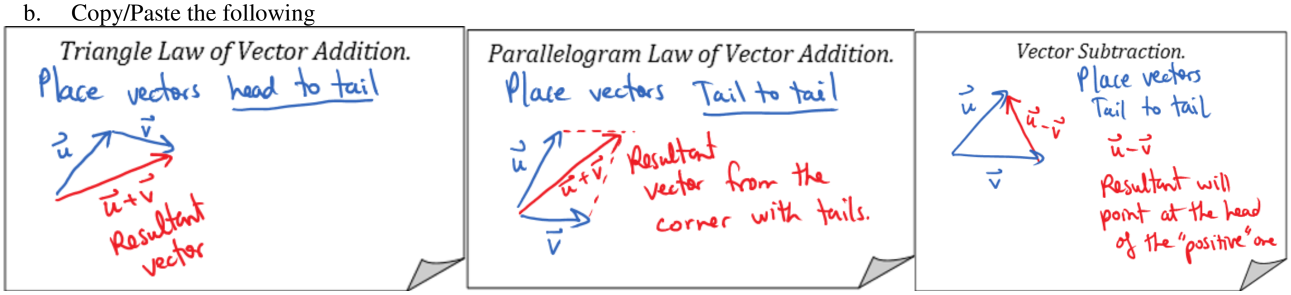
Vectors UNIT A

INTRODUCTION TO VECTORS – journal **NAME:___________________**

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

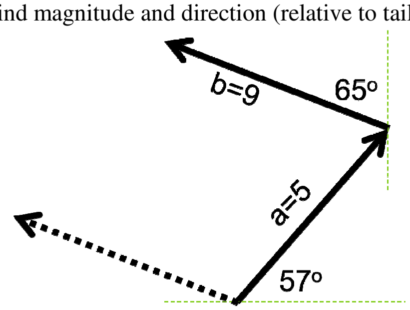
1. INTRODUCTION TO VECTORS
   a. Distinguish between vector and scalar, give examples
   b. Copy/Paste the following



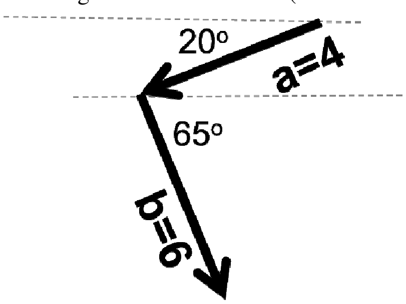
   c. Describe what angle between two tails will yield the following relationships, give reasons:

$\lvert\vec{a}+\vec{b}\rvert = \lvert\vec{a}\rvert + \lvert\vec{b}\rvert$	$\lvert\vec{a}+\vec{b}\rvert = \sqrt{\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2}$ **and** $\lvert\vec{a}+\vec{b}\rvert = \lvert\vec{a}-\vec{b}\rvert$	$\lvert\vec{a}+\vec{b}\rvert = \left\lvert\lvert\vec{a}\rvert - \lvert\vec{b}\rvert\right\rvert$	$\lvert\vec{a}+\vec{b}\rvert \le \lvert\vec{a}\rvert + \lvert\vec{b}\rvert$

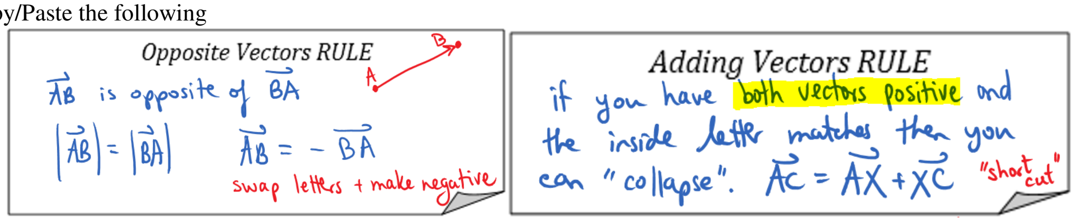
   d. Find magnitude and direction (relative to tail of original vectors) of the resultant ***difference*** $\vec{a}-\vec{b}$ using SineCosine METHOD



2. VECTORS in R^2 and R^3
   a. Record all formulas for 2D and 3D
   b. Draw a position vector of the point P(– 3, 7) and express it in polar form. Then state the unit vector in same direction as the position vector in component form.
   c. Express as a vector in component form
      i. $\vec{a} = (12, 330^o)$, in R^2
      ii. $\lvert\vec{u}\rvert = 8$, $\alpha = 60^o$, $\beta = 150^o$ in R^3 Explain how one can find the 3rd direction angle if given only two of them in R^3
   d. Find magnitude and direction (relative to tail of original vectors) of the resultant ***sum*** using COMPONENTS METHOD



3. VECTOR PROPERTIES & PROOFS
   a. Copy/Paste the following



To Find Position Vector

$\vec{OA} = (a_1, a_2)$ $\vec{OB} = (b_1, b_2)$

$\vec{OP} = \vec{AB} = (b_1 - a_1, b_2 - a_1)$

$\hat{u}$ Unit Vector

has a magnitude of ONE

$\hat{u} = \dfrac{\vec{u}}{|\vec{u}|}$

Properties of Vectors

$\vec{a}+\vec{b}=\vec{b}+\vec{a}$ commutative	$(kl)\vec{a}=k(l\vec{a})=l(k\vec{a})$ associative scalar
$\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$ additive identity	$(k+l)\vec{a}=k\vec{a}+l\vec{a}$ distribute vector
$\vec{a}+(-\vec{a})=(-\vec{a})+\vec{a}=\vec{0}$ additive inverse	$1\vec{a}=\vec{a}$ multiplicative identity
$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ associative vector	$(-1)\vec{a}=-\vec{a}$ opposite vector
$\lVert k\vec{a}\rVert = \lvert k\rvert \lVert\vec{a}\rVert$ scalar multiple	$0\vec{a}=\vec{0}$ zero vector
$k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$ distribute scalar	$\lVert\vec{0}\rVert=0$ magnitude of zero vector

b. Show a proof using vectors for:

If side BC of ΔABC is trisected by points P and Q, show that
$\vec{AB} + \vec{AC} = \vec{AP} + \vec{AQ}$

c. Prove the following using cosine law (DO NOT FOIL, can't until we learn about how to multiply vectors by dot and cross products)

$$\lVert\vec{a}+\vec{b}\rVert^2 + \lVert\vec{a}-\vec{b}\rVert^2 = 2\lVert\vec{a}\rVert^2 + 2\lVert\vec{b}\rVert^2$$

d. Redo the same question from assignment(below) using the above property without any rounding, compare with what you did in the assignment pg4.

Given $\lVert\vec{a}\rVert=10$, $\lVert\vec{b}\rVert=15$, and $\lVert\vec{a}+\vec{b}\rVert=20$, find $\lVert\vec{a}-\vec{b}\rVert$.

4. OPERATIONS WITH VECTORS

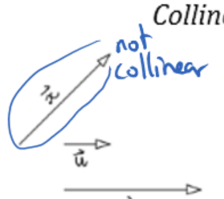
Clarify for yourself when position vectors must be added or and when they are subtracted (use the following examples to explain)

a. Given pt A and pt B, find $\vec{AB}$
b. Given parallelogram OAPB and points A and B, find point P (Notice one point of parallelogram is origin here)

5. SPANNING SETS

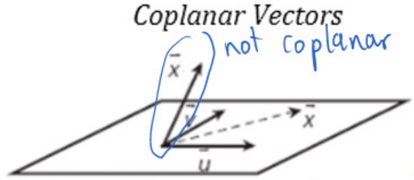
a. Explain Spanning Sets for lines, planes and 3-D space.
b. Copy/Paste the following

Collinear Vectors



$\vec{x}$ and $\vec{u}$ are collinear $\overset{\text{iff}}{\Leftrightarrow}$ there is a scalar k such that $\vec{x} = k\vec{u}$

Coplanar Vectors



$\vec{u}, \vec{v}$ and $\vec{x}$ are coplanar $\overset{\text{iff}}{\Leftrightarrow}$ there are scalars a and b such that $\vec{x} = a\vec{u} + b\vec{v}$

c. Do the following question and talk about what the vectors can span.

Prove that the vectors $\vec{a}=(-1,2,-3)$, $\vec{b}=(2,0,-1)$, and $\vec{c}=(-7,6,-7)$ are linear dependant. ∴ Coplanar