



Big idea

In this unit you will learn how to solve two equations with two unknowns. The unit is called linear systems since there will be more than one line involved. There are three methods you'll be shown in class: graphing method, elimination method, and substitution method.



Feedback & Assessment of Your Success

			Finished assignment pages?	Summarized notes in a journal?	How many extra practice questions did you try in each topic?	Tentative TEST date:
			Made corrections?	Added your own explanations?		
Date	Pages	Topics				Questions to ask the teacher:
	2-5	Linear Relations (gr.9 review) Journal #1				
	6-8	POI Graphing Method (MPM)				
2days	9-14	Solve Systems (MPM) Journal #2				
	15-16	Analyze Solutions (MPM) Journal #3				
2days	17-20	Problem Solving (MPM) Journal #4				

ASSIGNMENT Linear Relations gr9 Review (MPM)

SOLVING EQUATIONS, leave answers as exact fractions

1.

$$\frac{2}{3}t + \frac{4}{5} = -\frac{1}{2}$$

2.

$$\frac{1}{2}(z + 3) = \frac{1}{3}(z - 7)$$

3.

$$\frac{2x + 7}{5} = \frac{1 - 3x}{8}$$

4.

$$2(z + 3) - 5(6 - z) = 8(3z + 3) - 4(1 - 2z)$$

5.

$$\frac{m + 11}{3} + \frac{m - 2}{6} = \frac{2m - 1}{12}$$

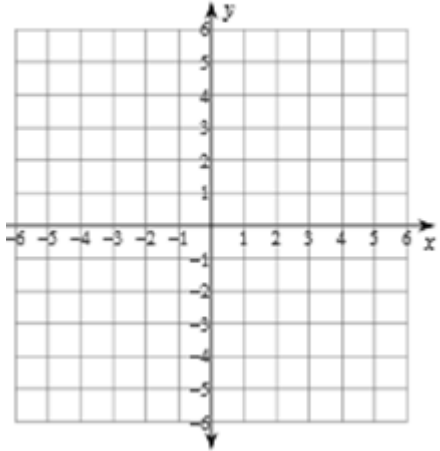
6.

$$\frac{2x + 4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$$

GRAPHING LINES

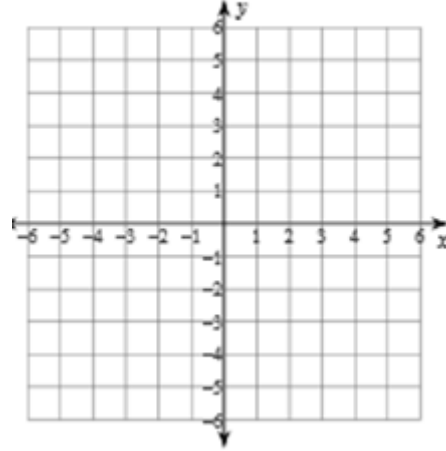
7. Using intercepts:

$$x + y = 3$$



8. Using slope and y-intercept:

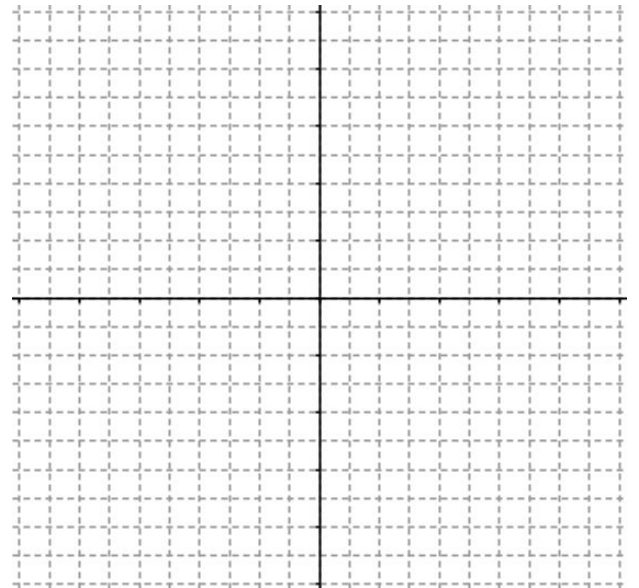
$$10x - 3y = 15$$



9. Find the point of intersection for these lines.
-
- Check your answer.

$$x + y = 10$$

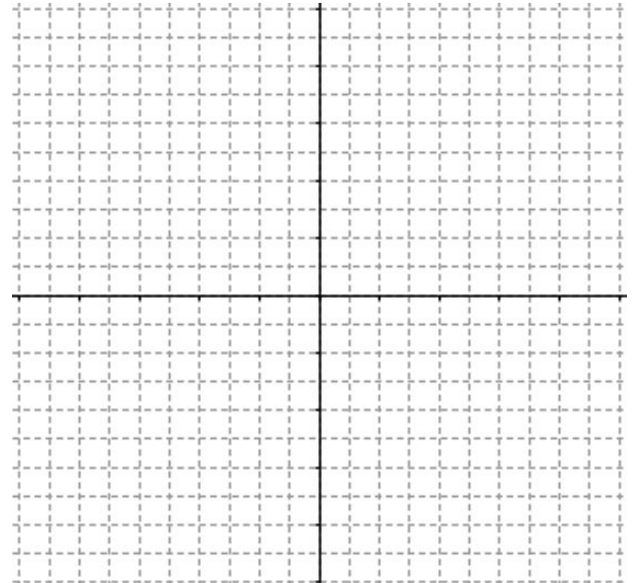
$$3x - 2y = 0$$



10. Find the point of intersection for these lines.
-
- Check your answer.

$$2x + 5y = -20$$

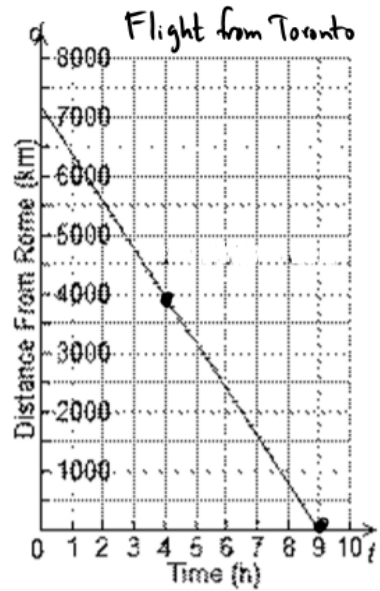
$$5x - 3y = -15$$



FINDING EQUATIONS OF LINES

11. $(2, 1)$ and $(-3, -14)$ use point-slope form

12.



13. The line goes through the point $(4, -1)$ and is perpendicular to the equation $y = 2x + 4$.
use slope yint form

TRANSLATING INTO SYMBOLS

Mathematics is a language like any other. Below is a statement in Russian that we will translate word by word to English.

ПРИВЕТ! КАК ДЕЛА?

Same idea is applied when translating English to Math. But first we'll need a Dictionary:

+	-	×	÷	=

When you see commas:

When you see the words "than" or "from":

When you see the words "per" or "each":

When you see a phrase:

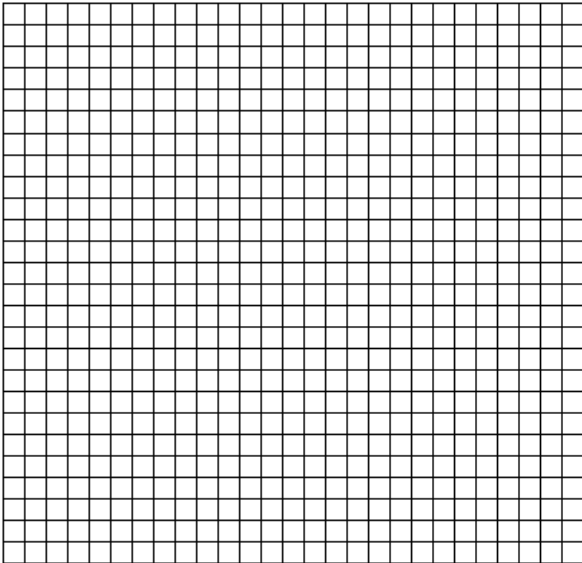
and when you see a sentence:

14. Translate each phrase into an algebraic expression.
- a) six more than three times a number
 - b) five less than one third a value
 - c) a number increased by four, times another number
15. Translate each phrase into an algebraic expression.
- a) three times a length
 - b) fifteen percent of an area
 - c) half a distance
16. Translate each sentence into an algebraic equation.
- a) Three times a value, decreased by four, is two.
 - b) One third a number, increased by two, is one.
 - c) One number is five times larger than two more than a second number.
 - d) The price of a meal, including fourteen percent tax, is ninety-five dollars and seventy-six cents.
17. Translate each sentence into an algebraic equation.
- a) At a school concert, 355 tickets were sold. There were 51 more student tickets sold than adult tickets.
 - b) A rectangle has a perimeter of 172 cm. The length of the rectangle is 23 cm longer than twice the width.
 - c) The sum of two times the smaller of two consecutive numbers and three times the larger number is 113.
 - d) Enrico weighs 7 kg more than Julian. The sum of their masses is 183 kg.

ASSIGNMENT P.O.I. – Graphing method (MPM)

Solve by graphing. Ensure you use appropriate scale, label axes, label lines and give the graph a title. Once POI is found, explain what it means.

1. For Megan’s sixteenth birthday, her parents are planning a party. Tony’s Pizzeria charges \$150 for the party room and \$12 per guest for the cost of food. Hamburger Joey charges \$180 for the party room and \$10 per guest.
 - a) Write a system of linear equations to represent the situation.



- b) Fill in the tables to help you graph the lines

Tony’s:

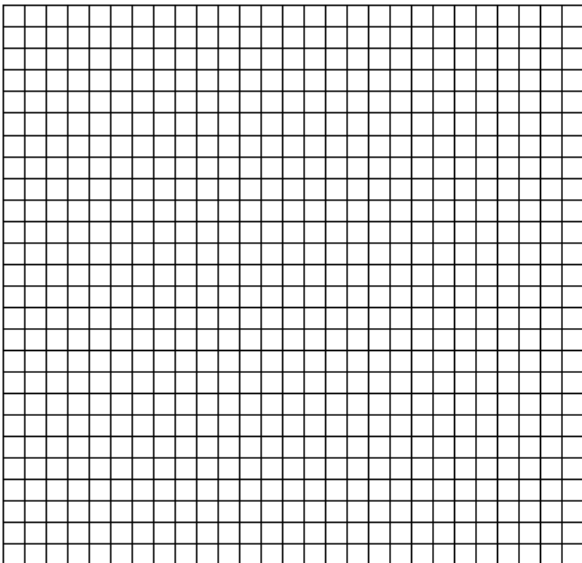
guests	total cost

Joey’s:

guests	total cost

- c) Find and check the point of intersection. What does this point of intersection represent?

2. Pin Town charges \$3 for shoe rental plus \$5 per game. Bowl-In-One charges \$5 for shoe rental plus \$4 per game. Let y represent the total cost for going bowling and x represent the number of games played.
 - a) Write a system of linear equations to represent the situation.



- b) Fill in the tables to help you graph the lines

Pin Town:

games	total cost

Bowl in One:

games	total cost

- c) Find and check the point of intersection. What does this point of intersection represent?

Solve by graphing. Ensure you use appropriate scale, label axes, label lines and give the graph a title. Once POI is found, explain what it means.

3. FunNGames Video rents game machines for \$10 and video games for \$3 each. Big Vid rents game machines for \$7 and video games for \$4 each. Let y be the total rental cost and x the number of games rented.

a) Write a system of linear equations to represent the situation.

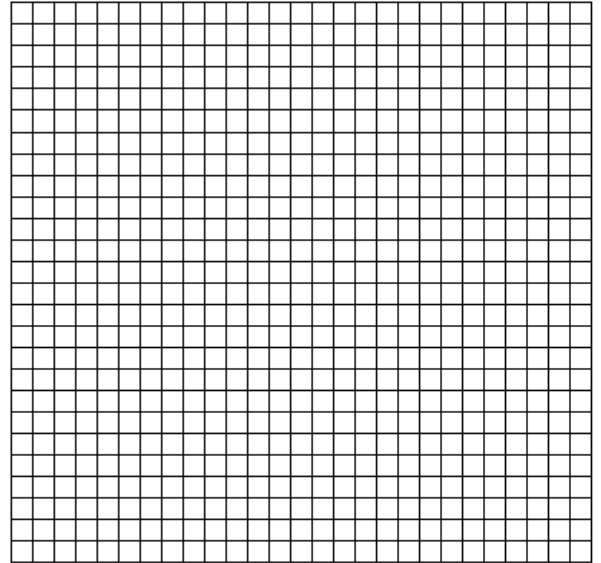
b) Fill in the tables to help you graph the lines

FunNGames:

games	total cost

Big Vid:

games	total cost



c) Find and check the point of intersection. What does this point of intersection represent?

4. Katrin is looking at banquet halls for her parents' anniversary party. Moonlight Hall charges a fixed cost of \$1000 plus \$75 per guest. Riverside Hall charges \$1500 plus \$50 per guest. Let C represent the total cost, and n represent the number of guests.

a) Write a system of linear equations to represent the situation.

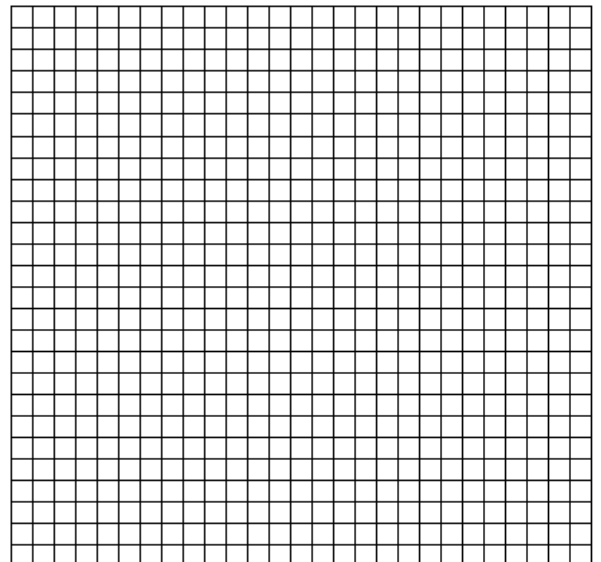
b) Fill in the tables to help you graph the lines

Moonlight:

guests	total cost

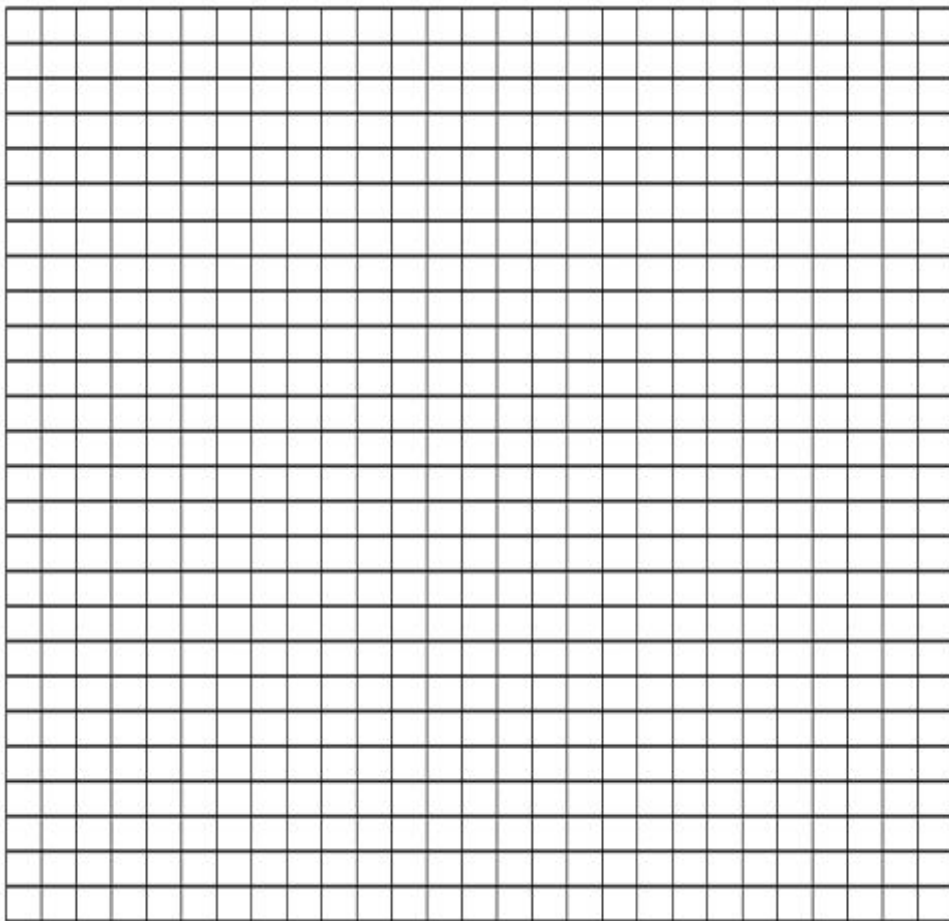
Riverside:

guests	total cost



c) Find and check the point of intersection. What does this point of intersection represent?

5. Charlene is looking into cell phone plans. Cell Plus gives unlimited minutes for \$50/month. A1 Cell offers a \$40 monthly fee, plus 5¢/min for any time over 300 min per month.
- Write a linear equation to represent the charges for each company.
 - Graph the two equations to find the point of intersection.
 - What does the point of intersection represent?
 - Which plan should Charlene choose if she estimates that she will use her phone 10 h per month? 6 h per month?



ASSIGNMENT Substitution & Elimination Methods (MPM)

Sometimes solving a linear system by graphing does not give accurate answers. There are two ways of solving a linear system algebraically:

- **Substitution Method**
- **Elimination Method**

Steps for substitution:

Steps of elimination:

1. Is $(2, -3)$ the solution for the following linear system? Explain how you can tell.

$$3x + 6y = -12$$

$$2y - 8x = -22$$

2. Solve by substitution. Check your solution.

$$4x + y = 0$$

$$x + 2y + 1 = 0$$

3. Solve by substitution. Check your solution.

$$2c - d + 2 = 0$$

$$3c + 2d + 10 = 0$$

4. Simplify each equation, and then solve the linear system by substitution.

$$3(x + 1) - 2(y - 2) = -6$$

$$x + 4(y + 3) = 29$$

5. Phoenix Health Club charges a \$200 initiation fee, plus \$15 per month. Champion Health Club charges a \$100 initiation fee, plus \$20 per month.
- Write a linear equation to represent the charges for each club.
 - Solve the linear system.
 - After how many months are the costs the same?
 - If you joined a club for only 1 year, which club would be less expensive?
6. The number of tickets sold for a school event is 330. Let a represent the number of adult tickets sold and s represent the number of student tickets sold. The cost of a student ticket is \$6 and the cost of an adult ticket is \$10. In total, \$2380 was taken in from ticket sales.
- Write a linear system to represent the information.
 - Solve the linear system to find the number of each type of ticket sold.

7. Solve the system using elimination.

$$0.6a - 0.2b = 1.8$$

$$0.5b - 0.4a - 0.7 = 0$$

8. Some provinces have names with First Nations origins. For example, “Ontario” comes from an Iroquois word meaning “beautiful water.” If the number of provincial names with First Nations origins is a , and the number with other origins is b , the numbers are related by the following equations.

$$a + b = 10$$

$$3a - 2b = 0$$

- a) Interpret each equation in words.
- b) Find the number of provinces that have names with First Nations origins.

9. Solve by elimination. Check the solution.

$$2(a - 4) + 5(b + 1) = 8$$

$$3(a - 1) - 2(b - 2) = -11$$

10. At Lisa's Sub Shop, two veggie subs and four roast beef subs cost \$34. Five veggie subs and six roast beef subs cost \$61. Write and solve a system of equations to find the cost of each type of sub.

11. Solve by elimination.

$$\frac{x-6}{4} + \frac{y+2}{3} = 0$$

$$\frac{x+1}{3} - \frac{y-5}{2} = 3$$

12. A weekend at Skyview Lodge costs \$360 and includes two nights' accommodation and four meals. A week costs \$1200 and includes seven nights' accommodation and ten meals. Write and solve a system of equations to find the cost of one night and the cost of one meal.
13. The Mackenzie, the longest river in Canada, is 1056 km longer than the Yukon, the second-longest river. The total length of the two rivers is 7426 km. Find the length of each river

ASSIGNMENT Analyze Solutions (MPM)

Show what happens if you try to solve these algebraically

1. **Independent Systems – Single Solution**Intersect in a single point
Single solution
Consistent

a)

$$\begin{aligned}x - 3y &= 11 \\ 2x + y &= 1\end{aligned}$$

Independent Systems – No SolutionDo not Intersect
Graphs are Parallel
False statement results after solving (like $2 = 3$)
Inconsistent

b)

$$\begin{aligned}x - y &= 1 \\ -3x + 3y &= 4\end{aligned}$$

Dependent Systems – Infinitely Many SolutionsLines on top of each other – Same graph
Equivalent equations
True statement (identity) results after solving (like $2 = 2$)
Consistent

c)

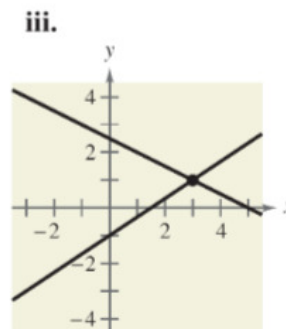
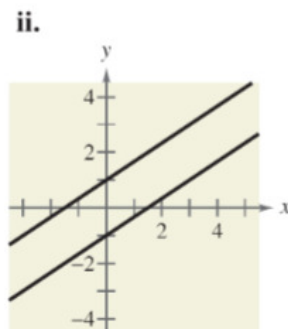
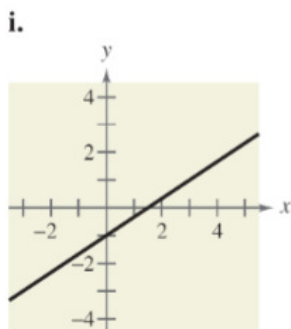
$$\begin{aligned}3x + y &= 1 \\ 6x + 2y &= 2\end{aligned}$$

2. Match each system of linear equations with its graph

a.
$$\begin{cases} 2x - 3y = 3 \\ -4x + 6y = 6 \end{cases}$$

b.
$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$$

c.
$$\begin{cases} 2x - 3y = 3 \\ -4x + 6y = -6 \end{cases}$$



3. Write an equivalent equation
 $y = -4x + 3$

4. Write a parallel but not equivalent equation
 $2x + 5y - 6 = 0$

5. The perimeter of a rectangle is 30 cm. Write an equation to represent this situation. Then, write an equivalent linear equation.

6. Cindy buys a large pizza for \$10 plus \$0.75 per topping. Write an equation to represent this situation. Then, write a NON equivalent linear equation that has the SAME slope.

How many solutions does each system have? Explain

7. $x + y = 7$
 $x - y = 10$

8. $2x - 4y = 8$
 $4 - x + 2y = 0$

9. $4x - y = 9$
 $12x - 6 = 3y$

10. $x(A + 3) - y = 10$
 $4x + y = 3$

Find A so that you have no solutions

11. $Ax - 3y = 4$
 $5x - 6y - 8 = 0$ Find A so that you have infinitely many solutions

12. $2Ax + 3y = 12$
 $(A - 2)x - y = -5$ Find A so that you have no solutions

ASSIGNMENT Problem Solving (MPM)

Money Problems

1. The cost of printing a magazine is based on a fixed set-up cost and the number of pages to be printed. One printing company charges a \$250 set-up fee and \$5/page, while a second company charges a \$400 set-up fee plus \$4/page.
 - a) Write an equation to represent the cost for each company. Define your variables.
 - b) Solve the linear system.
 - c) What does the point of intersection represent?
 - d) Which company should Richard choose to print 175 pages?
2. Joe invests a total of \$4000 in two plans. Part of the money is invested at 8% per year and the rest at 11% per year. The interest paid after 1 year on the 11% investment is \$212 more than the interest paid on the 8% investment. How much did Joe invest in each?

Geometry Problems

3. The perimeter of a rectangular pool is 38m. The length is 3m more than the width. Find the dimensions of the pool.
4. A 10m pipe is to be cut into two pieces of different length. One piece is 3m longer than the other. What are the lengths of the two pieces of pipe?
5. Linda has a garden. The width of her garden is 8 feet longer than the length of it. Around the garden she has a 2 foot wide sidewalk. The area of the sidewalk is 320 feet squared. What are the dimensions of the garden?

Mixture Problems

6. What volume, in millilitres, of a 60% hydrochloric acid solution must be added to 100 mL of a 30% hydrochloric acid solution to make a 36% hydrochloric acid solution?

Age Problems

7. Bill is 11 years younger than Sue. Nine years ago, Sue was twice as old as Bill. How old is each person now?
8. One type of granola is 30% fruit, and another type is 15% fruit. What mass of each type of granola should be mixed to make 600 g of granola that is 21% fruit?
9. Heidi's mother is twice as old as Heidi. The sum of their ages in 6 years will be 72. What are their present ages?

Motion Problems

10. Erika drove from Ottawa at 80 km/h. Julie left Ottawa 1 h later and drove along the same road at 100 km/h. How far from Ottawa did Julie overtake Erika?

Wind/Current Problems

11. Suppose it takes a small airplane flying with a head wind 16 hours to travel 1800 miles. When flying with a tail wind, the airplane can travel the same distance in only 9 hours. Find the speed of the wind and the speed of the plane without the wind
12. Debbie is driving from Moose Bay to Peggy's Cove, Larry is going in the opposite direction. Debbie drove her car for 3h and Larry drove his motorcycle for 5h when they passed each other. The distance between Moose Bay and Peggy's Cove is 420km. If Larry drove 16km/h slower than Debby, how fast did each of them go?
13. A boat goes 32 miles downstream in three hours. The return trip against the current takes sixteen hours. Find the rate of the boat in calm water and the rate of the current.

Rate Problems

14. Playing tennis burns energy at a rate of about 25 kJ/min. Cycling burns energy at about 35 kJ/min. Hans exercised by playing tennis and then cycling. He exercised for 50 min altogether and used a total of 1450 kJ of energy. For how long did he play tennis?
15. A street has a row of 15 new houses for sale. The middle house is on the most desirable piece of property and is the most expensive. The second house from one end costs \$3000 more than the first house, the third house costs \$3000 more than the second house, and so on, up to and including the middle house. The second house from the other end costs \$5000 more than the first house, the third house costs \$5000 more than the second house, and so on, up to and including the middle house. All the houses on the street cost a total of \$3 091 000. What is the selling price of the house at each end of the street? Explain and justify your reasoning.
16. **Other Problems**
The sum of two numbers is 56. One number exceeds the other number by 2. Find the two numbers