

Vectors UNIT C

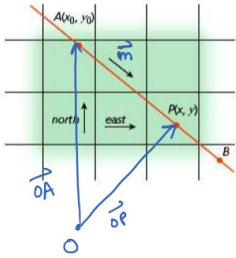
**LINES and PLANES – journal**

NAME: \_\_\_\_\_

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. LINES IN 2-DIMENSIONS

a. Copy/Paste the following

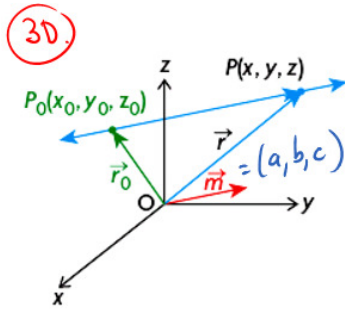


<p><b>Direction Vector of a Line</b></p> $\vec{m} = (a, b)$ $m = \frac{\text{rise}}{\text{run}} = \frac{b}{a}$ <p><b>Vector Equation of a Line</b></p> $\vec{OP} = \vec{OA} + \vec{AP} = \vec{OA} + k\vec{m}$ $(x, y) = (x_0, y_0) + k(a, b)$ <p><b>Parametric Equations</b></p> $x = x_0 + ka$ $y = y_0 + kb$	<p><b>Cartesian Equation of a Line</b></p> $Ax + By + C = 0$ $(A, B) = \vec{n}$ <p>is the normal (perpendicular) to the line</p>	<p><b>Angle between lines</b></p> $A_1x + B_1y + C_1 = 0 \text{ and } A_2x + B_2y + C_2 = 0$ $\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{ \vec{n}_1   \vec{n}_2 }$ <p>can also use direction vectors <math>\vec{m}_1</math> and <math>\vec{m}_2</math></p>
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b. Explain how to take a Grade 9 version of a line  $y = -\frac{2}{3}x + 8$  and change it into all the new forms above.

2. LINES IN 3-DIMENSIONS

a. Copy/Paste the following



**Vector Equation of a Line**  $\vec{OP} = \vec{OP}_0 + \vec{P}_0P$

$$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$$

**Parametric Equations of a Line**

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$

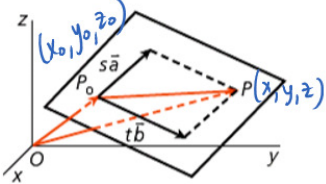
**Symmetric Equation of a Line**

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}, \quad \begin{matrix} a \neq 0 \\ b \neq 0 \\ c \neq 0 \end{matrix}$$

b. Write vector, parametric and symmetric equations for the line through the points A (5,1,-3) and B (4,5,-1).  
 c. Explain why it's not possible to do Scalar/Cartesian equation of a line in  $R^3$

3. PLANES

a. Copy/Paste the following

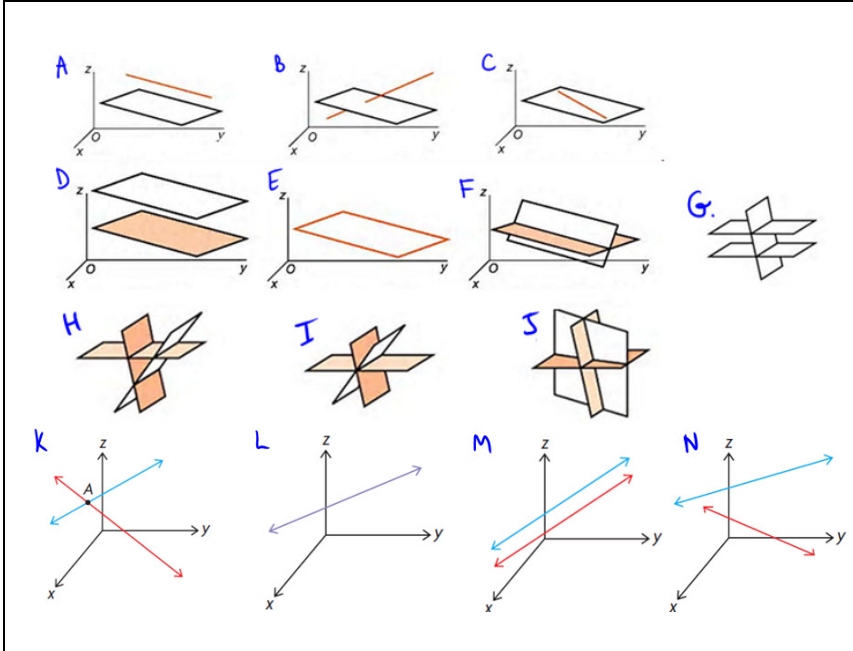


<p><b>Direction Vectors of a Plane</b></p> $\vec{a} = (a_1, a_2, a_3) \quad \vec{b} = (b_1, b_2, b_3)$ <p><b>Vector Equation of a Plane</b></p> $\vec{OP} = \vec{OP}_0 + \vec{P}_0P = \vec{OP}_0 + s\vec{a} + t\vec{b}$ $(x, y, z) = (x_0, y_0, z_0) + s(a_1, a_2, a_3) + t(b_1, b_2, b_3)$ <p><b>Parametric Equations of a Plane</b></p> $\begin{cases} x = x_0 + sa_1 + tb_1 \\ y = y_0 + sa_2 + tb_2 \\ z = z_0 + sa_3 + tb_3 \end{cases}$ <p>known "pt."</p> <p>must be non-zero and non collinear!!</p>	<p><b>Cartesian Equation of a Plane</b></p> $Ax + By + Cz + D = 0$ $\vec{n} = (A, B, C)$ <p><b>Intercept Equation of a Plane</b></p> $\frac{x}{x_0} + \frac{y}{y_0} + \frac{z}{z_0} = 1$ $\begin{matrix} x - mt(x_0, 0, 0) \\ y - mt(0, y_0, 0) \\ z - mt(0, 0, z_0) \end{matrix}$
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- b. Write vector, parametric and Cartesian equations for the plane which contains the points A (1,2,-3) , B (5,1,0) and C (3,2,-6).  
 c. Explain why it's not possible to do symmetric equation of a plane in  $\mathbb{R}^3$   
 d. Find the Intercept equation for question b. and describe a situation when it's impossible to find this.

4. INTERSECTIONS

Match each question to the given picture and solve it or discuss why there wouldn't be any solutions. If there is LineOfIntersection or PlaneOfIntersection, show how to find the vector forms



g) line  $L: \vec{r} = (1,2,3) + t(0,1,1), t \in \mathbb{R}$   
 $\pi: x + y - z - 3 = 0$ .

i)  $x + y - z = 1$   
 $x + y + z = 2$   
 $-2x - 2y + 2z = 3$

k)  $x + y - z = 2$   
 $x - 2y + z = 4$   
 $2x - 4y + 2z = 8$

m)  $(x, y, z) = (1 + t, 2 + t, -t)$   
 $(x, y, z) = (3 - 2u, 4 - 2u, -1 + 2u)$

a)  $L_1: \vec{r} = (0,1,2) + t(1,-1,2), t \in \mathbb{R}$   
 $L_2: \vec{r} = (-3,4,-4) + s(0,1,2), s \in \mathbb{R}$

b)  $\pi_1: x - 2y + 3z + 1 = 0$   
 $\pi_2: -3x + 6y - 9z - 3 = 0$

c)  $x - 3y - 2z = -9$   
 $2x - 5y + z = 3$   
 $-3x + 6y + 2z = 8$

d) line 1  $\vec{r} = (2, 1, 0) + t(1, -1, 1)$   
 line 2  $\vec{r} = (3, 0, -1) + s(2, 3, -1)$

e)  $L: \vec{r} = (-6,9,-1) + t(-2,3,1), t \in \mathbb{R}$   
 $\pi: -x + 2y + z + 4 = 0$ .

f)  $x = 1 + t, y = 1 + 2t, z = 1 - 3t$   
 $x = 3 - 2u, y = 5 - 4u, z = -5 + 6u$

h)  $\pi_1: x - y - 2z + 1 = 0$   
 $\pi_2: -4x + 4y + 8z - 3 = 0$

j)  $2x + y + z = 1$   
 $-x + y + z = -1$   
 $x + y + z = 0$

l)  $x + y + 2z = -2$   
 $3x - y + 14z = 6$   
 $x + 2y = -5$

n)  $L: \vec{r} = (3,0,0) + t(0,2,-3), t \in \mathbb{R}$   
 $\pi: -2x + 3y + 2z + 6 = 0$ .

5. DISTANCES

a. Copy/Paste the following

**METHOD 1**

**Shortest Distance**  
from point  $Q(x_1, y_1)$  to line in  $\mathbb{R}^2$   
 $Ax + By + C = 0$

Distance = shadow of  $\vec{PQ}$  on  $\vec{n}$   
 $= |\vec{PQ} \downarrow \vec{n}| = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$

$$= \frac{|(x_1 - x, y_1 - y) \cdot (A, B)|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|Ax_1 + By_1 - Ax - By|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

pt. not on the line.

**Shortest Distance**  
from point  $Q$  to line in  $\mathbb{R}^2$   
through  $P$  with direction vector  $\vec{m}$

$\vec{m} = (a, b, c)$   
 infinitely many  $\vec{n}$   
 can't do shadow of  $\vec{PQ}$  on  $\vec{n}$  here!

$$A \square = |\vec{m} \times \vec{PQ}|$$

$$\therefore \text{distance} = \text{height of } \square$$

$$= \frac{A \square}{\text{base}}$$

$$= \frac{|\vec{m} \times \vec{PQ}|}{|\vec{m}|}$$

don't reduce here!

**Shortest Distance**  
from point  $Q(x_1, y_1, z_1)$   
to plane in  $\mathbb{R}^3$   $Ax + By + Cz + D = 0$

distance = shadow of  $\vec{PQ}$  on  $\vec{n}$   
 $= |\vec{PQ} \downarrow \vec{n}|$

$$= \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$$

like before

$$= \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

- b. Find the distance from point  $Q(5, 8)$  to the line  $7x + y - 23 = 0$  using METHOD 1 & 2
- c. Calculate the distance between the two parallel planes  $2x - y + 2z + 4 = 0$  and  $2x - y + 2z + 16 = 0$
- d. Determine the distance between point  $Q(-2, 1, 0)$  and line  $\vec{r} = (0, 1, 0) + t(1, 1, 2), t \in \mathbb{R}$
- e. For question d. determine point  $R$  on the line at which minimum distance occurs (use METHOD 1 & 2 then check if your answer in d. is correct).

**METHOD 2:**

**Shortest Distance**  
from point  $Q$  to line in  $\mathbb{R}^2$  or  $\mathbb{R}^3$  through  $R$  with direction vector  $\vec{m}$

- convert line into parametric + vector forms (to see  $\vec{m}$ )
- general form of pt.  $R$  using parameter
- $\vec{m} \cdot \vec{QR} = 0$
- solve for parameter
- dist =  $|\vec{QR}|$

**Shortest Distance**  
from point  $Q(x_1, y_1, z_1)$   
to plane in  $\mathbb{R}^3$   $Ax + By + Cz + D = 0$

- convert plane into parametric + vector forms to see  $\vec{m}_1, \vec{m}_2$
- general form of pt.  $R$
- $\vec{m}_1 \cdot \vec{QR} = 0$
- $\vec{m}_2 \cdot \vec{QR} = 0$
- solve this system for 2 parameters
- dist =  $|\vec{QR}|$

**Finding Point at Minimum Distance**

Method 1  $\vec{OR} = \vec{OP} + \text{Proj}(\vec{PQ} \text{ on } \vec{m})$   
 $= \vec{OR} + \frac{\vec{PQ} \cdot \vec{m}}{|\vec{m}|^2} \vec{m}$

Method 2

- convert line to parametric and vector forms to see  $\vec{m}$
- state general pt.  $R$  using parameter
- $\vec{m} \cdot \vec{QR} = 0$  solve for parameter

6. MATRICES (AP) - OPTIONAL

Show the steps of using a matrix to solve, then interpret the result

$$3x - 2y + z - 1 = 0$$

$$x + 3y - 2z + 7 = 0$$

$$10x - 3y + z + 4 = 0$$