## Big idea

Did you know that bees use vectors? A honey bee that has found a beautiful meadow full of ripe flowers must come back to the hive and communicate that information. A bee must tell its fellows in what direction and how far to travel to get to the meadow, they can even compensate for any wind direction in their communications! (Show a video from youtube.)

You will be introduced to the idea of a directed line segment, called a vector. You will explore vectors in their geometric and algebraic form, and will learn the notation used to describe vectors. You will then study vector addition and properties and how to understand vectors in 2D and 3D spaces. Vectors will enable you to define a line in 2D or 3D space. After which you can then solve where two such lines meet, if ever (unit 2).

## Feedback & Assessment of Your Success

**Tentative TEST date:** ____________________

| **Date** | **Pages** | **Topics** | **Finished assignment pages?** **Made corrections?** | **Summarized notes in a journal?** **Added your own explanations?** | **How many extra practice questions did you try in each topic?** | **Questions to ask the teacher:** |
|---|---|---|---|---|---|---|
| 1.5days | 2-4 | Introduction to Vectors (MCV)<br>Journal #1 | | | | |
| 2days | 5-8 | Vectors in $R^2$ and $R^3$ (MCV)<br>Journal #2 | | | | |
| 2days | 9-11 | Vector Properties & Proofs (MCV)<br>Journal #3 | | | | |
| 1.5days | 12-14 | Operations with Vectors (MCV)<br>Journal #4 | | | | |
| | 15-16 | Linear Combinations and Spans (MCV)<br>Journal #5 | | | | |

# ASSIGNMENT Introduction to Vectors (MCV)

1. A **vector** is a quantity that requires both a ____________________ and a ____________________ for a complete description.

A **scalar** is only ________________________, it can be completely specified by just one number.

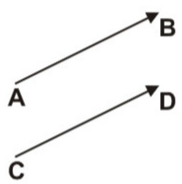
**Geometric Vectors** are vectors not related to any ________________________________

**Algebraic Vectors** are described by their ____________________ relative to a reference system (frame).

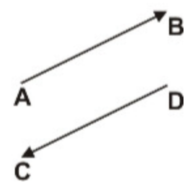
The **magnitude** is the _______________, or size, or norm, or intensity of the vector

2. Classify as vector (V) or scalar (S)
   a) Mass of moon
   b) Acceleration of a drag racer
   c) Velocity of a wave on the beach
   d) Speed of light
   e) Force of gravity
   f) Magnetic field of earth
   g) Area of a rectangle
   h) Temperature of a swimming pool
   i) Your weight
   j) Rocket launched 150m directly up

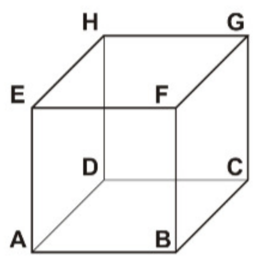
3. Two vectors are **equivalent** or **equal** if they have ____________________ magnitude and direction.




4. Two vectors are called **opposite** if they have the same magnitude and opposite ________________.

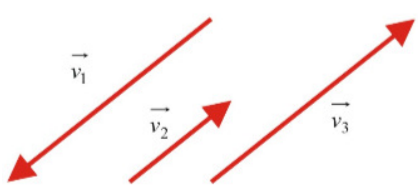



5. Find three pairs of equivalent vectors in the next diagram:




6. Find three pairs of opposite vectors in the previous diagram.

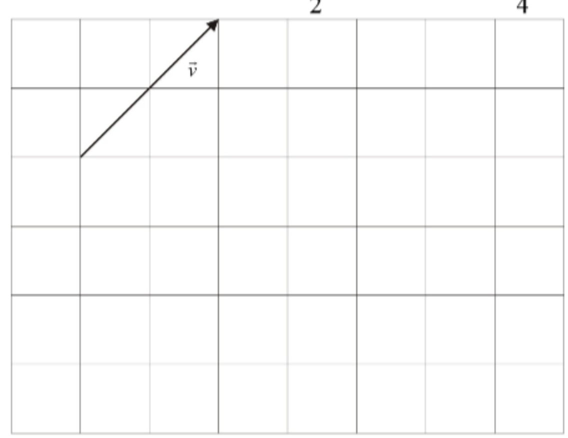
7. Two vectors are **parallel** or **collinear** if their directions are either the _______________ or ______________



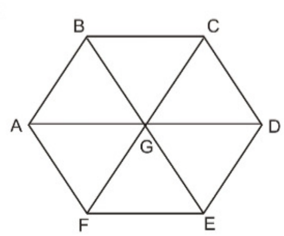

8. **Scalar Multiples of Vectors**

Given the vector $\vec{v}$, draw the following vectors:

a) $2\vec{v}$ b) $-3\vec{v}$ d) $\frac{1}{2}\vec{v}$ e) $-\frac{1}{4}\vec{v}$

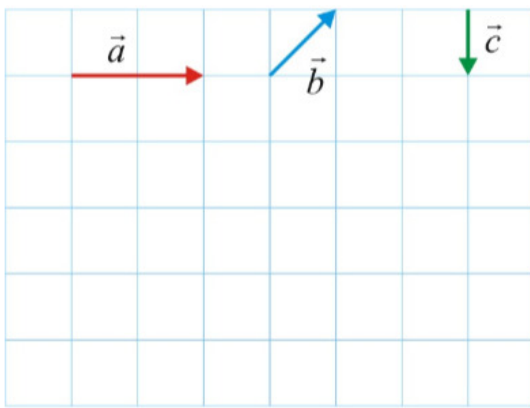



9. Use the following diagram and identify three vectors parallel to $\overrightarrow{AG}$.



10. **Vector Addition – Triangle Rule**

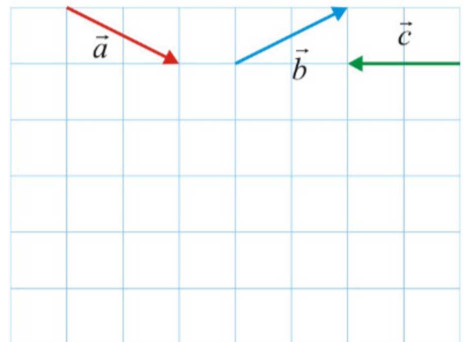
Use the following diagram and the triangle rule compute the required operations.




a) $\vec{a}+\vec{b}$ b) $\vec{b}+\vec{c}$ c) $\vec{a}+\vec{c}$

11. **Vector Addition – Parallelogram Rule**

Use the parallelogram rule to compute the required operations:

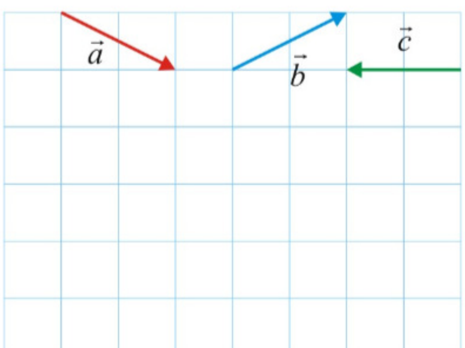



a) $\vec{a}+\vec{b}$ b) $\vec{b}+\vec{c}$ c) $\vec{a}+\vec{c}$

12. **Vector Subtraction**

Compute the required operations.

a) $\vec{a}-\vec{b}$ b) $\vec{b}-\vec{c}$ c) $\vec{a}-\vec{c}$

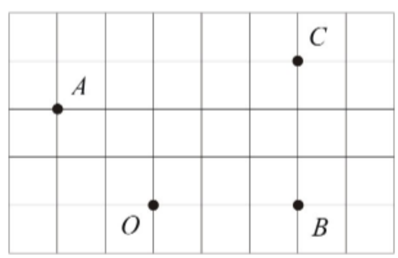



13. The **position vector** is the directed line segment from the ________________ of the coordinate system

The **displacement vector** is the directed line segment from ______________________________.

The **angle between two vectors** is always between ___________________________

14. Draw position vectors $\overrightarrow{OA}$, $\overrightarrow{OC}$ and displacement vectors $\overrightarrow{BA}$, $\overrightarrow{BC}$




15. Analyze the following specific cases of addition of two vectors. Explain and make a diagram.

a) $\theta = 0$

b) $\theta = \pi / 2$

c) $\theta = \pi$

16. Give an <u>informal proof</u> /justification of the triangle inequality: $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$.

17. Given an informal proof / justification of

if $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then $\vec{a} \perp \vec{b}$

18. Given the magnitude of two vectors $\|\vec{a}\|=4$ and $\|\vec{b}\|=7$, and the angle between them when placed tail to tail as being $\theta=60°$, find the magnitude of the vector sum $\vec{s}=\vec{a}+\vec{b}$ and the direction (the angles between the vector sum and each vector). Draw a diagram.

19. Given the magnitude of two vectors $\|\vec{a}\|=10$ and $\|\vec{b}\|=14$, and the angle between them when placed tail to tail as being $\theta=120°$, find the magnitude of the vector difference $\vec{d}=\vec{a}-\vec{b}$ and the direction (the angles between the vector sum and each vector). Draw a diagram.

20. Given $\|\vec{a}\|=4$, $\|\vec{b}\|=6$, and $\|\vec{a}+\vec{b}\|=8$, find the angle between the vectors $\vec{a}$ and $\vec{b}$ (when placed tail to tail) $\theta=\angle(\vec{a},\vec{b})$.

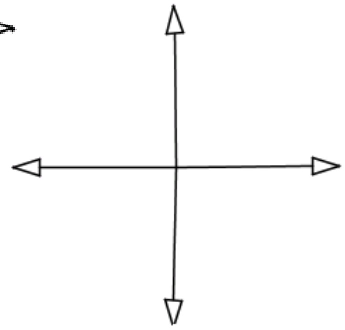
21. Given $\|\vec{a}\|=10$, $\|\vec{b}\|=15$, and $\|\vec{a}+\vec{b}\|=20$, find $\|\vec{a}-\vec{b}\|$.

## ASSIGNMENT Vectors in $R^2$ and $R^3$ (MCV)

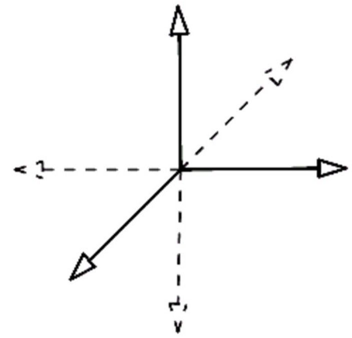
1. Describe what $x = 3$ would mean in each dimension.

**R – line**

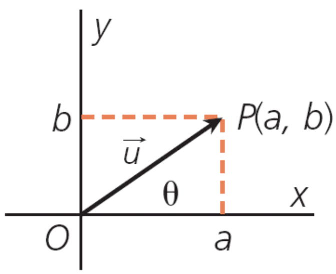
**$R^2$ – plane**



**$R^3$ – space**



2. 2D info for **Position Vectors**




Standard Basis Vectors:

Position Vector in
-linear combination form:

-component form:

-polar form

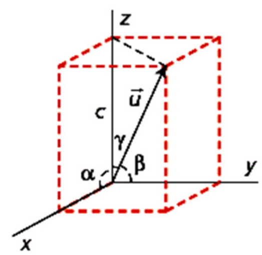
Direction angle

Magnitude:

Unit Vector:

Formulas for **Displacement Vectors** from points: $A(x_1, y_1)B(x_2, y_2)$

3. 3D info for **Position Vectors**




Standard Basis Vectors:

Position Vector in
-linear combination form:

-component form:

Direction angles

Magnitude:

Unit Vector:

Formulas for **Displacement Vectors** from points:
$A(x_1, y_1, z_1)B(x_2, y_2, z_2)$

4. Draw the following vectors

**Quadrant Bearing (Compass)** form:

$\vec{b} = 5m[N45°E]$

**True (Azimuth) Bearing** Angles:

$\vec{v} = 5m/s\ [120°]$

**Polar Coordinates** form:

$\vec{b} = (4,198°)$

**Scalar/Component Coordinates** form:

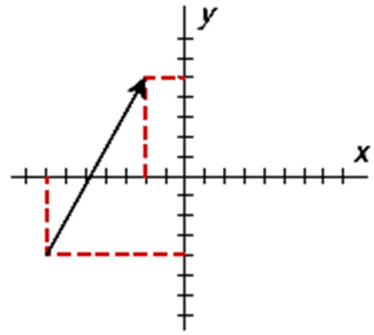
$\vec{b} = (-4,-7)$

**Linear Combination** of Standard Basis Vectors

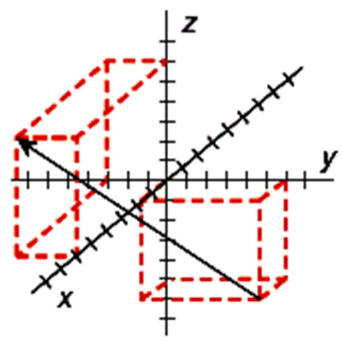
$\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$

5. Reposition each of the following vectors so that its initial point is at the origin, and determine its components.

a)




b)




6. Do the required conversions.

a) Convert $\vec{a} = (10,120°)$ to the scalar coordinates.

b) Convert $\vec{b} = (-4,-7)$ to the polar coordinates.

7. Convert each vector.

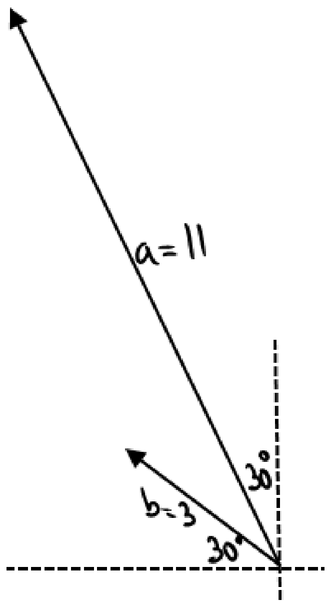
a) $\vec{v} = 5m/s[210°]$ (to quadrant bearing)

b) $\vec{d} = 25m[N30°W]$ (to true bearing)

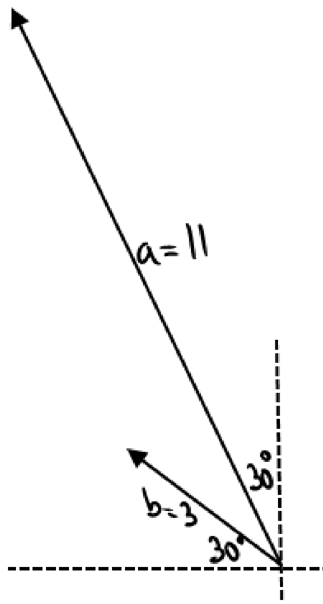
8. Fill in the chart

| | **Geometric Vectors METHOD #1 with SinCosLaws:** | **Algebraic Components METHOD #2 with polar form:** |
|---|---|---|
| Pros | | |
| Cons | | |

9. Find $\vec{b} - \vec{a}$ using Geometric vectors METHOD #1




10. Find $\vec{b} - \vec{a}$ using Algebraic Vectors METHOD #2



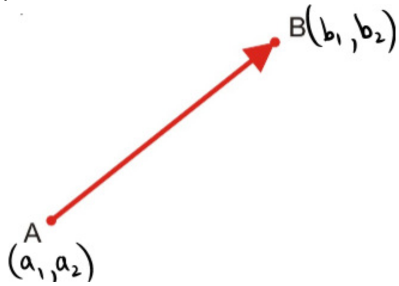
11. A student travels to school by bus, first riding 2 km west, then changing buses and riding a further 3 km north. Find the resultant geometric vector

12. A search and rescue aircraft, travelling at a speed of 240km/h, starts out at a heading of $N20^o W$ . After travelling for one hour and fifteen minutes, it turns to a heading of $N80^o E$ and continues for another 2 hours before returning to base.

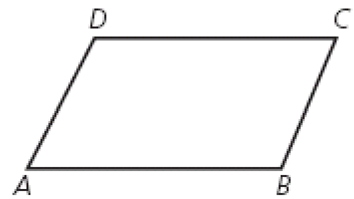
a) Determine the displacement vector for each leg of the trip. – use a method of your choice
b) Find the total distance the aircraft travelled and how long it took.

## ASSIGNMENT Vector Properties & Proofs (MCV)

1. Show (geometrically and algebraically) how to find the related position vector of any vector between points?



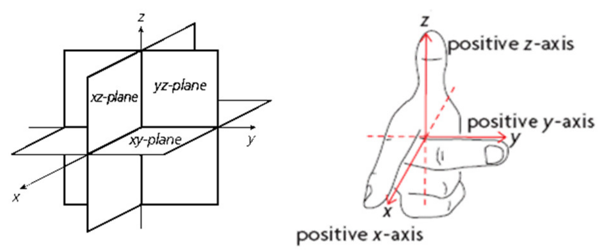

2. In parallelogram ABCD, find the difference $\overrightarrow{AB} - \overrightarrow{AD}$
   a. geometrically
   b. algebraically



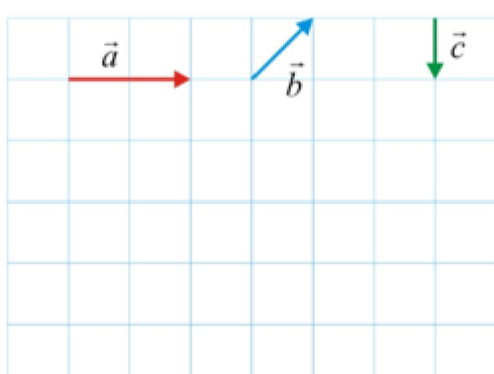

3. An **unit vector** is a vector having a magnitude of __

   Notation & Formula:

4. **Standard Basis Vectors** are unit vectors in the ___________ direction of x. y and z axes




5. Prove the associative law geometrically



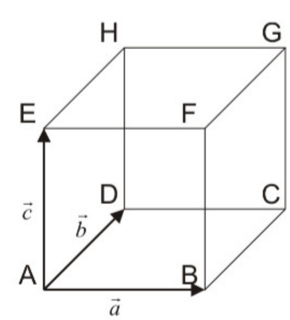

6. A cube is constructed from three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, as shown in the left figure. Express in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$:

   a) $\overrightarrow{AF}$

   b) $\overrightarrow{CF}$

   c) $\overrightarrow{AG}$

   d) $\overrightarrow{BH}$



7. Given $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$ , $\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$ , write the following expressions in terms of the vectors $\vec{i}$ , $\vec{j}$ , and $\vec{k}$ .

a) $\vec{a} + \vec{b}$

b) $2\vec{a} - 3\vec{b}$

8. If $\vec{a} = 2\vec{i} - 3\vec{j}$ and $\vec{b} = \vec{i} + \vec{j}$ , find $\vec{i}$ and $\vec{j}$ in terms of $\vec{a}$ and $\vec{b}$ .

9. If $\vec{x}$ and $\vec{y}$ are two unit vectors with an angle of $30°$ between them, find the magnitude and direction of the vector $3\vec{x} - 5\vec{y}$ .

10. Given $\|\vec{u}\| = 8m$ and $\|\vec{v}\| = 12m$ , $\|\vec{u} + \vec{v}\| = 16$ , determine the magnitude and the direction of the vector $2\vec{u} - 3\vec{v}$ .

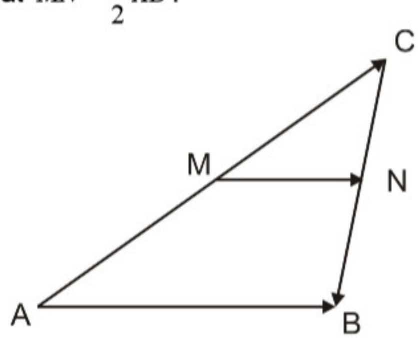
11. Show a <u>formal proof</u> that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular when $|\vec{a}|=|\vec{b}|$.

12. Consider the triangle $\Delta ABC$ and the point $O$ defined by $\overrightarrow{AO}=\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{3}$. Let $M$ be the midpoint of $BC$.

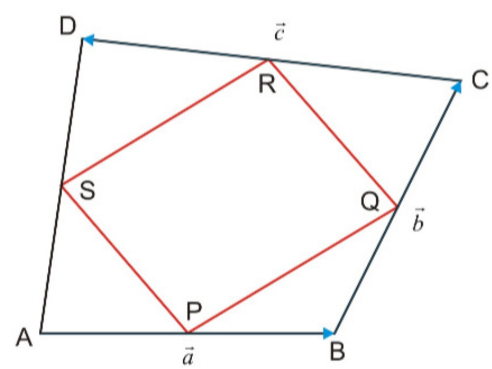
a) Prove that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\vec{0}$.

b) Prove that $\overrightarrow{AM}=\dfrac{3}{2}\overrightarrow{AO}$.

13. Consider the triangle $\Delta ABC$. Let $M$ be the midpoint of $AC$ and $N$ be the midpoint of $BC$. Prove that $\overrightarrow{MN}=\dfrac{1}{2}\overrightarrow{AB}$.

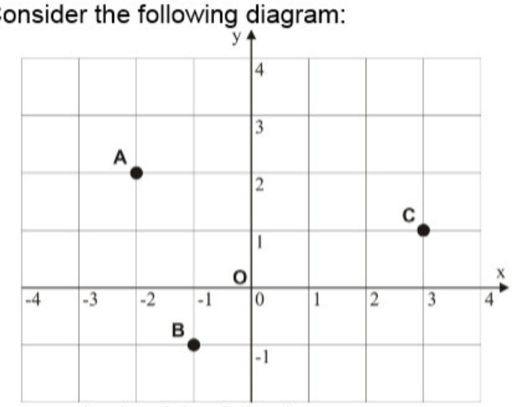



14. Consider a polygon $ABCD$ and let $P$, $Q$, $R$, and $S$ be the midpoints of $AB$, $BC$, $CD$, and $DA$ respectively. Prove that $PQRS$ is a parallelogram.



## ASSIGNMENT Operations with Vectors (MCV)

1. Consider the following diagram:



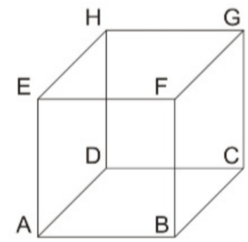

Find the magnitude of the following vectors:

a) $\overrightarrow{OA}$

b) $\overrightarrow{AB}$

c) $\overrightarrow{BC}$

2. Consider the cube $ABCDEFGH$ with the side length equal to $10cm$.




Find the magnitude of the following vectors:

a) $\overrightarrow{AB}$

b) $\overrightarrow{BD}$

c) $\overrightarrow{BH}$

3. Find scalar coordinates of the vector given $|\vec{u}| = 20$ $\alpha = 120^o$, $\beta = 45^o$

4. Find the components of the unit vector with the direction opposite to $\overrightarrow{XY}$ where $X(7,4,-2)$ and $Y(1,2,1)$. Then find direction angles.

5. Find the sum of the vector $\vec{a} = 3\vec{i} - 5\vec{j}$ and $\vec{b} = (-2,7)$

6. Find the magnitude of the difference $\vec{a} - \vec{b}$ between the vector $\vec{a} = \vec{i} - \vec{j}$ and $\vec{b} = (1,2,-1)$.

7. Given $\vec{a} = (1,-2,0)$, $\vec{b} = (0,-2,-3)$, and $\vec{c} = (-1,0,2)$, find the vector $\vec{d} = \vec{a} - 2\vec{b} + 3\vec{c}$.

8. Given $\vec{a} = (-2,1)$, $\vec{b} = (1,-3)$ solve for $\vec{x}$ the following vector equation $2\vec{a} - 3\vec{x} = \vec{b}$.

9. Given $A(-2,3)$, $B(-1,-2)$, and $D(4,2)$, find the point $C$ such that the polygon $ABCD$ is a parallelogram.

10. Find an unit vector parallel to the vector $\vec{a} = (3,-4)$.

11. Find the perimeter of the triangle $\Delta ABC$ where $A(0,1)$, $B(2,3)$, and $C(-1,-2)$.

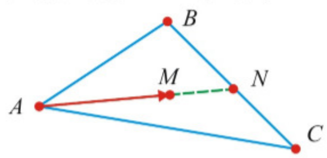
12. Find the point on the y-axis that is equidistant from the points (2, – 1, 1) and (0, 1, 3)

13. Prove that the midpoint of the segment line $AB$ is given by:

$$\overrightarrow{OM}=\frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}=\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2},\frac{z_A+z_B}{2}\right)$$

14. Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ where $P(2,0)$, $Q(0,3)$, and $R(-3,-4)$.

15. Consider the triangle $\Delta ABC$ where $A(-1,-4,1)$, $B(2,-3,0)$, and $C(-4,1,2)$.



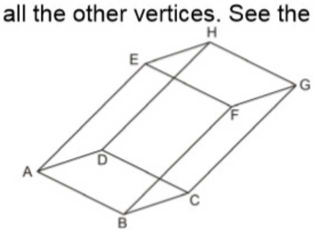

a) Find the centroid $M$ of the triangle.

b) Use the result at part a) to show that $\overrightarrow{MA}+\overrightarrow{MB}+\overrightarrow{MC}=\vec{0}$.

c) Find the midpoint $N$ of the side $BC$.

d) Show that $\overrightarrow{AN}=3\overrightarrow{MN}$.

16. The shape $ABCDEFGH$ is a parallelepiped. Given $A(0,1,3)$, $B(1,0,2)$, $C(1,2,0)$, and $E(4,4,4)$, find the coordinates of all the other vertices. See the figure below.



## ASSIGNMENT Linear Combinations and Spanning Sets (MCV)

1. **Parallel/Collinear Vectors** can be written as ______________________________ of each other
Ex.

2. **Coplanar Vectors** can be written as ______________________________ of each other.
Vectors can also be said to be **Linearly Dependent**
Ex.

3. **Spanning Set for 1D – LINE in $R^2$ or $R^3$** **Spanning Set for 2D – PLANE in $R^3$** **Spanning Set for 3D – SPACE=$R^3$**

Determine if the following $\vec{x}$ and $\vec{u}$ are collinear, then explain what the two vectors can span.

4. $\vec{x} = 4\hat{i} - 8\hat{j}$
$\vec{u} = 6\hat{i} - 12\hat{j}$

5. $\vec{x} = (10, -8, 3)$
$\vec{u} = (5, -4, 6)$

6. Prove that the points $A(2,-1,0)$, $B(-1,0,2)$, and $C(0,1,2)$ are not collinear.

7. Using vectors, demonstrate that the three points $A(5,-1), B(-3,4)$ and $C(13,-6)$ are collinear.

Determine if the following $\vec{u}$ , $\vec{v}$ and $\vec{w}$ are coplanar, then explain what the 3 vectors can span.

8. $\vec{u} = (3, -1, 4)$

$\vec{v} = (6, -4, -8)$

$\vec{w} = (7, -3, 4)$

9. $\vec{u} = (1, 3, 2)$

$\vec{v} = (1, -1, 1)$

$\vec{w} = (5, 1, -4)$