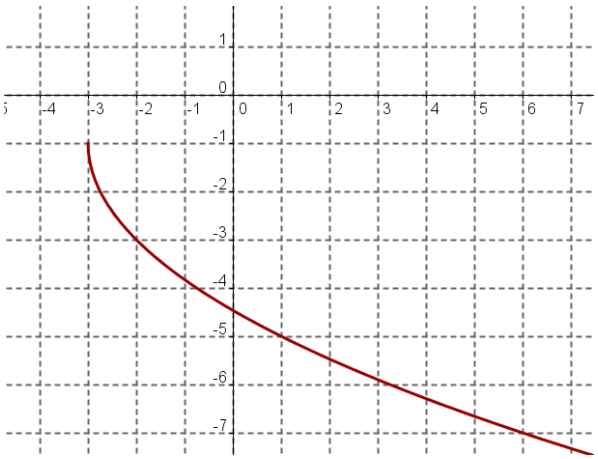


Sketch

11. $$y = \frac{1}{-(x+2)^2 - 3}$$

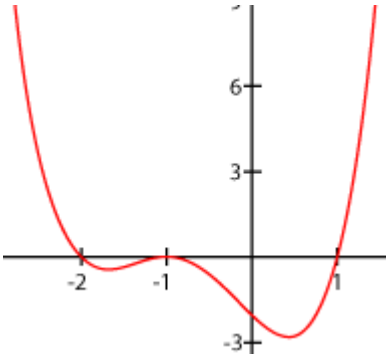
12. $$h(x) = \frac{1}{(x-1)^2(x+3)(x-5)}$$

For the following graphs

a) State a family of equations that would describe the original given graphs

b) Sketch Both EVEN and ODD Roots of these graphs, do it separately or use different colours

13.



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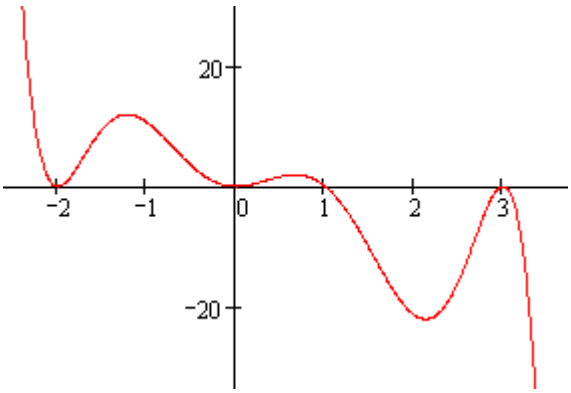
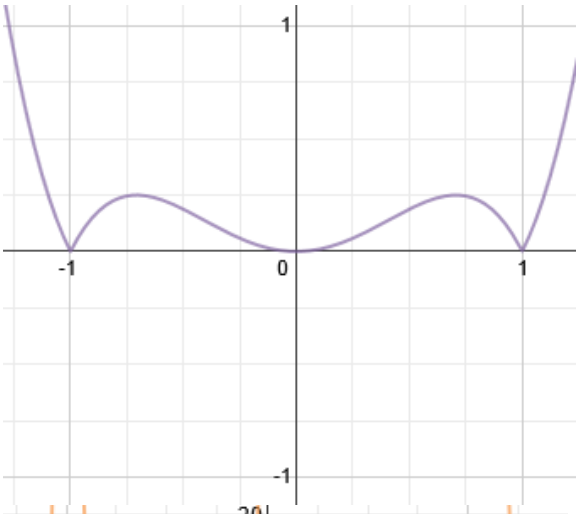


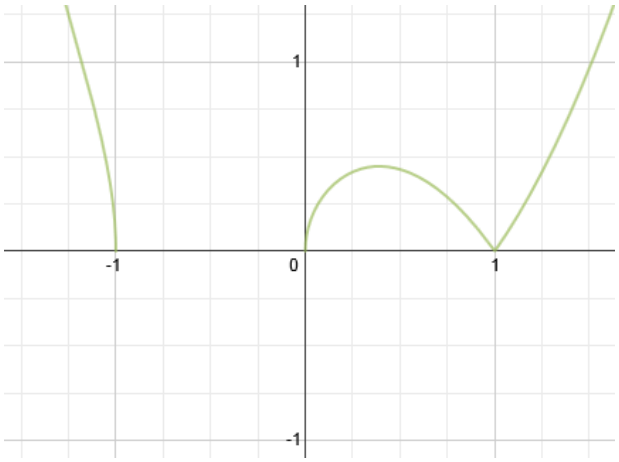
Match the following graphs to their equations. There are more choices for equations given, so think!

A.	$\frac{1}{(x(x-1)^3(x+1)^2)}$	B.	$\frac{1}{(x(x-1)^2(x+1))}$	C.	$\sqrt{(x-1)^2(x)(x+1)}$	D.	$\sqrt[3]{(x-1)^2x(x+2)}$
E.	$\lvert(x-1)x(x+1)^2\rvert$	F.	$\sqrt[3]{-(x-1)x(x+1)^2}$	G.	$\lvert(x-1)x^2(x+1)\rvert$	H.	$\sqrt{(x-1)x(x+1)}$

15.



16.



17.

18.