Review

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1. Find the coordinates of the point of intersection of the following lines.
   $L_1: \frac{x+3}{-1} = \frac{y-7}{4} = \frac{z-2}{1}$ and L_2: line through the points A (0 , 2 , 1) and B (-4 , 4 , 1)

2. The point A (1 , 4 , 2) and the direction vector $\vec{d} = [2, -1, 0]$ are given.
   a. Find a vector equation of the line L_1 passing through point A with direction vector $\vec{d}$.
   b. Find three other points on L_1.
   c. Find the parametric equations of L_1.
   d. Find the symmetric equations of L_1.

3. Which two of the following lines are the same? Explain.
   a. $[x, y, z] = [1, 2, 3] + s[3, -1, 2]$
   b. $[x, y, z] = [-2, 3, 1] + t[-3, 1, -2]$
   c. $[x, y, z] = [7, 0, 6] + p[-3, -1, -2]$

4. The points A (1 , 2 , 3) , B (-1 , 3 , 2) , and C (3 , -2 , -1) are given.
   a. Determine a vector equation of the line that passes through A and that is parallel to the segment BC.
   b. Determine the parametric equations of the line that passes through B and that is parallel to the segment AC.
   c. Determine the symmetric equations of the line that passes through C and that is parallel to the segment AB.

5. Find the scalar equation of each plane.
   a. The plane with normal vector $\vec{n} = [4, -1, 9]$ passing through the point R (2 , -1 , -1).
   b. The plane passing through S (4 , 0 , -1) and containing the direction vectors
      $\vec{m}_1 = [2, 1, 5]$ *and* $\vec{m}_2 = [-3, 0, 1]$.
   c. The plane passing through the points A (4 , -5 , 1) , B (2 , 3 , 3) , and C (0 , 2 , -4).

6. Find the scalar equation of the plane containing the point A (-3 , 1 , 2) and that is parallel to the lines
   $\frac{x+3}{1} = \frac{y}{2} = \frac{z-5}{3}$ *and* $x = 2, \frac{y+1}{-2} = \frac{z+3}{1}$.

7. Find the scalar equation of the plane through the points P (2 , 2 , 2) and Q (3 , 2 , 1) , and that is perpendicular to the plane $4x - y + 2z - 7 = 0$.

8. Determine, if possible, the point(s) of intersection of each line and each plane. If there is a point of intersection, find the angle between the line and the plane.
   $\frac{x+4}{2} = \frac{y+2}{3} = \frac{z-3}{2}$ *and* $3x - y + 2z - 3 = 0$

9. Two lines with the following symmetric equations are given.

   $L_1: \frac{x-4}{-1} = \frac{y-2}{-2} = \frac{z+3}{2}$ *and* $L_2: \frac{x+6}{-2} = \frac{y+2}{2} = \frac{z-3}{1}$

   a) Prove that L_1 and L_2 are skew lines.
   b) Find the equations of two parallel planes containing L_1 and L_2.

10. Find vector and symmetric equations for the line of intersection of each pair of planes.

$\pi_1:\ 3x + 2y - z = 0$ *and* $\pi_2:\ 2x + 2y - 3z = 0$

11. Find the equation of the plane that passes through the line of intersection of the planes $3x + 4y - z + 5 = 0$and $2x + y + z + 10 = 0$, and satisfies each condition.
   a) It passes through the point (-2 , 5 , 1)
   b) It is perpendicular to the plane $6x + y + 2z - 5 = 0$

12. The equations of three planes are given.

$\pi_1 : x + 2y + 3z = -4$

$\pi_2 : x - y - 3z = 8$

$\pi_3 : 2x + y + 6z = -14$

   a) Show that the three planes intersect at a single point.
   b) Find the coordinates of the point of intersection.

13. Show that the following planes form a triangular prism.

$\pi_1 : 3x + 2y + z = 0$

$\pi_2 : x + 2y + 3z = 4$

$\pi_3 : x + y + z = 16$

14. Solve each linear system.

$2x + 3y = -4$

$3x + y = 1$

15. Solve each system of equations completely. Give a geometric interpretation of the solution.

$x + 2y + 3z = 5$

$2x - y - 4z = -10$

$5x + 7y + 6z = 7$

16. Explain why two skew lines cannot be coplanar but vectors collinear with the skew lines are always coplanar.

17. Consider the point A(–2, –1, 1) and the plane defined by $6x - 7y + 2z = 0$.

a) Determine a point, X, on the plane.

b) Determine the magnitude of the projection of $\dot{AX}$ on the normal vector to the plane.

c) Explain the significance of the magnitude from part b.

18. Determine which pairs of lines are skew lines. Prove your answer in each case.

a) L_1: $x = -2t, y = 3 + 4t, z = -1 + 9t$

L_2: $\dfrac{x-5}{-2} = \dfrac{y-6}{4} = \dfrac{1-z}{-9}$

b) L_1: $x = -2t, y = 3 + 4t, z = -1 + 9t$

L_2: $x = 3 + k, y = -1 + 2k, z = -1 + 2k$

19. Two planes, π_1 and π_2, intersect in the line $\dfrac{x-11}{-2} = \dfrac{2y+2}{4} = \dfrac{3-z}{-1}$.

Point A(1, 2, 3) lies on π_1, and point B(1, 0, 1) lies on π_2. Determine the scalar equations of the two planes.

20. Analyze the normal vectors of the following planes and describe the geometric situation of each system. Include a geometric interpretation of each solution.

a) $7x - 5y + 4z - 9 = 0$
$-x + 3y + 5z - 2 = 0$
$5x + y + 14z - 20 = 0$

b) $7x - 5y + 4z - 9 = 0$
$-5x + 4y + z - 3 = 0$
$x + 2y - z + 1 = 0$

c) $-3x + 2y - z + 10 = 0$
$6x - 4y + 2z - 10 = 0$
$5x + 7y + 9z - 14 = 0$

d) $5x - 9y + 2z - 3 = 0$
$4x + 2y - 3z + 4 = 0$
$9x - 7y - z + 1 = 0$

21. Determine the solution of each system. If the solution is not unique, express the solution in parametric form.

a) $3x - 2y + 4z - 3 = 0$
$-x + 4y + 8z + 1 = 0$
$\frac{1}{2}x + y + 2z - 35 = 0$

b) $6x - 2y + 3z - 4 = 0$
$5x + y - 2z + 8 = 0$
$-3x - 7y - 24 = 0$

22. Determine two unit vectors that are collinear with the line of intersection of the planes $2x + 3y + z + 3 = 0$ and $3x + 5y + 2z + 4 = 0$.

23. Suppose three colours of paint are purchased to decorate the interior of a building. The retail store sells the paint in 1-L cans. White paint costs \$4.25/L, green paint costs \$5.25/L, and brown paint costs \$6.15/L. The total number of cans of paint purchased is 57. The volume of the white paint purchased is twice the volume of the green paint purchased.

a) Determine the number of cans of each colour of paint purchased in terms of the number of cans of brown paint.

b) At a different retail store, the cost is \$3.50/L for white paint, \$4.00/L for green paint, and \$6.00/L for brown paint. The total cost is \$237.00. Use this information together with your solution in part a to determine the number of cans of each colour of paint that are required.

24. If a line and its normal vector with tail at (0, 0) intersect at N(–2, 5), determine the equation of the line.

25. Determine the distance from the point A(4, –5) to the line $2x + 7y + 4 = 0$.

26. Define the normal axis of a plane $ax + by + cz + d = 0$ as the line passing through (0, 0, 0) that has a direction vector equal to the normal vector of the plane, that is, $[a, b, c]$. Find the intersection of the plane $2x - y - z + 5 = 0$ and its normal axis.

27. State, in general terms, the equation of a plane parallel to both the x-axis and the z-axis.

28. Determine the intersection of each line with the plane $7x - 23y - 17z + 42 = 0$.

a) the line with point A(1, –1, 2) and direction cosines $\cos\alpha = \frac{1}{3}$, $\cos\beta = \frac{-2}{3}$, and $\cos\gamma = \frac{2}{3}$

b) $L: \frac{x+2}{5} = \frac{y+1}{3} = \frac{z-3}{-2}$

29 Consider the following planes.

$$\pi_1 : x - 2y - z - 4 = 0$$

$$\pi_2 : (x, y, z) = (0, 2, -1) + r(-1, 3, 3) + t(1, 1, 0) \qquad s, t \in \mathbb{R}$$

$$\pi_3 : \begin{cases} x = -2 + 4s - 2t \\ y = s - 3t \\ z = 5 + 2s + 4t \end{cases} \qquad s, t \in \mathbb{R}$$

$$\pi_4 : 2(x-1) + 3(y-5) - 4(z+1) = 0$$

a) Find the angle between the planes π_1 and π_2, the planes π_1 and π_3 and the planes π_1 and π_4.

b) Find which planes are parallel or perpendicular with π_1.

c) Find, if possible, the intersection of the planes π_1 and π_2, the planes π_1 and π_3 and the planes π_1 and π_4.

too long can skip → d) Find the relative position of each pair of planes. ie. intersection of all pairs.

e) Determine the distance from each plane to the point $P(1,2,-3)$.

f) Determine the distance between the pair of planes π_1 and π_3.

g) For each of the points $A(1,7,2)$, $B(15,9,-7)$ and $C(1,1,1)$, determine to which plane (if any) they belong.

h) Find the point on each plane that is closest to the point $P(1,4,2)$.

i) For each plane, find the equation of the line passing through the point $P(1,2,-3)$ and perpendicular to the plane.

30. Consider the line $L : 5 - x = y + 4 = \dfrac{z+3}{2}$.

a) Find the point Q on the line L that is closest to the point $P(8,9,1)$.

b) Find the distance between the point P and the line L.

c) Find the equation of a line perpendicular to L and passing through the point P.