LIN SYs UNIT

| Review of Gr 9 notes |
| :--- |
| point slope form |
| POI - by graphing, by substitution, by elimination |
| Analyze systems - no sol, infinite sol, one sol |
| - what happens algebraically, graphically |
| - find constant given conditions |
| Word problems - different types |
|  |

## Algebra UNIT

## NAME:

Vocabulary
Expand - different types

- visual representation with rectangle area


## GCF, Grouping

Trinomials - simple, complex

- visual dimensions of rectangle

Special Products -perfect square trinomials (visual)

- difference of squares (visual)
- sum of cubes (SOAP)
- difference of cubes

Factoring with fractions
When to use what
Complete the square - with fractions

- visual representation

TRIG UNIT NAME:

Angle facts
Congruent triangles - define, symbol

- what conditions prove them
- what conditions do NOT + why (SSA and AAA) - proof

Similar triangles - define, symbol

- what conditions prove them - proof
- solve using proportions

SOH CAH TOA - how it comes from similar triangles

- define opp, adj, hyp, focus angle
- solve for side
- solve for angle

Labeling A,B,C versus a,b,c
Angles of elevation, depression
Sine Law - formulas when each one is best to use (proof)

- solve ex

Cosine Law - formulas when each one is best to use (proof) - solve ex

When to use what?
Word Problem
Bearings - true bearing

- compass bearing


## QUAD UNIT NAME:

Is it Quadratic or not? - graph, table, equation
Vocabulary
Transformations
Sketching with image points $(x, y) \rightarrow(x-h, a y+c)$
Different forms - vertex, factored, standard
Find equations
Sketching - with step pattern

- with zeros, a.of.s., opt.val.

Solve equations - by factoring

- by SAMDEB if variable appears once
- by completing square and SAMBED
- by quadratic formula (proof)
\# of solutions - using discriminant
- using sketch

Word Problems - discuss related equation

- deciding on tool to use based on key words

Setting up equations - revenue type

$$
\begin{aligned}
& \text { - fence/rope off area type } \\
& \text { - free fall type } \\
& \text { - other types? }
\end{aligned}
$$

## GEOMETRY UNIT NAME:

## Slope, Midpoint, Distance

Classify - quadrilaterals

- triangles

Circles - at origin, not at origin

- find equation from sketch
- find centre and radius by completing sq

Special lines (action plans, fully solve at least one)

- tangent
- median
- perpendicular bisector
- altitude line

Define - Circumcentre, orthocenter, centroid
Shortest distance - action plan

## Introvectors UNIT NAME:

| Definitions, notation <br> Addition $\quad-$ head to tail (triangle law) <br> $\quad-$ tail to tail (parallelogram law) |
| :--- |
| Subtraction - head to tail when adding the opposite |
| Scalar multiple of vector, unit vector <br> Properties: commutative, associative, distributive <br> Notes on finding magnitudes (when to add, subtract, use Pythagorean/SohCahToa, <br> Cosine/Sine, why sine law should be avoided) <br> Example of calculation if vectors are on a grid <br> Elements needed for proofs <br> Show a proof and explain it <br> Velocity ex - relative velocity <br> $\quad-$ swimmer/current with right triangle <br> $\quad \quad$ plane/wind with non-right triangle <br> Newton's $1^{\text {st }, ~ 2 ~} 2^{\text {nd }}, 3^{\text {rd }}$ laws |
| Free body diagrams \& vectors at equilibrium |
| Force - tension ex |
| $\quad$ - ramp ex |

## AlgVectors UNIT NAME:

Convert 2D vectors: alg to geo, geo to alg

- Geometric(true/compass)
- Algebraic (cartesian/linear combination)

How does one find components of AB vector if you're given point $\mathrm{A} \& \mathrm{~B}$
Convert 3D alg to geo, geo to alg
Algebraic calculations for

- find wind vector given ground velocity of plane
- find direction of boat and magnitude of resultant
- find resultant and equilibrant force from multiple forces

Linear combinations

- write a vector using non-standard vectors as the basis
- identify if vectors are collinear or coplanar or neither
- identify the span of given points and/or vectors and find the basis set

Dot product

- properties
- alg formula and ex
- geo formula and ex

Projections formulas

- scalar alg, geo with visual
- vector alg, geo with visual with arrow

Cross product

- properties
- alg formula with ex
- geo formula with ex AND discuss right hand rule

Applications

- work formula and ex
- area/volume formulas and ex
- torque formula and ex

Example of a proof that uses properties of dot or cross product

LINESPLANES UNIT
NAME:
Equations of Lines in 2D

- grade 9, vector, parametric, symmetric, cartesian
- example(s)

Equation of Lines in 3D

- vector, parametric, symmetric
- why cartesian impossible
- example(s)

Equation of Plane

- vector, parametric, cartesian
- why symmetric is not done for planes
- example(s)

Show visuals of all types of intersections and key notes for them

- example of POI from 2 lines in vector form - K
- example of POI from planes - J
(OPTIONAL include solution using matrices)
- example of LOI from 3 non-coincident planes - I
- example of PLNOI recorded in vector form - E
- example of when you get a contradiction and how to decide what scenario it is (A/D/G/H/M/N)
Shortest distances formulas/steps for all types
Examples
${ }^{-p t}$ to line in 2D with scalar projection
- pt to line in 3D with parallelogram
-pt to line in 3D using parameter - and find pt at min distance
-find pt at minimum distance with vector projection


## FUNCTIONS UNIT

| Function or not |
| :--- |
| - from graph |
| - from equation |
| - from mapping diagram/table |
| - from real life scenario |
| Inequalities |
| - linear |
| - nonlinear |
| Symbols and shorthand |
| Domain from equation |
| - with denom |
| - with sq root |
| - otherwise |
| Domain \& Range from graph |
| - set notation |
| - interval notation |
| Function notation |
| - numeric inputs |
| - expression input |
| - formulas in function notation |
| Parent function shapes |
| Transformations |
| - step by step order |
| - image points |
| Inverses |
| - graphs |
| - equations |
| - one to one functions and unique inverses |
|  |

## RAtionalAlg UNIT NAME:

Exponent laws

- negative exponent law
- zero exponent law
- multiplication/division laws
- power of power
- new exponent law
- mix

Factoring Review

- GCF, grouping
- diff of sq, criss cross
- GCF smallest power out

Simplifying Rational Expressions

- restrictions
- simplify with monomials
- simplify with polynomials
- mult/div rationals
- find LCD with monomials
- find LCD with polyn
- add/subt rationals


## Exponentials UNIT NAME:

New Exponent Law
Radicals

- reduce sq roots
- reduce other roots
- add/subtract radicals
- mult radicals
- divide radicals
- rationalize monomial denom
- rationalize binomial denom

Solve equations with variable in exponent

- match bases
- trial and error
- using replacement to make things easier

Solve equations with variable in the base

- with one sq root
- with two sq roots
- with any rational power

Parent Exponential Growth/Decay graph

- transformations
- graph using all transformations
- simplify to have exponent only $x$ then graph

Find equations

- from table/graph
- with HA given/not

Word Problems

- key info to know
- double/triple/half-life types
- \% inc/dec types
- solve for $y$ given $x$
- solve for $x$ given $y$
- find initial value given another point
- find half-life period given two points


## QUAD+CONICS UNIT NAME:

Quadratic Forms vertex/factored/standard

- what each form tells you
- find $y$-int/x-int/vertex from all forms
- graphing

Find equation

- given transformations
- given y-int and two more general points
- given axis of symmetry, one x-int and one more general point
- given two irrational x-int and one more general point

Inverse of Quad

- find inverse from graphs and domain restriction for a unique inverse
- find inverse from equations

Conics (AP ONLY)

- circle
- ellipse vertical/horizontal
- hyperbola vertical/horizontal
- parabola vertical/horizontal
- find equations from graphs
- distinguish which type the equation is

Non-linear systems and find POI
Number of solutions

- relate to \# of zeros of related equation
- find missing constant to satisfy a specific \# of solutions required

Word problems

- revenue
- profit
- fence/rope off area
- falling object under gravity
- type that involves finding vertex
- type that involves finding $y$-int
- type that involves finding $x$-int of related equation

| DISCRETE UNIT NAME: |
| :--- |
| Pascal tri, expand \& simplify <br> Definitions, notation <br> Sequences, develop formulas <br> - Explicit: arith, geo, neither <br> - Recursive: arith, geo, neither <br> - word prob <br> Find \# of terms <br> Given two terms find other info <br> Series, develop formulas <br> - arith <br> - geo <br> - word prob <br> Finance, what letters stand for, develop formulas <br> - simple, compound, continuous <br> - annuities: PV, FV <br> - when to use what <br> - how to deal with complications <br> $\quad$ - change in frequency ex <br> - change in something else ex |

## RotationalTrig UNIT Name:

## Vocabulary

- CAST
- quadrant \#'s
- acute/obtuse/reflex/quadrantal angles
- terminal arm
- initial position for math versus physics
- positive/negative rotation for math versus physics
- coterminal angles
- principal angle
- related acute/reference angle $\theta_{r}$
- why the $\theta_{r}$ must be near x-axis

Trig ratios

- primary ones
- secondary ones
- new definitions using $\mathrm{x}, \mathrm{y}, \mathrm{r}$ for both primary one and secondary ones
- equivalent ratios

Special triangles and where they come from
FIND RATIOS

- given quadrantal angles
- given special triangle angles
- given non-special angles (use calc)

FIND ANGLES

- given ratio that creates a quadrantal angle
- given ratio that creates a special triangle angle
- given ratio that creates a non-special angle (use calc)
- know how to give list(s) of coterminal angles
- know how to create a sequence equations for the lists

Find coordinates of new point after rotation
Trig identities

- new reciprocal ones
- quotient ones
- Pythagorean types
- how to prove a complicated type from the above types
- strategies: LCD/distribute denom, factor/expand, multiply by conjugate to get squares Cosine Law \& Sine Law
Ambiguous Case
- drawing accurate height to see how many (if any) triangles are possible
- using sine law to find \# of triangles
- using cosine law to find \# of triangles

Solving 3D trig

## SINUSOIDALs UNIT

NAME:
Definitions

- periodic
- sinusoidal
- period
- axis
- amplitude
- range
- find these terms from graphs or description

Sinusoidal graphs

- parent sine and cosine graphs
- state period, $\mathrm{max} / \mathrm{min}$, sentence to memorize for each parent
- transformations
- sketch transformed version using image points
- sketch transformed version using key characteristics

Find equations

- from graph or table
- sine or cosine versions
- with/without reflections

Word Problems

- solve for y given x
- solve for x given y using several methods
FUNCTIONS 12 UNIT NAME:

| Combining Functions |
| :--- |
| - notation, graphs |
| Solving Inequalities |
| - linear, nonlinear |
| Absolute Values, conjunction vs disjunction |
| - evaluating, solving equations, solving inequalities, sketching |
| Piecewise Functions |
| - evaluating, sketching, writing absolute value as piecewise |
| - find equations from graph, find equations from word problems |
| Function Properties |
| - intercepts, critical points, asymptotes, inc/dec intervals, end behavior |
| - odd/even/neither symmetry |
| $\quad$ |
| $\quad$ - graphically |
| $\quad$ algebraically |

## Polyn UNIT

NAME:
(AP ONLY)Complex numbers and their operations (reduce, multiply, divide, distance to origin)
Sketching power functions
Sketching polynomials

- from standard form, discuss all possibilities
- from factored form, discuss how to find degree without expanding

Finding polynomial equation from a table with constant differences
Families of polynomial equations
Division of polynomials

- long division, pros and cons
- synthetic division, pros and cons
- find the missing thing in the statement:
dividend $=($ quotient $)($ divisor $)+$ remainder
by comparing coefficients
Theorems, difference between factor of polynomial and root of polynomial
- remainder theorem
- factor theorem

Factoring Polynomials with the use of

- rational root theorem
- factor theorem

Find all types of roots by using

- fundamental theorem of algebra
- imaginary root theorem
- irrational root theorem

Solve polynomial inequalities

- sketching method
- table of intervals method

Word problems
Composing Polynomials with Roots, Rationals, Absolute Values

- graphs
- find equations

RATIONALS UNIT NAME:
Review how to work with polynomials inside fractions

- mult/div - discuss need to factor first since one can't cancel terms but can cancel factors
- add/subt - discuss how to find LCD properly and note that expanding is only done to
clean up bracket in a bracket, otherwise leave things in factored form
- discuss restrictions - hole versus VA

Sketching Algorithm

- domain/holes/VA
- HA/OA
- x -int/ y -int
- behaviour near zeros cut/bounce/bend
- behaviour near VA odd/even symmetry

Other sketching notes

- sketching reciprocal of polynomial method is helpful if there are no VAs
- some rational functions can be converted to transformed version of $1 / x$

Find equations given graphs
Solve Equations algebraically

- rational type
- irrational type

Solve equations/inequalities graphically - two graphs overtop of each other
Solve inequalities

- rational type
- (AP ONLY)irrational type

Word Problems - motion, work, \%

## RadianTrig UNIT NAME:

| Radians |
| :--- |
| - what are they, discuss why there is no unit |
| - drawing radians (decimal versions and fractions of pi) |
| - converting |
| Doing trig with radians |
| - find output values of special triangle angles/quadrantal angles |
| - find input angles from given ratios (ensure to show quadrantal angle type, special triangle |
| angle type, and neither of those type, also ensure to show all possible solutions sequence |
| equations) |
| Working with radians |
| - arc length/radius/angle relationship and word problems |
| - sector area, where formula comes from and word problems |
| - linear/angular speed relationships and word problems |
| Graphs |
| - how do reciprocal trig functions relate to regular trig functions? |
| - how do inverse trig functions relate to the original trig functions? |
| - transformations of all of the above |
| - find equations of all of the above |

Trigid UNIT NAME:
Trig identities
stating equivalent expression

- odd/even properties
- complementary/cofunction properties
- similar shaped graph properties
- shift by a period
- related acute angle symmetry properties
finding exact values
proofs
Solve trig equations - with k value, with use of identity/factoring, pseudo quadratic type
Sinusoidal word problems, using radians
- sketch and find equation
- find output value and several input values
(AP ONLY) Trig inverses
- simplify expressions
- graphs

LOGS UNIT

## NAME:

Exponent Laws

- zero, negative, rational powers
- for rational powers indicate when to restrict domain or insert absolute values - mult, div, power rules

Show examples of simplifying expressions using exp laws
Exponential versus Logarithmic graphs (grow/decay, int, asymptotes, domain, range)
Logarithmic Laws

- changing forms $\exp \leftrightarrow \log$
- composing exp with log rules
- mult, div, power rules
- change of base rule

Show examples of expanding/condensing using log laws
Solve equations

- exponential type (matching bases with exp laws, convert to log)
- logarithmic type (condensing with log laws, convert to exp)

Word problems

- earthquake, sound, pH
- \% inc/dec, double/half-life

Some common mistakes with log laws:
$\log \left(\frac{x}{6 y}\right)=\log x-\log 6 y$
$\frac{\log x}{\log 6 y} \neq \log x-\log 6 y$
$\left(\log _{2} 3 x\right)^{4} \neq 4 \log _{2} 3 x$
$\begin{aligned} \log _{2} 3 x^{4} & \neq 4 \log _{2} 3 x \\ \log _{2}(3 x)^{4} & =4 \log _{2} 3 x\end{aligned}$
$\log _{3}(5 x+2 x) \neq \log _{3}(5 x)+\log _{3}(2 x)$
$(2 \cdot 3)^{2}=2^{2} \cdot 3^{2}$
$(2+3)^{2} \neq 2^{2}+3^{2}$
$\sqrt{25 \bullet 9}=5 \bullet 3$
$\log \left(\frac{x}{6 y}\right) \neq \log x-\log 6+\log y$
$\log \left(\frac{x}{6 y}\right) \neq \frac{\log x}{\log 6 y}$ $\frac{\log x}{\log 6 y}=\log _{6 y} x$
$\log _{3}((5 x)(2 x)) \neq\left(\log _{3} 5 x\right)\left(\log _{3} 2 x\right)$ $\left(\log _{3} 5 x\right)\left(\log _{3} 2 x\right) \neq \log _{3}(5 x+2 x)$ $\log _{3}((5 x)(2 x))=\log _{3} 5 x+\log _{3} 2 x$
$2 \cdot 3^{x} \neq 6^{x}$
$2^{x} \cdot 3^{x}=6^{x}$
$\sqrt{25-9} \neq 5-3$

COMBINEFUNCT+ROC UNIT NAME:
Difference Quotient and how it changes for AROC vs IROC
Other words used that may still mean to find rate of change, be specific to type
Examples of algebra to get division by $h$ to cancel:

- expanding
- LCD
- rationalizing

Using rates of change on graphs

- describe what is occurring from a given graph
- relate the following to the d-t graph: direction, concavity, slope

Combine functions with - add/subt, mult/div, composition
What effect each one of the above has on the graph and its domain
What is the difference between absolute max and local max (or min)
Under what conditions may there be no absolute max (or min)
Show an example how to find extreme values without a graph
LIMITS UNIT

| Limits |
| :--- |
| - what is a one sided limit \& notation |
| - conditions for the existence of a limit at a point |
| - examples of when limits do not exist (oscillation, jump, no other side exists, approach $\infty$ ) |
| Continuity |
| - conditions for continuity at a point |
| - example of finding continuous intervals |
| - classifying discontinuity as jump//scillation/essential/removable |
| - find a constant to ensure continuity |
| Limit Laws |
| - sum of functions example |
| - mult by constant example |
| - product of functions example |
| - quotient of functions example |
| - compositions if outer function is continuous example |
| - (AP ONLY)compositions if functions are discontinuous examples |
| Vocabulary undefined, defined, indeterminate, determinate |
| Find limits of indeterminate forms |
| - evaluate/sketch ex |
| - factor ex |
| - LCD ex |
| - rationalize ex |
| - multiply by conjugate ex or change of variable ex |
| - abs val rewritten as piecewise ex |
| - dividing by highest power rational ex |
| - dividing by highest power irrational ex |
| - compare rates of growth ex |
| - (AP ONLY)squeeze theorem ex |
| - (AP ONLY)sinx/x, damped wave function ex |
| - (AP ONLY)using trig id ex |
| IVT (AP ONLY) |
| - theorem conditions and conclusion |
| - visual that shows the theorem |
| - example |

## DERIV UNIT

Derivative at a point/differentiable at a point versus
derivative function/differentiable function

- notation
- direct definition
- alternate definition

Define and show pictures of: critical pt, max $/ \min$ H.T t , $\max / \mathrm{min}$ V.T. cusp, sharp pt, V.T.
Inf pt, H.T. saddle/inf pt, general inf pt, VA odd symm, VA even symm, HA, hole, jump
Sketching

- derivative
- antiderivative

Rates of Change

- AROC
- IROC

Derivative Rules

- constant
- power
- constant multiple
- sum/difference
- chain example with prime notation
- chain example with Leibniz notation
- product
- quotient
- trig derivatives
- implicit derivatives
- exponential and log derivatives
- using log laws to simplify derivative taking and $\mathrm{x}^{\wedge} \mathrm{x}$ types
- (APonly)derivative at a point of an inverse
- (APonly)inverse trig derivatives

Differentiability conditions and theorem

- find constant to ensure differentiability

Applications

- equation of tangent line
- equation of normal line


## App 1 Deriv UNIT

NAME:
Compare absolute max with local max
EVT - theorem conditions and conclusion, visual that shows the theorem, example of applying the theorem
(APonly)MVT- theorem conditions and conclusion, visual that shows the theorem,
example of applying the theorem
(APonly)Rolle's Theorem
Clarify how to find:

- Critical points vs Possible Inf pt vs Actual Inf pt
- Increase vs Decrease intervals
- Concave Up vs Concave Down intervals

Classify Critical points

- using $1^{\text {st }}$ derivative test
- using $2^{\text {nd }}$ derivative test

Sketching Algorithm, example
Applications to Motion

- Displacement, Velocity, Acceleration
- describe what's occurring given graph
- draw deriv/antideriv and interpret

Other application interpretation of deriv

## App2 Deriv UNIT NAME:

Optimization three different examples
Steps:

- diagram of one scenario and draw another scenario that may be better, from this identify what changes or what are the variables
- labels on diagram or let statements for the variables
- come up with equation(s) relating all variables together
- identify what variable will be the output to maximize/minimize
- combine equations if needed, so that there's only two variables (input and output)
- find domain in terms of input variable
- find derivative w.r.t. input variable
- find critical points, justify Max/Min ( $1^{\text {st }}$ deriv test/2 $2^{\text {nd }}$ deriv test/EVT if have closed interval)
- answer the question with units

Related Rates three different examples
Steps:

- what are the givens with units? what is required to find with units? identify the 'AT' what number?
- diagram of original with variables, indicating the parts that will change with arrows, note the ones that stay constant
- A.T.I. diagram with all information found for the 'AT' number
- come up with equation(s) relating all variables together (usually same equation as used in the A.T.I. equation to find values, but now just with variables)
- *for an easier time later, combine equations, so that there's only one variable per side
- find derivative implicitly w.r.t. time
- substitute the numbers from the A.T.I. diagram and solve for the unknown rate of change
- answer the question with units
(AP ONLY) L'Hopital's Rule
- forms applicable for L'H
- forms that can be manipulated to apply L'H
- 4 different examples
(AP ONLY) Approximations
- Linearization
- Differentials


## UNIVERSITY CALCULUS MATH JOURNAL CHECKLISTS

INTEGRATION UNIT NAME:

| Antiderivatives/indefinite integration |
| :--- |
| - notation, common rules |
| - discuss how to handle constants (+C, riders, rule constants, correction constants) |
| - using initial conditions to find C in a differential equation ie. Particular solutions |
| Area under curve |
| - real life application - how to deal with units |
| - curvy graphs approximations: LRAM, RRAM, TRAPEZOID, MRAM, SIMPSON |
| Working with Sigma |
| - properties, using properties to evaluate a sum, integration from 1st principles using sums |
| - convert sigma sum to what it would be as a definite integral of a function |
| Definite integration |
| - notation with sums vs integral Leibnitz notation |
| - signed area found from left to right and vice versa |
| - graphs that create a regular geometric shape with the x-axis |
| - integration properties and how to use them |
| Summary of methods so far and why FTC is needed |
| FTC part 2 |
| - discuss the need for 1.integrand to be continuous, 2.knowing the antiderivative function |
| FTC part 1 |
| - explain how to define a function of an integral i.e. the accumulation function |
| - find derivative of an integral with variables on BOTH limits of integration |
| Total Area vs Net accumulation (needed for displacement vs distance later) |
| Average Value of a Function |

## App 1 InTEg UNIT NAME:

[^0]
## App2 InTEG UNIT NAME:

Area between two curves steps and key notes

- using vertical slices ex
- using horizontal slices ex
- explain why sometimes you can't slice enclosed region both ways

Volume, steps and key notes

- using disc ex
- using washer ex
- using shell ex
- slicing ex with regular shape cross-sections

Slope Fields \& DE - define each, why important to learn

- discuss general solution vs particular sol
- example of the field
- example of using the field to draw the unique antiderivative curve given a point

Word problems with DE

- growth/decay ex (where rate is proportional to original quantity)
- limited growth ex (where rate is proportional to remaining room for growth)

OTHERS (include? if you're going into that specific field)
Physics:

- Newton's law of cooling ex, forensics ex
- motion ex without air resistance ex, with air resistance ex
- work done by constant force ex, by variable force ex
- pressure of fluid on a horizontal surface ex, on a vertical surface ex


## Biology:

- viral/bacterial growth ex
- medicine in bloodstream ex
- mixing solutions ex
- cardiac output ex

Economics:

- continual compounding interest ex
- producer surplus vs consumer surplus ex
- marginal revenue or marginal cost ex


[^0]:    Deciding when to use what version of the formula

    - MVT for derivatives or MVT for integrals
    - average value of a function or average change in a function
    - net change in a function or accumulation at one point

    Applications

    - interpretations of the integral
    - questions about inc/dec/CU/CD/InfPt
    - everything you need to know about motion

    U Substitution for integrals where you recognize chain rule (inner function's derivative multiplied)

    - how to deal with definite integrals with U-sub

    Other strategies

    - work backward by pattern recognition, ensuring you take care of constants properly
    - factor/complete the square to match to an inverse trig
    - use trig identities/long divide/expand/distribute denominator for pattern recognition - multiply by a factor equivalent to ONE
    - do implicit derivative on U-sub

    Using TI-89 calculator

