Review

March 30, 2014 4:00 PM

MHF

1. $f(x) = 1000(1.3)^x$
   Find instantaneous rate of change at x=0 using the three approximate methods, ensure that one of your answer is accurate to at least two digits.

2. Find exact instantaneous rate of change of the following
   using difference quotient

   i) $f(x) = \frac{2}{x^2}$ at $x = 1$

   ii) $f(x) = 2x^3 - 1$ at $x = 3$

   iii) $y = 4\sqrt{x-2}$ at $x = 5$

3. Use the Difference Quotient to determine the value of a so that the instantaneous rate of change of the function $h(x) = x^2 + 3x + 2$ at $x = a$ is -1.

4. The deer population of a country is modeled by $P(t) = 20t^2 + 200t + 10500$, where $P(t)$ is the size of the population and t is the number of years since 1995.
   a. Do you expect the rate of change to remain the same for this function or not? Explain.
   b. Calculate average rate of change in each time period
      i. 1995 – 2000
      ii. 2005 – 2015

MCV + AP

1.

a) Explain in your own words what is meant by the equation $\lim_{x \to 2} f(x) = 4$. Is it possible for this statement to be true and yet $f(2) = 5$? Explain. What graphical feature would be manifested in this situation?

b) Explain what it means to say that $\lim_{x \to 1^-} f(x) = 3$ and $\lim_{x \to 1^+} f(x) = 6$.

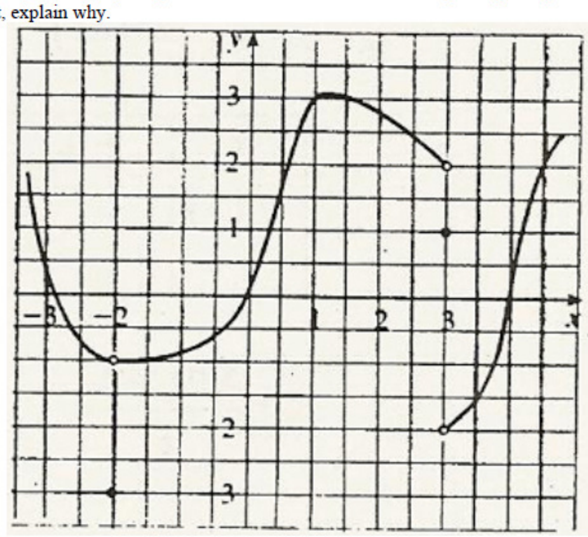
In this situation, is it possible that $\lim_{x \to 1} f(x)$ exists? Explain. What graphical feature would be manifested in this situation?

c) Explain the meaning of each of the following, then sketch a graph of a function exhibiting the indicated behavior.

i) $\lim_{x \to -2} f(x) = \infty$

ii) $\lim_{x \to -3^+} g(x) = -\infty$.

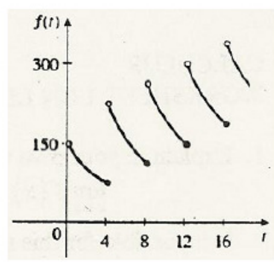
2.) For the function f whose graph is given at below, state the value of the given quantity, if it exists. If it does not exist, explain why.



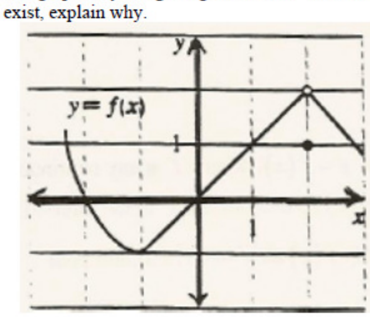
(a) $\lim_{x\to 1} f(x) =$ (b) $\lim_{x\to 3^-} f(x) =$ (c) $\lim_{x\to 3^+} f(x) =$ (d) $\lim_{x\to 3} f(x) =$

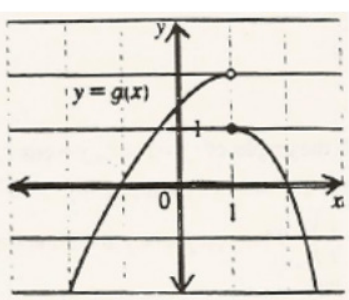
(e) $f(3) =$ (f) $\lim_{x\to -2^-} f(x) =$ (g) $\lim_{x\to -2^+} f(x) =$ (h) $\lim_{x\to -2} f(x) =$ (i) $f(-2) =$

3 A patient receives a 150-mg injection of a drug every four hours. The graph at right shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find $\lim_{t\to 12^-} f(t)$ and $\lim_{t\to 12^+} f(t)$ and then explain the significance/meaning of these one-sided limits in terms of the injections.



4.) The graphs of f and g are given below. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





(a) $\lim_{x\to 2}[f(x)+g(x)] =$ (b) $\lim_{x\to 1}[f(x)+g(x)] =$ (c) $\lim_{x\to 0}[f(x)g(x)] =$

(d) $\lim_{x\to -1} \frac{f(x)}{g(x)} =$ (e) $\lim_{x\to 2} x^3 f(x) =$ (f) $\lim_{x\to 1} \sqrt{3+f(x)} =$

(5) Using trig manipulation, find the following limits. Show all steps.

(a) $\lim_{x\to 0}\frac{\sin 2x}{x}=$ (b) $\lim_{x\to 0}\frac{\sin x}{2x^2-x}=$ (c) $\lim_{x\to 0}\frac{x+\sin x}{x}=$

(d) $\lim_{x\to 0}\frac{\sin^2 x}{x}=$ (e) $\lim_{x\to 0}\frac{3\sin 4x}{\sin 3x}=$ (f) $\lim_{x\to 0}\frac{x^2}{1-\cos x}=$

(6) For each of the following find:

(a) find $\lim_{x\to\infty} f(x)$

(b) $\lim_{x\to-\infty} f(x)$

~~(c) Identify all horizontal asymptotes.~~

i) $f(x)=\frac{3x+1}{|x|+2}$

ii) $f(x)=\frac{-2x^2+4}{\sqrt{4x^4+8x^2+1}}$

iii) $f(x)=\frac{4x^2-3x+5}{2x^3+x-1}$

(7.) For each of the following, find the limit.

(a) $\lim_{x\to -3}\frac{1}{(x+3)^2}=$

(b) $\lim_{x\to 2}\frac{6-3x}{|2x-4|}=$

(c) $\lim_{x\to 1}\frac{2x+10}{x^2|x+5|}$

(d) $\lim_{x\to -\frac{\pi}{2}^+}\sec x=$

(e) $\lim_{t\to 2}\frac{t^3+2t^2-13t+10}{t^3+4t^2-4t-16}=$

(f) $\lim_{x\to 5}\frac{\frac{2}{x+3}-\frac{1}{4}}{x-5}=$

g) If $g(x)=\begin{cases}5-2x, & x>1\\ 4, & x=1\\ 4-x, & x<1\end{cases}$ find:

i) $\lim_{x\to 5} g(x)$ ii) $\lim_{x\to 1^-} g(x)$

iii) $\lim_{x\to 1^+} g(x)$ iv) $\lim_{x\to 1} g(x)$

h) $\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}=$

i) $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)=$

j) If $3x \le f(x) \le x^3+2$, evaluate $\lim_{x\to 1} f(x)$

8. Use the IVT to show that there is a solution to the given functions on the given intervals. Be sure to test your hypothesis, show numeric evidence, and write a concluding statement. No Calculator

(d) $\cos x = x,\ (0,1)$ (b) $\ln x = e^{-x},\ (1,2)$

9. Determine the values of x for which the function $f(x)=\begin{cases}\frac{1}{x}, & x<1\\ x^2, & 1\le x<2\\ \sqrt{8x}, & 2<x\le 8\\ 8.0001, & x>8\end{cases}$ is continuous.

10. If f and g are continuous functions with $f(3)=5$ and $\lim_{x\to 3}\left[2f(x)-g(x)\right]=4$, find $g(3)$.

11. For $f(x)=\begin{cases}cx^2-3, & x\le 2\\ cx+2, & x>2\end{cases}$, find the value of c to make f continuous at $x=2$.

Multiple Choice

use technology or graphing calc.

____ 1. (Calculator permitted) $\lim_{x\to 0}\frac{\sin 2x-2x}{x^3}=$

(A) DNE (B) 1 (C) $-\frac{4}{3}$ (D) ∞ (E) $-\infty$

____ 2. Let f be the function defined by the following.

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

____ 3. If $f(x)=\begin{cases}\ln x, & 0<x\le 2\\ x^2\ln 2, & 2<x\le 4\end{cases}$, then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

____ 4. If f is continuous on $[-4,4]$ such that $f(-4)=11$ and $f(4)=-11$, then which must be true?

(A) $f(0)=0$ (B) $\lim_{x\to 2} f(x)=8$ (C) There is at least one $c\in[-4,4]$ such that $f(c)=8$

(D) $\lim_{x\to 3} f(x)=\lim_{x\to -3} f(x)$ (E) It is possible that f is not defined at $x=0$

______ 5 Let $h(x)$ be a continuous function. Selected values of h are given in the table below.

x	2	3	4	5	7
$h(x)$	2	5	k	4	3

For which value of k below will the equation $h(x) = \frac{2}{3}$ have **at least two solutions** on the closed interval $[2, 7]$?

(A) 0 (B) $\frac{3}{4}$ (C) $\frac{7}{9}$ (D) $\frac{2}{3}$ (E) $\frac{11}{18}$

______ 6 If $f(x) = \sqrt{1-x^2}$, which of the following is NOT true?

(A) Domain of f is $[-1,1]$ (B) $[f(x)]^2 + x^2 = 1$ (C) Range of f is $[0,1]$

(D) $f(x) = f(-x)$ (E) The line $y = 1$ intersects the graph of f at two points