PreCalculus UNIT 5

GRAPHING TRIG (MCR + MHF) – journal questions

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. INTRO to SINUSOIDAL graphs (MCR)
   a. DEFINITIONS
      - Sinusoidal –means graph is in the shape of a sine graph or a cosine graph, wavelike in structure.
        (Look at List B you've done in UNIT F journal to see that other trig graphs are not wavelike and thus are not sinusoidal)
      - Periodic vs Non periodic comparison of graphs (then use your periodic graph example to answer the following)
      - Period definition and show how to calculate it using:
        $\boxed{p = \text{last horiz pt of cycle - first horiz pt of the same cycle}}$
        (Make a note that period is always positive)
      - Axis definition and show how it can be found by $\boxed{axis = c = \dfrac{Max + Min}{2}}$
      - Amplitude definition and show how it can be found by either one of the following:
        $\boxed{\begin{array}{l} a = Max - c \\ a = c - Min \\ a = \dfrac{Max - Min}{2} \end{array}}$ (Make a note that amplitude is always positive)
      - Range is found from Max and Min $\boxed{R = \{Min \le y \le Max\}}$

   b. Sketch one cycle of the PARENT SINE AND COSINE shapes separately, (see the left side pictures in #2 below). Label the five points of the cycle as coordinate points. Explain how input angle, x (in degrees), relates to the output ratio, y, from the unit circle calculations. Underneath each graph record the following properties:
      - Period, Axis, Amplitude
      - Domain, Range
      - Odd or Even or Neither symmetry
      - Things to memorize: "Sine starts at the axis and goes up", "Cosine starts at Max"

2. TRANSFORMATIONS of Sine or Cosine $y = a\sin(k(x-d)) + c$ (MCR)
   a. Copy the following into your journal

- Sketch3 horizontal dotted lines and label the values on them (Don't bother drawing the x and y axes)

  $\text{MAX} = c + |a|$ – – – – – – – – – – – –

  $\text{axis} = c = \dfrac{\text{MAX + MIN}}{2}$ – – – – – – – – – – – –

  $\text{MIN} = c - |a|$ – – – – – – – – – – – –

  recall that

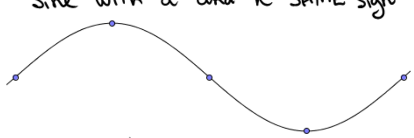
  $\text{amplitude} = |a|$

  $\text{period for sin/cos/csc/sec} = p = \dfrac{360^o}{|k|} = \dfrac{2\pi}{|k|}$

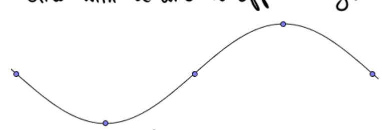
  $\text{period for tan/cot} = p = \dfrac{180^o}{|k|} = \dfrac{\pi}{|k|}$

- Decide on the shape of the wave and sketch it between the lines you just drew

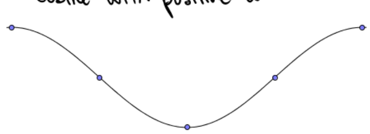
  Sine with a and K SAME sign

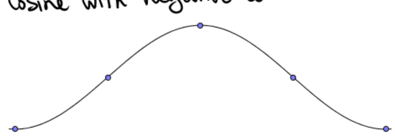
  Sine with a and K opposite signs

  Cosine with positive a

  Cosine with negative a

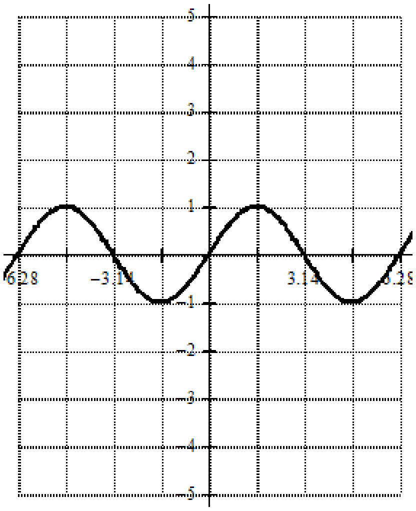
- Label horizontal points as follows
  first $= d$
  last $= d + period$
  middle = average of edges (or count by $\frac{1}{4}period$)

   b. Show how to sketch i.) $y = 2\sin(300^o - 3^o x) - 8$ and ii). $y = -35\cos\left(\dfrac{90^o - 2x}{3}\right) + 5$

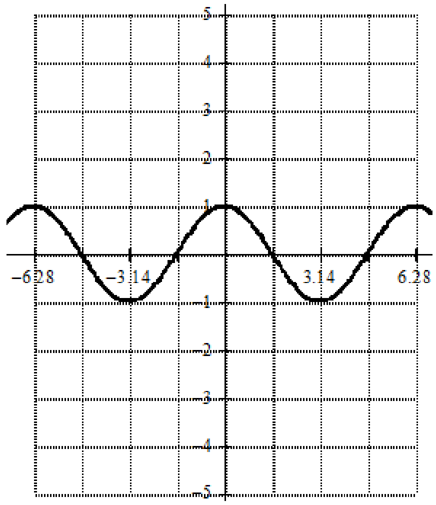
3. SECONDARY and Tangent Trig Graphs (MHF)

a. Copy the following parent primary trig functions, then overtop sketch the reciprocal (secondary) trig functions:

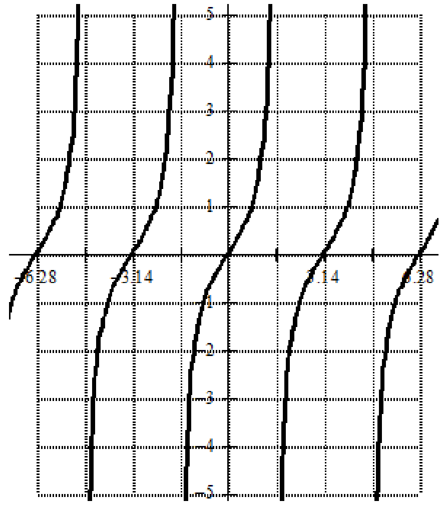
$\frac{1}{f(x)} = \frac{1}{\sin x} =$



$\frac{1}{g(x)} = \frac{1}{\cos x} =$



$\frac{1}{h(x)} = \frac{1}{\tan x} =$



b. For each of the graphs above (except sin and cos was done in #1b) include:

- Period, Domain, Range
- Odd or Even or Neither symmetry
- Things to memorize for $y = a\tan(k(x-d)) + c$: "Tan increases, d is middle", "Cot decreases, d is VA"

c. Explain and show how to sketch the following TRANSFORMED Tangent & Secondary Trig Graphs:

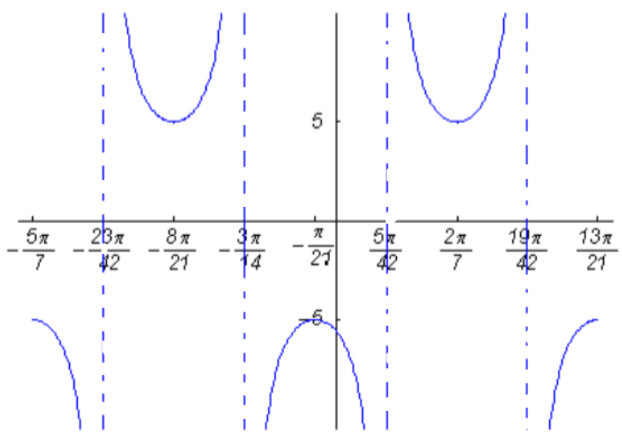
i. $y = 4\tan(2x - \pi) + 1$

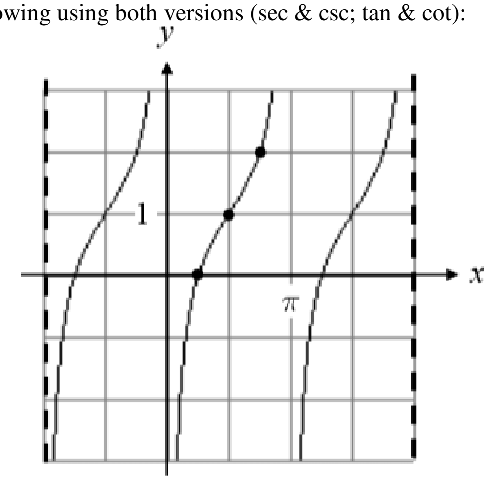
ii. $y = -5 + \cot(\frac{x}{2} + 30^o)$

iii. $y = 0.5\csc(\pi x - 6\pi) + 3$

iv. $y = 15\sec(\frac{\pi}{2}x + \pi) - 9$

d. Explain and show how to FIND EQUATIONS, in radians, for the following using both versions (sec & csc; tan & cot):





4. SINUSOIDAL WORD PROBLEMS

a. Copy the following (MCR)

A rung on a hamster wheel, with a radius of 25 cm, is travelling at a constant speed. It makes one complete revolution in 3 sec. The axle of the hamster wheel is 27 cm above the ground.

i. Explain how to find the period and the k value given revolutions per second (using degrees). Sketch height of the rung above the ground versus time, beginning when the rung is closest to the ground. Then find the equation (discuss the differences if you use sin or cos)

ii. What is the height of the rung at 2 sec? Show how to find this using both sine and cosine equation.

iii. Give a list of all possible times at which the rung is 5 cm high. Explain how to find this using both sine and cosine equations. Include explanation of steps.

b. Copy the following (MHF)

A nail is stuck in the tire of a car that moves at 12km/hr. The diameter of the wheel is 60 cm

Explain how to sketch a graph for the following if the nail starts at the top of the wheel.

i. Height versus "distance travelled by a point on the wheel" – graph. Come up with a sine equation for it in radians. Then find height after the car drove 1km.

ii. Height versus time in seconds – graph. You will have to convert km to cm and hours to sec. Come up with a cosine equation for it in radians. Then find the times, accurate to 2 decimals, at which the nail is at 10 cm above the ground. Do all the work in radians please.

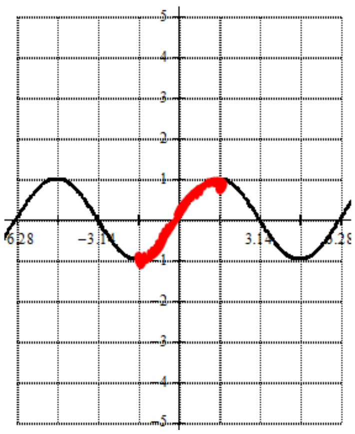
5. INVERSE Trig Functions

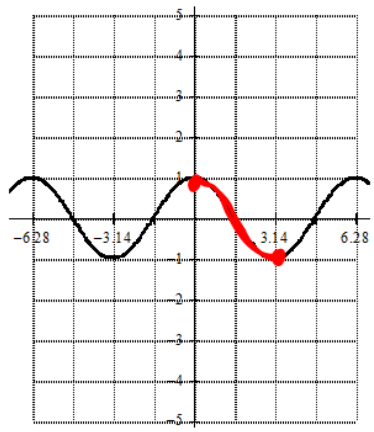
a. Use graphing technology like www.desmos.com to compare the following,

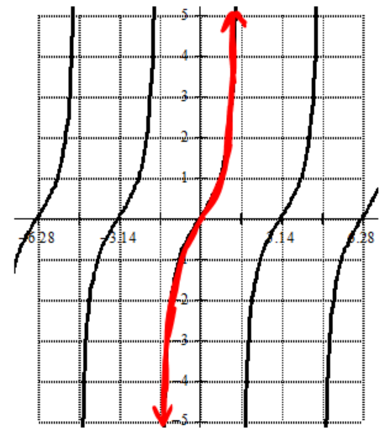
$$y = \sin^{-1} x,\ y = (\sin x)^{-1},\text{ and } y = \sin\left(x^{-1}\right)$$

then give a reason why $\arcsin x$ is a better notation for inverse sine than $\sin^{-1} x$:

b. Copy the following trig functions. Sketch the inverse RELATIONS overtop of the given graphs. Then separately, not over top of these, redraw the inverse trig FUNCTIONS for the one-to-one shaded portions of the graphs:







c. For each of the FUNCTION graphs above include:

- Domain, Range
- Odd or Even or Neither symmetry
- Note: How our calculator is programmed "arcsine and arctangent outputs give angles in quadrant I and II", "arccosine output gives angles in I and II",

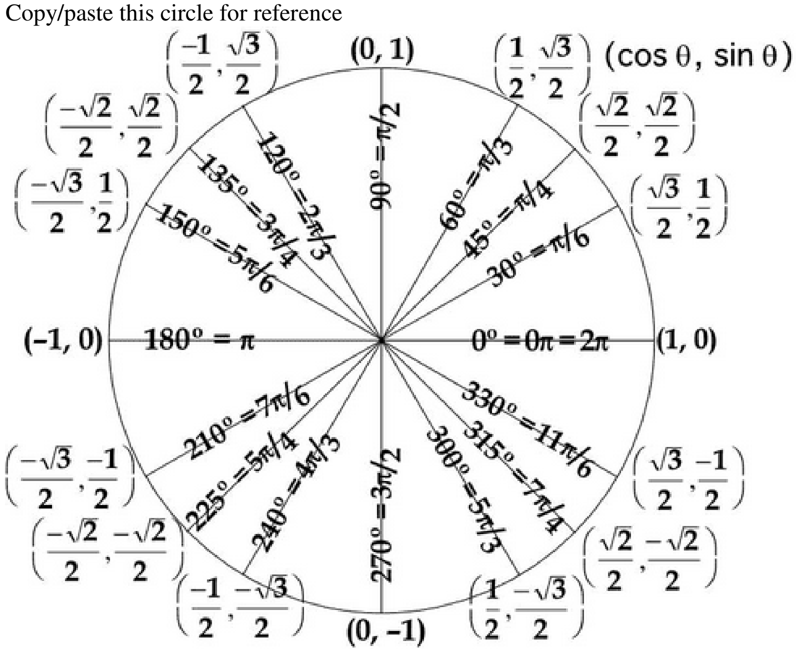
d. Try the following on the calculator:

A $\sin^{-1}\left(\sin\frac{\pi}{10}\right) = \boldsymbol{\frac{\pi}{10}}$ B $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \boldsymbol{\frac{\pi}{6}}$ C $\cos\left[\cos^{-1}(0.2)\right] = \mathbf{0.2}$ D $\cos\left[\cos^{-1}(10)\right]$ is **undefined**

Note, that sine and arcsine appear to cancel for A, but not for B; and cosine and arccosine may sometimes cancel like in C, or be sometimes undefined like in D. The 'moral of this story' is work things out - inside to out - don't cancel.

Talk about number of solutions if you have to apply the inverse function versus if it is already written (relate it to applying a square root yourself versus it being already written).

Copy/paste this circle for reference



e. Explain how to find the value of

$$\cos\left(\operatorname{Arctan}\left(-\sqrt{3}\right) + \operatorname{Arcsin}\frac{1}{2}\right)$$

f. Explain how to simplify the following using a right triangle

$$\cot\left(\cos^{-1}\frac{x}{3}\right)$$