Name____ Date_______________________ Period_____

Worksheet 9.1—Sequences & Series: Convergence & Divergence
Show all work. No calculator except unless specifically stated.

Short Answer

1. Determine if the sequence $\left\{\dfrac{\ln n}{n^2}\right\}$ converges.

2. Find the nth term (rule of sequence) of each sequence, and use it to determine whether or not the sequence converges.

(a) $2, \dfrac{3}{4}, \dfrac{4}{9}, \dfrac{5}{16}, \dfrac{6}{25}, \ldots$

(b) $1, \dfrac{1}{2}, \dfrac{1}{6}, \dfrac{1}{24}, \dfrac{1}{120}, \ldots$

3. Use the *n*th Term Divergence Test to determine whether or not the following series converge:

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1+3n^2+n^3}{4n^3-5n+2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{2n!+1}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{(n+2)!}{10n!}$

4. (Calculator Permitted)

(a) What is the sum of $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

(b) Using your calculator, calculate S_{500} to verify that the SOPS (sum of the partial sums) is bounded by the sum you found in part (a). (Calculator entry shown at right.)

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sum(seq(1/(N+1)-
1/(N+3),N,1,500)
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5. Use the indicated test for convergence to determine if the series converges or diverges. If possible, state the value to which it converges.

(a) Geometric Series: $3+\frac{15}{4}+\frac{75}{16}+\frac{375}{64}+\cdots$

(b) Geometric Series: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\cdots$

(c) p-series: $\sum_{n=1}^{\infty} n^{-2/3}$

(d) Integral Test: $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$

(e) Direct Comparison: $\sum_{n=1}^{\infty} \frac{e^n}{n}$

(f) Direct Comparison: $\sum_{n=1}^{\infty} \frac{3^n}{7^n + 1}$

(g) Limit Comparison: $\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$

(h) Limit Comparison: $\sum_{n=1}^{\infty} \frac{n+5}{3n\left(4^n\right)}$

(i) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

(j) Ratio Test: $\sum_{n=1}^{\infty} \frac{2}{n^2}$

(k) AST: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

(l) AST: $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{2n}$

(m) Direct Comparison: $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(n) Any viable method: $\sum_{n=1}^{\infty} \frac{(-1)^n \left(4^n\right)}{n!}$

6. (Calculator permitted) To five decimal places, find the interval in which the actual sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ is contained if S_5 is used to approximate it.

Determine whether or not the series converge using the appropriate convergence test (there may be more than one applicable test.) State the test used. If possible, give the sum of the series.

7. $\sum_{n=0}^{\infty}\left(\frac{2}{7}\right)^n$

8. $\sum_{n=1}^{\infty}\frac{4}{n^3}$

9. $\sum_{n=1}^{\infty}\frac{n^2}{5^n}$

10. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^5+5}}$

11. $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

12. $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots$

13. $2+\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots$

14. $\sum_{n=1}^{\infty}\frac{5n^2-6n+3}{n^3-7n+8}$

15. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{\sqrt{n}}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + 4}{2^n}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{8n^3 - 6n^5}{12n^4 + 9n^5}$

18. $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5 + 2}}$

19. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{3n+4}}$ converges absolutely, converges conditionally, or diverges.

20. What is the sum of the following:

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{3}{2^n}$ (b) $\displaystyle\sum_{n=2}^{\infty}\left(-\frac{3}{2}\right)^{-n}$ (c) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{3}{(2n-1)(2n+1)}$

21. (Calculator Permitted) Consider the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n}$

(a) Show that the series is absolutely convergent.

(b) Calculate S_6, the sum of the first six terms. Round your answer to three decimal places.

(c) Find the number of terms necessary to approximate the sum of the series with an error less than 0.001

22. If the series $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, determine which of the following series must diverge Justify each answer as to why or why not.

(a) $\sum_{n=1}^{\infty} a_n^2$ (b) $\sum_{n=1}^{\infty} |a_n|$ (c) $\sum_{n=1}^{\infty} (-1)^{2n} a_n$ (d) $\sum_{n=1}^{\infty} (-a_n)$

23. Classify any of the following convergent series as absolutely or conditionally convergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n\sqrt{n}}$ (b) $\sum_{n=0}^{\infty} (-1)^n e^{-n}$ (c) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ (d) $\sum_{n=1}^{\infty} \left(-\frac{\pi}{e}\right)^{-n}$ (e) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+3}$

Multiple Choice:

24. S_2 is used to approximate $S=\sum_{n=1}^{\infty}\frac{4}{n^2}$. Which interval gives an upper and lower bound for this sum?

(A) $\frac{41}{9}\le S\le\frac{49}{9}$ (B) $\frac{53}{9}\le S\le\frac{58}{9}$ (C) $\frac{49}{9}\le S\le\frac{53}{9}$ (D) $\frac{58}{9}\le S\le\frac{62}{9}$ (E) Diverges

25. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

26. If $\lim_{b\to\infty}\int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges (B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges (C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges (E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ diverges

27. $\lim_{x\to 1} \dfrac{\int_1^x e^{t^2}\,dt}{x^2-1}$ is

(A) 0 (B) 1 (C) $\dfrac{e}{2}$ (D) e (E) nonexistent

28. For what integer $k,\ k>1$, will both $\sum_{n=1}^{\infty} \dfrac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\dfrac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

① $\left\{\frac{\ln n}{n^2}\right\}$ converge or diverge?

$\lim_{n\to\infty} \frac{\ln n}{n^2} = 0$, so $\left\{\frac{\ln n}{n^2}\right\}$ converges to zero

② (a) $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \dots = \left\{\frac{n+1}{n^2}\right\}$

$\lim_{n\to\infty} \frac{n+1}{n^2} = 0$, so sequences converges

(b) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \dots = \left\{\frac{1}{n!}\right\}$

$\lim_{n\to\infty} \frac{1}{n!} = 0$, so sequence converges

③ (a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

$\lim_{n\to\infty} \frac{n^3+3n^2+1}{4n^3-5n+2} = \frac{1}{4} \neq 0$

So series diverges by n^{th} term test

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$

So series May or May Not converge (n^{th} term test inconclusive)

(c) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

$\lim_{n\to\infty} \frac{n!}{2\cdot n!+1} = \frac{1}{2} \neq 0$

So series diverges by nth term test

(d) $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!} = \sum_{n=1}^{\infty} \frac{(n+2)(n+1)\cancel{n!}}{10\cancel{n!}} = \sum_{n=1}^{\infty} \frac{n^2+3n+2}{10}$

$\lim_{n\to\infty} \frac{1}{10}(n^2+3n+2) = \infty$

So series diverges by n^{th} term test

④ $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$

(a) Telescoping series

$= \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) + \dots$

$= \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6} \approx 0.833}$

(b) $S_{500} = 0.8293532299 < 0.833$

*from calculator

sum(seq(1/(X+1)−1/(X+3),X,1,500)

List/Math/5   List/OPS/5

⑤ (a) $3 + \frac{15}{4} + \frac{75}{16} + \frac{375}{64} + \dots$

$= 3\left(1 + \frac{5}{4} + \frac{25}{16} + \frac{125}{64} + \dots\right)$

$= 3\left(\left(\frac{5}{4}\right)^0 + \left(\frac{5}{4}\right)^1 + \left(\frac{5}{4}\right)^2 + \left(\frac{5}{4}\right)^3 + \dots\right)$

$= 3\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n, \quad |r| = \frac{5}{4} > 1$

So series is Divergent Geometric Series

⑤ (b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$

$= \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots$

$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n, \quad |r| = \frac{1}{2} < 1$

So convergent Geometric Series

Converges to $\frac{1/2}{1-1/2} = \boxed{1}$

5(c) $\sum_{n=1}^{\infty} n^{-2/3} = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$p = \frac{2}{3} < 1$, so series is a divergent p-series

5(d) $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$

for all $n > k, \exists k \in \mathbb{Z}^+$, this series is Decreasing, Continuous, and Positive

$3\int_1^{\infty} \frac{n}{2n^2+3}\,dn$

$= \frac{3}{4}\ln|2n^2+3|\Big|_1^{\infty}$

$= \frac{3}{4}\left[\ln(\infty) - \ln 5\right] = \infty \Rightarrow$ Diverges

So the series diverges too!

5 (e) $\sum_{n=1}^{\infty} \frac{e^n}{n}$, compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent harmonic series.

Since $\frac{1}{n} \le \frac{e^n}{n}$ $\forall n \ge 1$

$\sum_{n=1}^{\infty} \frac{e^n}{n}$ diverges too!

5 (f) $\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$, compare to $\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n = \sum_{n=1}^{\infty} \frac{3^n}{7^n}$, a convergent geometric series.

Since $\frac{3^n}{7^n+1} \le \frac{3^n}{7^n}$ $\forall n \ge 1$,

$\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$ converges too!

5 (g) $\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$,

compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent series

$$\lim_{n\to\infty} \left(\frac{3n+6}{7n^2-5n+1} \cdot \frac{n}{1}\right)$$

same as dividing by $\frac{1}{n}$

$$= \lim_{n\to\infty} \frac{3n^2+6n}{7n^2-5n+1} = \frac{3}{7}$$

where $\frac{3}{7}$ is finite & positive, so

$\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$ diverges too!

5 (h) $\sum_{n=1}^{\infty} \frac{n+5}{3n(4^n)}$, compare to $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$

$= \sum_{n=1}^{\infty} \frac{1}{4^n}$, a convergent geom. series.

$$\lim_{n\to\infty} \left(\frac{n+5}{3n(4^n)}\right)\left(\frac{4^n}{1}\right)$$

$$= \lim_{n\to\infty} \frac{\cancel{(4^n)}(n+5)}{\cancel{(4^n)}(3n)} = \frac{1}{3} > 0$$

So $\sum_{n=1}^{\infty} \frac{n+5}{3n(4^n)}$ converges too!

5 (i) $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

same as dividing by $\frac{n^3}{n!}$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^3 \cdot \cancel{n!}}{n^3 \cdot (n+1) \cdot \cancel{n!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n^3 + \cdots}{n^4 + n^3} \right| = 0 < 1$$

So $\sum_{n=1}^{\infty} \frac{n^3}{n!}$ converges

5 (j) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

$$\lim_{n\to\infty} \left| \frac{2}{(n+1)^2} \cdot \frac{n^2}{2} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2n^2}{2n^2 + \cdots} \right| = 1$$

so Ratio Test is inconclusive

* use Limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ to show it converges.

5 (k) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

*Alternating Series

$\lim_{n\to\infty} \frac{1}{n} = 0$ and

$\{\frac{1}{n}\}$ is decreasing

So $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges

(conditionally convergent)

5 (l) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{2n}$

*Alternating Series

$\lim_{n\to\infty} \frac{n+3}{2n} = 1 \neq 0$

So diverges by n^{th} term test

(Alt. series test is inconclusive)

5 (m) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, a convergent p-series

$\frac{\sin n}{n^2} \le \frac{1}{n^2}$ since $\sin n \le 1 \ \forall n \in \mathbb{R}$,

So $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges too!

5 (n) $\sum_{n=1}^{\infty} \frac{(-1)^n (4^n)}{n!}$ *converges by A.S.T. since $\frac{4^n}{n!}$ is decreasing $\forall n \ge k, \exists k \in \mathbb{Z}^+$

or by Ratio Test (more fun!)

$\lim_{n\to\infty} \left| \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n} \right|$ *don't need alternators on Ratio test because of abs. values.

$\lim_{n\to\infty} \left| \frac{\cancel{4^n} \cdot 4 \cdot \cancel{n!}}{\cancel{4^n} \cdot (n+1) \cdot \cancel{n!}} \right|$

$= \lim_{n\to\infty} \left| \frac{4}{n+1} \right| = 0 < 1$ So converges

6 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$S_5 = \frac{1}{1^2} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25}$

$= 0.838611111 = \frac{3019}{3600}$

$|R_5| \le |a_6| = \left| \frac{-1}{36} \right| = \frac{1}{36}$ ← Abs. value of 1st unused term

So $S \in \left[\frac{3019}{3600} - \frac{1}{36}, \frac{3019}{3600} + \frac{1}{36} \right]$

or $S \in [0.81083, 0.86638]$

(actual sum $\approx \underset{S_{999}}{\underset{\parallel}{0.822}}$)

7 $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$, convergent geom series with $|r| = \left|\frac{2}{7}\right| = \frac{2}{7} < 1$.

series converges to $\frac{1}{1 - 2/7} = \boxed{\frac{7}{5}}$

8 $\sum_{n=1}^{\infty} \frac{4}{n^3} = 4\left[\sum_{n=1}^{\infty} \frac{1}{n^3}\right]$ → convergent p-series with $p = 3 > 1$ (or Limit Comparison test)

9 $\sum_{n=1}^{\infty} \frac{n^2}{5^n}$, $\lim_{n\to\infty} \left| \frac{(n+1)^2}{5^{n+1}} \cdot \frac{5^n}{n^2} \right|$

$= \lim_{n\to\infty} \left| \frac{(n^2+2n+1)\cancel{5^n}}{n^2 \cdot 5 \cdot \cancel{5^n}} \right| = \frac{1}{5} < 1$

So series converges by Ratio Test.

(10) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$

Compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$, a Convergent p-series.

since $\dfrac{1}{\sqrt[3]{n^5+3}} \le \dfrac{1}{n^{5/3}}$ $\forall n \ge 1$,

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$ converges by Direct Comparison

(Limit Comparison works too!)

(12) $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$

$= \displaystyle\sum_{n=5}^{\infty} \frac{1}{n}$ which is the divergent harmonic series

(14) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$, compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent harmonic series

$\displaystyle\lim_{n\to\infty}\left(\frac{5n^2-6n+3}{n^3-7n+8}\cdot\frac{n}{1}\right) = 5 > 0$

So $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$ diverges by the Limit Comparison Test.

(16) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+4}{2^n}$ *Diverges by nth term test OR

compare with $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n = \sum_{n=1}^{\infty} \frac{3^n}{2^n}$, a divergent geom series.

Since $\dfrac{3^n+4}{2^n} \ge \dfrac{3^n}{2^n}$, $\displaystyle\sum \frac{3^n+4}{2^n}$

diverges by Direct Comparison Test

(11) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$\displaystyle\lim_{n\to\infty} \frac{n^n}{n!} = \infty$

So series diverges by n^{th} term test

(13) $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

$= 2 + \left(\frac{1}{4}\right)^0\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)^1\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)$

$= 2 + \frac{1}{2}\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$ convergent geometric series with $|r| = \left|\frac{1}{4}\right| = \frac{1}{4} < 1$

converges to $2 + \frac{1}{2}\left[\dfrac{1}{1-\frac{1}{4}}\right] = 2 + \frac{2}{3} = \boxed{\frac{8}{3}}$

(15) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$, $\cos n\pi = -1, 1, -1, 1$ for $n = 1, 2, 3 \ldots$

* Alternating series

$\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ and $\frac{1}{\sqrt{n}}$ is decreasing

So $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$ converges by the Alternating Series Test

(17) $\displaystyle\sum_{n=1}^{\infty} \frac{8n^3-6n^5}{12n^4+9n^5}$

$\displaystyle\lim_{n\to\infty} \frac{8n^3-6n^5}{12n^4+9n^5} = -\frac{6}{9} \ne 0$

So series diverges by n^{th} term test

(18) $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{3n+1}{n^5+2}}$ compare with $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{1}{n^4}}=\sum_{n=1}^{\infty}\frac{1}{n^2}$ a convergent p-series.

$$\lim_{n\to\infty}\left(\sqrt{\frac{3n+1}{n^5+2}}\cdot\sqrt{\frac{n^4}{1}}\right)$$

$$=\sqrt{\lim_{n\to\infty}\left(\frac{3n^5+n^4}{n^5+2}\right)}$$

$$=\sqrt{3}>0$$

So $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{3n+1}{n^5+2}}$ converges too! by Limit Comparison Test

(19) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[5]{3n+4}}$

the series converges by Alt. Series test.

* test for Abs convergence (w/o alternator)

$\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{3n+4}}$ compare with $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{1/5}}$, a divergent p series.

$$\lim_{n\to\infty}\frac{\sqrt[5]{n}}{\sqrt[5]{3n+4}}=\sqrt[5]{\lim_{n\to\infty}\left(\frac{n}{3n+4}\right)}=\sqrt[5]{\frac{1}{3}}>0$$

So $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{3n+4}}$ diverges

So $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[5]{3n+4}}$ converges conditionally

(20) (a) $\displaystyle\sum_{n=0}^{\infty}\frac{3}{2^n}$ (convergent geom series)

$$=3\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$$

$$S=3\left[\frac{1}{1-1/2}\right]=\boxed{6}$$

(b) $\displaystyle\sum_{n=2}^{\infty}\left(-\frac{3}{2}\right)^{-n}$

$$=\sum_{n=2}^{\infty}\left(-\frac{2}{3}\right)^n$$ (convergent geom/alt series)

$$S=\frac{\overset{\text{1st term}}{4/9}}{1-(\underset{r}{-2/3})}=\left(\frac{4}{9}\right)\left(\frac{3}{5}\right)=\boxed{\frac{4}{15}}$$

(c) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$ (telescoping series)

$$=\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\dots$$

$$=\frac{1}{2}+\frac{1}{3}=\boxed{\frac{5}{6}}$$

(d) partial fraction decomp.

$$\sum_{n=1}^{\infty}\frac{3}{(2n-1)(2n+1)}=3\sum_{n=1}^{\infty}\left(\frac{1/2}{2n-1}-\frac{1/2}{2n+1}\right)$$

$$=\frac{3}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$=\frac{3}{2}\left[\left(1-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{7}}-\frac{1}{9}\right)+\dots\right.$$

$$=\frac{3}{2}(1)=\boxed{\frac{3}{2}}$$

(21) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

(a) for Abs convergence, $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{2^n}\right| = \sum_{n=1}^{\infty} \frac{1}{2^n}$ must converge.

$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ which is a <u>convergent</u> <u>geometric series</u> with $|r| = \frac{1}{2} < 1$

so $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ <u>converges absolutely</u> (with or without the help of the alternator)

(b) $S_6 = -\frac{1}{2^1} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \frac{1}{2^6} = -0.328125 = -\frac{21}{64}$

$\approx \boxed{-0.328}$

(c) for error to be less than 0.001, the magnitude of the 1st <u>unused</u> term is the partial sum must be < 0.001

trial & error: $\frac{1}{2^6} = 0.0156 \not< 0.001$, $\frac{1}{2^7} = 0.0078 \not< 0.001$, $\frac{1}{2^8} = 0.0039 \not< 0.001$

$\frac{1}{2^9} = 0.00195 \not< 0.001$, $\boxed{\frac{1}{2^{10}} = 0.00097 < 0.001}$

so S_9 approximates S to within 0.001, so <u>9 terms</u> are needed.

(22) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, so $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges

(a) $\sum_{n=1}^{\infty} a_n^2$ <u>could converge</u> if $a_n^2 < a_n$

ex) $a_n = \left(\frac{2}{3}\right)^n$

$(a_n)^2 = \left(\frac{4}{9}\right)^n$

(b) $\sum_{n=1}^{\infty} |a_n|$ <u>MUST diverge</u> by def of Abs convergence

(c) $\sum_{n=1}^{\infty} (-1)^{2n} a_n = \sum_{n=1}^{\infty} \left((-1)^2\right)^n a_n = \sum_{n=1}^{\infty} a_n$ which <u>converges</u>

(d) $\sum_{n=1}^{\infty} (-a_n) = -\sum_{n=1}^{\infty} a_n$ which <u>converges</u>

so only (b) <u>MUST</u> diverge.

(23) (a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n\sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \left(\frac{n-1}{n^{3/2}}\right)$

* <u>Converges conditionally by</u> Alt. series test

* $\sum_{n=1}^{\infty} \left(\frac{n-1}{n^{3/2}}\right)$ Diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$, a divergent p-series

(b) $\sum_{n=0}^{\infty} (-1)^n e^{-n}$

* $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges by Ratio Test

So $\sum_{n=0}^{\infty} (-1)^n e^{-n}$ <u>converges absolutely</u>

(c) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

* <u>converges conditionally</u> by Alt. series test

* $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ <u>diverges</u> by comparison to the harmonic series

$\frac{\ln n}{n} > \frac{1}{n} \ \forall n \geq 2$

(23) (d) $\sum_{n=1}^{\infty}\left(-\frac{\pi}{e}\right)^{-n}$

$= \sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{e}{\pi}\right)^n$

* $\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^n$ converges by the Geom Series Test, $|r|=\frac{e}{\pi}<1$,

So $\sum_{n=1}^{\infty}(-1)^n\left(\frac{e}{\pi}\right)^n$ converges absolutely (to $\frac{-e/\pi}{1+e/\pi}$)

(e) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{\sqrt{n}}{n+3}\right)$

* converges conditionally by Alt. Series Test

* $\sum_{n=1}^{\infty}\frac{n^{1/2}}{n+3}$ diverges by comparison with $\sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$, a divergent p-series.

(24) $S=\sum_{n=1}^{\infty}\frac{4}{n^2}$ (convergent p-series)

$=4\sum_{n=1}^{\infty}\frac{1}{n^2}$

$S_2=4\left[\frac{1}{1}+\frac{1}{4}\right]=4+1=5$

$R_2=|a_3|=\frac{4}{3^2}=\frac{4}{9}$

$S\in\left[5-\frac{4}{9},\ 5+\frac{4}{9}\right]$

$S\in\left[\frac{41}{9},\ \frac{49}{9}\right]$

so $\boxed{\frac{41}{9}\le S\le\frac{49}{9}}$ $\boxed{A}$

(25) Which converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}\rightarrow$ diverges by n^{th} term test since $\lim_{n\to\infty}\frac{n}{n+2}=1\neq0$

✓ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}\rightarrow$ conditionally convergent harmonic series by Alt. Series Tst.

III. $\sum_{n=1}^{\infty}\frac{1}{n}\rightarrow$ divergent harmonic series

So only II converges $\boxed{B}$

(26) $\lim_{b\to\infty}\int_1^b\frac{1}{x^p}dx$ is finite so $\int_1^{\infty}\frac{1}{x^p}dx$ converges,

So $p>1$. Which MUST be true?

✓ (A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges $\rightarrow$ TRUE by Integral Test.

$\boxed{\text{So answer is } A}$

(B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges $\rightarrow$ False, see (A)

(C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ converges $\rightarrow$ Not always true. False if $n\le3$

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges $\rightarrow$ Not always true. False if $n\le2$

(E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ Diverges, false. by comparison, if $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges, $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ must too!

(27) (Review Question) d/o

$$\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2-1}$$

2nd Fun. thm of Calculus

by L'Hôp: $\lim_{x\to 1} \frac{e^{x^2}}{2x} = \boxed{\frac{e}{2}}$ $\boxed{C}$

(28) For what K, $K>1$ will both $\sum_{n=1}^{\infty} \frac{(-1)^{Kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{K}{4}\right)^n$ converge?

* $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ will only converge (conditionally) if $(-1)^{kn}$ alternates, so K must be an odd integer >1

$\boxed{K = 3, 5, 7, 9, 11, \ldots}$

* $\sum_{n=1}^{\infty} \left(\frac{K}{4}\right)^n$ will only converge by geometric series test if $\frac{K}{4} < 1$, $K < 4$, (but >1)

$\boxed{K = 3, 2}$

The only value that satisfies both scenarios is $\boxed{K=3}$ $\boxed{D}$