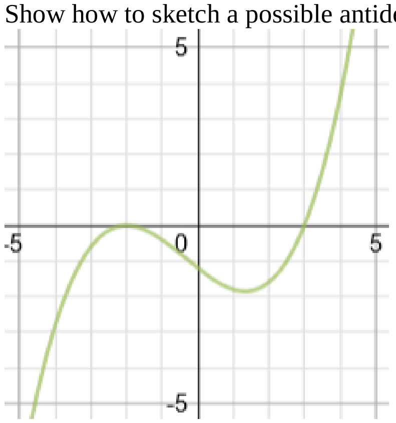
Calculus UNIT 5

INTEGRATION (AP)– journal questions

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. ANTIDERIVATIVES & INDEFINITE INTEGALS
   a. Define the antiderivative, talk about correct notation.
   b. Show how to sketch a possible antiderivative of



   c.

SKETCHING ANTIDERIVATIVES

Drawing f **from** f' - Split domain at the following x values: zeros, turning points/cusps, discontinuities. Then use the following chart.

$f(x)$	$f'(x)$
• MAX Turning point (has HT)	• +, Zero, –
• MIN Turning point (has HT)	• – , Zero, +
• Inflection point (but not VT or HT)	• Turning point NOT AT ZERO
• Saddle point (is a Pt. of Inf. too, also has HT)	• Turning point or cusp AT ZERO – , Zero, – *or* + , Zero, +
• Vertical tangent (is a Pt. of Inf. too)	• VA with even symmetry
• Vertical Cusp (is a MAX/MIN pt too)	• VA with odd symmetry
• Sharp point but not with ∞ slopes	• Hole with Jump discontinuity
• Inc (slope is pos)	• Pos (above x-axis)
• Dec (slope is neg)	• Neg (below x-axis)
• CU	• Inc (slope is positive)
• CD	• Dec (slope is negative)
• VA	• Stays VA
• HA at any constant	• Becomes HA at y=0
• Flat graph at any constant	• Becomes flat graph at y=0
• If graph resembles x^n polyn	• Will resemble x^{n-1} polyn

Drawing f' **from** f - Split domain at the following x values: all the critical points (turning points, vertical tangents, cusps, sharp points, discontinuities) and inflection points.

   d. Copy/Paste the following

COMMON INTEGRALS (the ones with * still need to be developed with methods other than just one step antiderivative)

$\int k\,dx = kx + C$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$

$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$

* $\int \ln(x)\,dx = x\ln(x) - x + C$

$\int e^x\,dx = e^x + C$

$\int a^u\,du = \frac{a^u}{\ln a} + C$

$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$

$\int \cos x\,dx = \sin x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

* $\int \csc^2 x\,dx = -\cot x + C$

* $\int \tan x\,dx = \ln|\sec x| + C$

* $\int \cot x\,dx = \ln|\sin x| + c$

* $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

* $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$

$\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}}dx = \sec^{-1} x + C$

NOTES on CONSTANTS

$$\int 7(4+9x)^{\frac{3}{2}}dx$$

$$= 7\left(\frac{2}{5}\right)\frac{1}{9}(4+9x)^{\frac{5}{2}} + C$$

When integrating by pattern recognition, you will collect no more than three different types of scalar/constant multiples out in front of your antiderivative:

- 7 = constant multiples that were there in the original integrand (I call these “riders”)
- 2/5 = constant multiples generated from an integration rule like the power rule (I call these “rule” constants)
- 1/9 = constant multiples that “correct” any unwanted constant multiple generated by the derivative of the “inside function.” These values will always be the reciprocal of the unwanted value (I call these “corrections.”)

   e. Find

$$\int\left(x^3 + \frac{1}{(7x-5)^3} + e^{8x} + 4^{3x+2} + 2\cos(5x) + \frac{1}{16+4x^2}\right)dx$$

   f. Explain why you’d be stuck for this one $\int e^{x^2}dx$

and be ok for this one $\int 2xe^{x^2}dx$

2. ESTIMATING AREAS

a. Copy/Paste the following

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x\sum_{k=0}^{n-1} f(x_k) \qquad R_n = \Delta x\sum_{k=1}^{n} f(x_k)$$

Midpoint Rule

$$M_n = \Delta x\sum_{k=0}^{n-1} f(\frac{x_k + x_{k+1}}{2})$$

Simpson Rule for even n

$$S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n))$$

b. Read up on Trapezoidal and Simpson's Approximations at http://tutorial.math.lamar.edu/Classes/CalcII/ApproximatingDefIntegrals.aspx and www.mrsk.ca/AP/LESSONtrapezoidSimpsons.pdf then discuss i) what shapes each method uses, ii) order the approximation methods from least to best iii) record all the ERROR BOUND formulas (you may need to research the Left and Right approximation error bounds yourself.

c. The table below gives the velocity of a car as a function of time.

Time *t* (hr)	1	1.25	1.5	1.75	2
Velocity *v* (km/hr)	60	75	80	68	55

i. Approximate $\int_1^2 v(t)dt$ using LRAM, RRAM, TRAP – show your solution clearly so looking back you'd know where numbers are coming from.

ii. Approximate $V'(1.5)$

iii. Explain what physical quantities your answers in i) and ii) are approximating.

3. SIGMA LIMITS

a. Refer to your journal PRECALCULUS UNIT 3 about Sigma notation and remind yourself how to simplify finite sums

i. Simplify using properties of sums then find the limit

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[\left(\frac{i}{n}\right)^3 + 1\right]$$

ii. Record the given expression in i).as a definite integral

b. Find the area under $f(x) = 1 - x^2$ between $x = 0$ and $x = 1$ using sums.

c. Explain what must be done to find the area between curve and x-axis same function as b). except if interval changed from $[0,1]$ to $[0,2]$. No need to solve, just set up.

4. DEFINITE INTEGRALS

a. Copy/Paste the following

INTEGRATION PROPERTIES

If *f* and *g* are integrable on given intervals

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$, — order of integration
2. $\int_a^a f(x)dx = 0$, — zero width interval
3. $\int_a^b c\,dx = c(b-a)$, c is any constant — horizontal line gives rectangle area
4. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$ — sum/difference (note no product/quotient property!)
5. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, c is any constant — constant multiple
6. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ — additivity of adjacent intervals

b. Given

$$\int_{-2}^{2} f(x)\,dx = 4,\ \int_{2}^{5} f(x)\,dx = -3,\ and\ \int_{-2}^{2} g(x)\,dx = 9$$

Find, if possible, state what property is being used

(i) $\int_{5}^{2} f(x)\,dx$ (ii) $\int_{-2}^{5} f(x)\,dx$ (iii) $\int_{-2}^{2} [3f(x)+2g(x)]\,dx$

(iv) $\int_{0}^{2} f(x)dx$ (v) $\int_{-4}^{4} h(x)\,dx$ (vi) $\int_{2}^{2} f(x)dx + \int_{-4}^{4} 6dx$

c. Explain the statement "we can use area to help us find definite integrals, and we can use definite integrals to help us find areas!"

5. FTC, MVT & AVERAGE VAL

a. Copy/Paste the following

MORE INTEGRATION PROPERTIES

7. If $f(x) \ge 0$ for $a \le x \le b$, then $\int_a^b f(x)dx \ge 0$

If $f(x) \ge g(x)$ for $a \le x \le b$, then $\int_a^b f(x)dx \ge \int_a^b g(x)dx$

domination property

8. If $\min \le f(x) \le MAX$ for $a \le x \le b$,

then $\min(b-a) \le \int_a^b f(x)dx \le MAX(b-a)$.

Max/Min inequality

9. $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$

net area ≤ total area

10. $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx \text{ if } f \text{ is even } \rightarrow f(-x)=f(x) \\ 0 \quad \text{ if } f \text{ is odd} \rightarrow f(-x)=-f(x) \end{cases}$

Symmetry

11.

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$ or $\int_a^b f(x)dx = \int_a^b f(a-b-x)dx$

Fundamental Theorem of Calculus (FTC): Suppose f is continuous on $[a,b]$ Define F as:

part1.

$F(x) = \int_a^x f(t)dt,\ a \le x \le b$ then F is continuous on [a,b] and differentiable on (a,b), and

$$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x).$$

part2.

$\int_a^b f(x)dx = F(b) - F(a)$,

where F is any antiderivative of f.

ACCUMULATION up to pt b of a Function F(x) from a

$$F(b) = F(a) + \int_a^b F'(x)dx$$

Mean Value Theorem for Derivatives

If f is a function that is continuous on [a, b] and differentiable on (a, b), then there is a number c ∈ (a, b) such that:

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$ which can be rearranged to $f(b) - f(a) = f'(c)(b - a)$

Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$\int_a^b f(x)\,dx = f(c)(b-a).$ OR $f(c) = f_{ave}$

AVERAGE VALUE of a Function f(x) on [a,b]

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

AVERAGE CHANGE in a Function F(x) on[a,b]

$$aroc = \frac{1}{b-a}\int_a^b F'(x)dx \quad \text{OR} \quad aroc = \frac{F(b)-F(a)}{b-a}$$

NET CHANGE in a Function F(x) between a and b

$$\int_a^b F'(x)dx = F(b) - F(a)$$

b. Apply PART 2 of FTC to find exact value of:

$$\int_{1/2}^{1/e}\left(10x^4-2\left(1-x^2\right)^{-1/2}-\frac{5}{x}\right)dx$$

c. Determine b so that the average value on [1, b] is 10 for $f(x)=2x+3$

6. FTC & MVT CONTINUED

a. Apply PART 1 of FTC

i. $\frac{d}{dx}\int_2^x \cos(3t)dt$ do this one by first integrating (using FTC Part 2), then taking the derivative. Explain the shortcut (FTC part 1) that would work even if you don't know the antiderivative like the next question.	ii. $\frac{d}{dx}\int_x^3 \ln(t^3+t)dt$	iii. $\frac{d}{dx}\int_5^{\cos x} 2tdt$ do this one by first integrating (using FTC Part 2), then taking the derivative. Explain the shortcut (FTC part 1) that would work even if you don't know the antiderivative like the next question.	iv. $\frac{d}{dx}\int_{x^2}^{\sin x}\sqrt{1+t^3}dt$

b. Determine c so that f(c) is the average or mean value of $f(x)=3x^2+2x+1$ on [1, 4]

c. Explain why checking domain is important in $\int_1^4 \frac{1}{2x-4}dx$ what would be the answer to this?

d. Solve the differential equation

$$\frac{dy}{dx}=e^x+20\left(1+x^2\right)^{-1} \text{ and } y(0)=-2$$

i. Using separation of variables (like it was done in first topic of assignment of this unit)	ii. Using the principle of accumulation $y(x)=y(a)+\int_a^x y'(t)dt$

7. INTERPRET INTEGRALS

a. Copy/Paste the following

EVERYTHING you need to know about MOTION	*ALSO look at journal UNIT 3 question #4.*
To find **position function** if you are given the velocity function and some value of position $s(b)=s(a)+\int_a^b v(t)dt$	To find the **total distance travelled** on [a, b] **TDT on [a, b]** = $\int_a^b \lvert v(t)\rvert dt$ For **displacement** remove absolute values
If you are asked for **average velocity** AND you are only given a table with s(a) and s(b) then: Average velocity on [a, b] = $\frac{s(b)-s(a)}{b-a}$ If you are asked for average velocity AND you are given the velocity function then: Average value of the velocity on [a, b] = $\frac{1}{b-a}\int_a^b v(t)dt$	If you are asked for **average acceleration** AND you are only given a table with v(a) and v(b) then: Average acceleration on [a, b] = $\frac{v(b)-v(a)}{b-a}$ If you are asked for average acceleration AND you are given the acceleration function then: Average acceleration on [a, b] = $\frac{1}{b-a}\int_a^b a(t)dt$

b. The rate at which the world's oil supply is being consumed is continually increasing given by $r(t)=1.2e^{0.001t}$ millions of barrels/year where t=0 is start of year 2000.

i. Write an expression representing the total quantity of oil used between 2000 and 2010. No need to solve.

ii. Write an expression representing the average quantity of oil used between 2005 and 2010. No need to solve.

iii. If the quantity used in 2000 was 77 million barrels what was the total quantity of oil used in 2012?