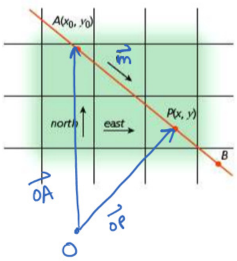
Vectors UNIT C

LINES and PLANES – journal

NAME:____________________

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. LINES IN 2-DIMENSIONS
   a. Copy/Paste the following



Direction Vector of a Line

$\vec{m} = (a, b)$ $\qquad m = \frac{rise}{run} = \frac{b}{a}$

Vector Equation of a Line

$\vec{OP} = \vec{OA} + \vec{AP} = \vec{OA} + k\vec{m}$

$(x,y) = (x_0, y_0) + k(a, b)$

stretch factor = parameter, $k \in \mathbb{R}$

Parametric Equations

$x = x_0 + ka$

$y = y_0 + kb$

Symmetric Equation

$\frac{x - x_0}{a} = \frac{y - y_0}{b}$

if $a, b \neq 0$

Cartesian Equation of a Line

$Ax + By + C = 0$

$(A, B) = \vec{n}$ is the normal (perpendicular) to the line

$\vec{m}$ $\quad \vec{n}$

Angle between lines $A_1x+B_1y+C_1=0$ *and* $A_2x+B_2y+C_2=0$

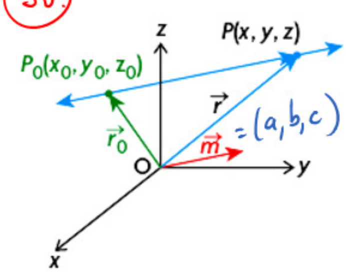
$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

can also use direction vectors $\vec{m}_1$ and $\vec{m}_2$

   b. Explain how to take a Grade 9 version of a line $y = -\frac{2}{3}x + 8$ and change it into all the new forms above.

2. LINES IN 3-DIMENSIONS
   a. Copy/Paste the following

3D



Vector Equation of a Line $\quad \vec{OP} = \vec{OP_0} + \vec{P_0P}$

$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Parametric Equations of a Line

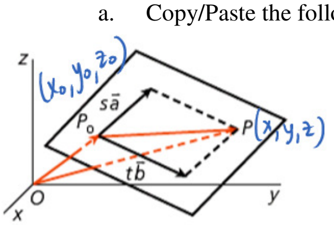
$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$

Symmetric Equation of a Line

$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$, $a \neq 0$, $b \neq 0$, $c \neq 0$

   b. Write vector, parametric and symmetric equations for the line through the points A (5,1,-3) and B (4,5,-1).
   c. Explain why it's not possible to do Scalar/Cartesian equation of a line in R^3

3. PLANES
   a. Copy/Paste the following



Direction Vectors of a Plane

$\vec{a} = (a_1, a_2, a_3)$ $\vec{m}_1$ $\qquad \vec{b} = (b_1, b_2, b_3)$ $\vec{m}_2$

Vector Equation of a Plane

$\vec{OP} = \vec{OP_0} + \vec{P_0P} = \vec{OP_0} + s\vec{a} + t\vec{b}$

$(x, y, z) = (x_0, y_0, z_0) + s(a_1, a_2, a_3) + t(b_1, b_2, b_3)$

known "pt."

must be non-zero and non collinear!!

Parametric Equations of a Plane

$$\begin{cases} x = x_0 + sa_1 + tb_1 \\ y = y_0 + sa_2 + tb_2 \\ z = z_0 + sa_3 + tb_3 \end{cases}$$

Cartesian Equation of a Plane

$Ax + By + Cz + D = 0$

$\vec{n} = (A, B, C)$

Intercept Equation of a Plane

$\frac{x}{x_0} + \frac{y}{y_0} + \frac{z}{z_0} = 1$

x-int $(x_0, 0, 0)$

y-int $(0, y_0, 0)$

z-int $(0, 0, z_0)$

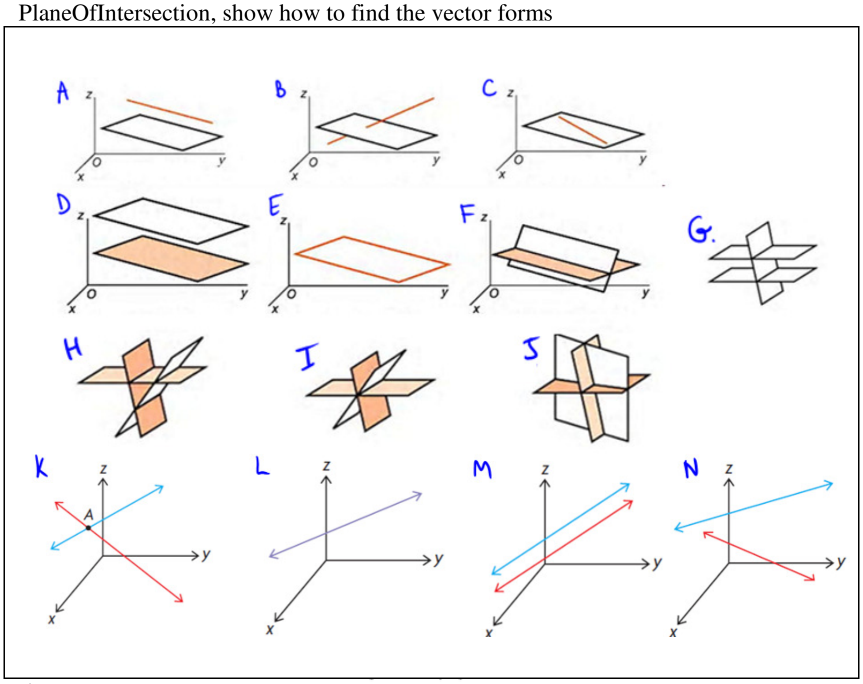
b. Write vector, parametric and Cartesian equations for the plane which contains the points A (1,2,-3) , B (5,1,0) and C (3,2,-6).

c. Explain why it's not possible to do symmetric equation of a plane in R^3

d. Find the Intercept equation for question b. and describe a situation when it's impossible to find this.

4. INTERSECTIONS

Match each question to the given picture and solve it or discuss why there wouldn't be any solutions. If there is LineOfIntersection or PlaneOfIntersection, show how to find the vector forms



a) $L_1: \vec{r} = (0,1,2) + t(1,-1,2), \quad t \in R$
$L_2: \vec{r} = (-3,4,-4) + s(0,1,2), \quad s \in R$

b) $\pi_1: x - 2y + 3z + 1 = 0$
$\pi_2: -3x + 6y - 9z - 3 = 0$

c) $x - 3y - 2z = -9$
$2x - 5y + z = 3$
$-3x + 6y + 2z = 8$

d) **line 1** $\vec{r} = (2, 1, 0) + t(1, -1, 1)$
line 2 $\vec{r} = (3, 0, -1) + s(2, 3, -1)$

e) $L: \vec{r} = (-6,9,-1) + t(-2,3,1), \quad t \in R$
$\pi: -x + 2y + z + 4 = 0$.

f) $x = 1 + t,\ y = 1 + 2t,\ z = 1 - 3t$
$x = 3 - 2u,\ y = 5 - 4u,\ z = -5 + 6u$

g) line $L: \vec{r} = (1,2,3) + t(0,1,1), \quad t \in R$
$\pi: x + y - z - 3 = 0$.

h) $\pi_1: x - y - 2z + 1 = 0$
$\pi_2: -4x + 4y + 8z - 3 = 0$

i) $x + y - z = 1$
$x + y + z = 2$
$-2x - 2y + 2z = 3$

j) $2x + y + z = 1$
$-x + y + z = -1$
$x + y + z = 0$

k) $x + y - z = 2$
$x - 2y + z = 4$
$2x - 4y + 2z = 8$

l) $x + y + 2z = -2$
$3x - y + 14z = 6$
$x + 2y = -5$

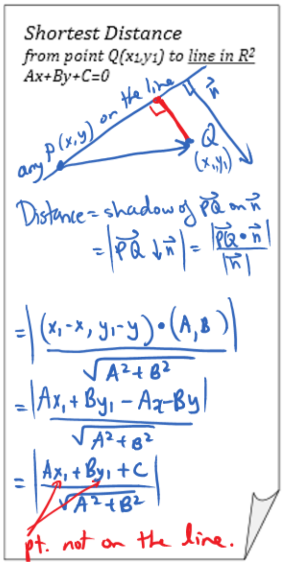
m) $(x, y, z) = (1 + t, 2 + t, -t)$
$(x, y, z) = (3 - 2u, 4 - 2u, -1 + 2u)$

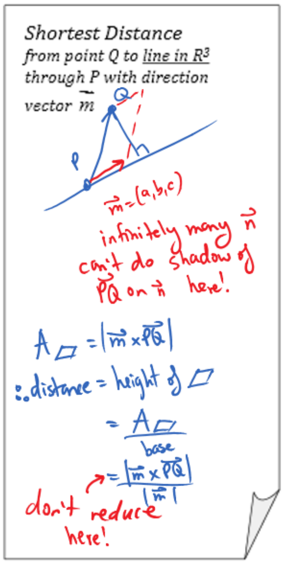
n) $L: \vec{r} = (3,0,0) + t(0,2,-3), \quad t \in R$
$\pi: -2x + 3y + 2z + 6 = 0$.

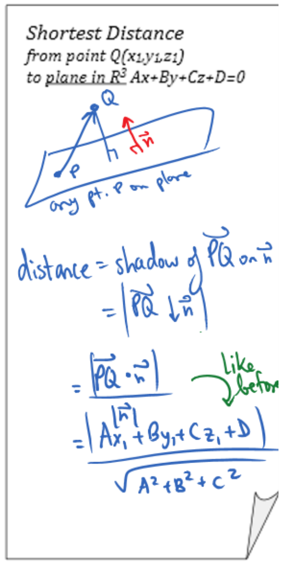
5. DISTANCES

a. Copy/Paste the following

METHOD 1

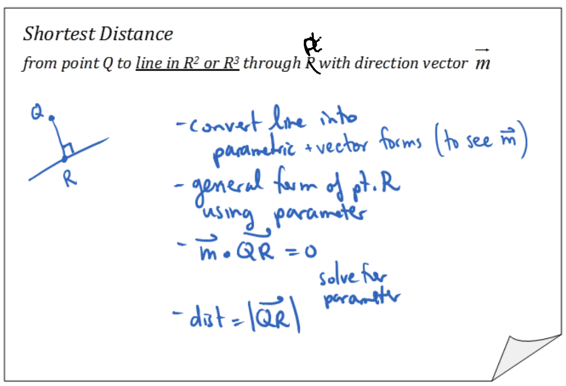


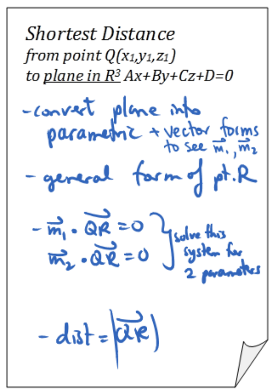


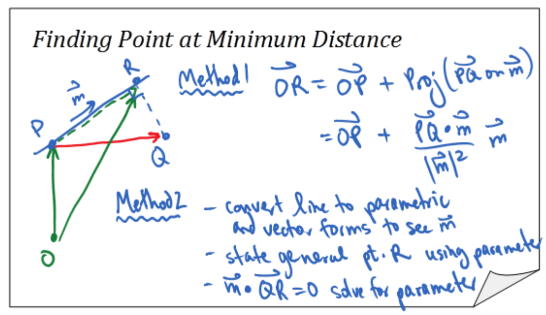


- b. Find the distance from point Q(5,8) to the line $7x + y - 23 = 0$ using METHOD 1 & 2
- c. Calculate the distance between the two parallel planes $2x - y + 2z + 4 = 0$ and $2x - y + 2z + 16 = 0$
- d. Determine the distance between point Q(-2,1,0) and line $\vec{r} = (0,1,0) + t(1,1,2), t \in R$
- e. For question d. determine point R on the line at which minimum distance occurs (use METHOD 1 &2 then check if your answer in d. is correct).

METHOD 2:







6. MATRICES (AP) - OPTIONAL

Show the steps of using a matrix to solve, then interpret the result

$$3x - 2y + z - 1 = 0$$

$$x + 3y - 2z + 7 = 0$$

$$10x - 3y + z + 4 = 0$$