Big idea

In this unit you will learn how trigonometry can be used to model wavelike relationships. These wavelike functions are called sinusoidals (they come from either sine or cosine equations). You will study key properties that these functions have and use these properties to sketch functions to model real life situations and to solve trigonometric equations. You will also learn about graphs of tangent, cosecant, secant and cotangent; you will graph these functions with and without transformations. Finally you will look over inverse trig graphs and use that knowledge to simplify expressions that have arcsine, arccosine and arctangent. This simplification will help you later when you study more advanced type of integrals that involve trig substitution.

Feedback & Assessment of Your Success

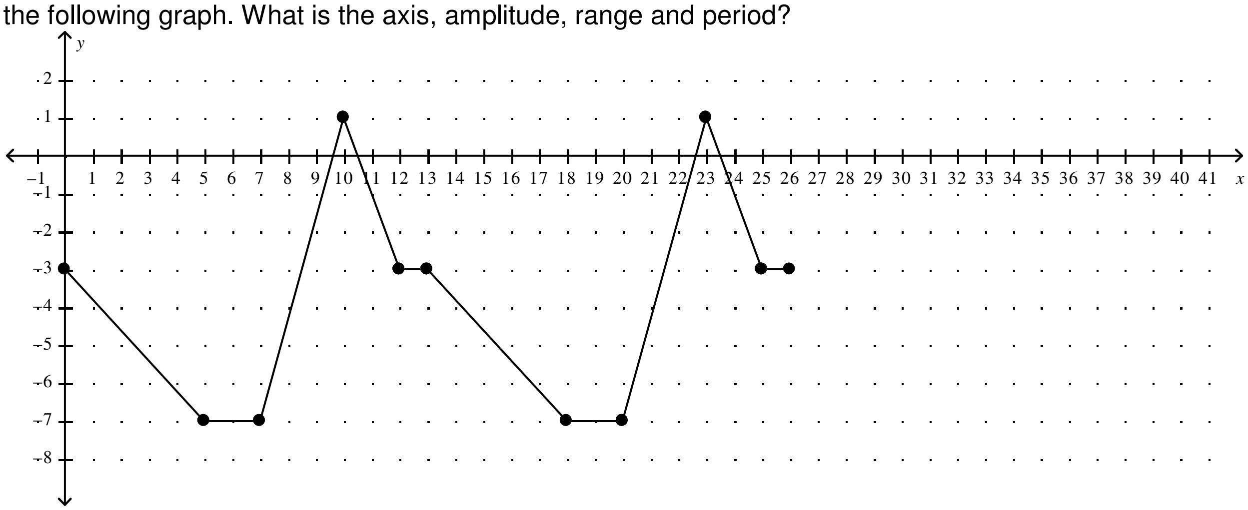
Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: ____________________ Questions to ask the teacher:
	2-4	Intro to Sinusoidals (MCR) Journal #1				
1.5days	5-8	Transformations of Trig Functions (MCR+ MHF) Journal #2				
1.5days	9-11	Finding Equations (MCR + MHF) Journal #3				
2days	12-18	Problem Solving (MCR+ MHF) Journal #4				
	19-20	Inverse Trig Journal #5				

ASSIGNMENT Intro to Sinusoidals (MCR)

1. There are many situations in real life that repeat in cycles.For example, tides, daylight hours, temperature for the year, heartbeat, volume of air in lungs, rides on ferris wheels, the list can go on. This trend that repeats in cycles is called **periodic** phenomenon. The length of the cycle is called the **period**. The average value of peaks and troughs is the **axis** of the function, and the distance from the axis to the maximum, or from the axis to the minimum is called the **amplitude**.

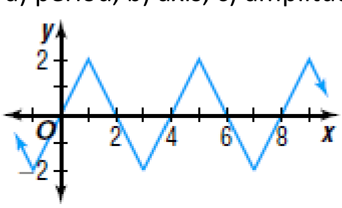
   Summarize the equations that you can use to find the period from the graph, the axis, the amplitude and the range.

2. The movement of a factory machine that cuts grooves in metal to create a required template pattern is shown on the following graph. What is the axis, amplitude, range and period?



Find the a) period, b) axis, c) amplitude and d) range of each periodic graph

3.



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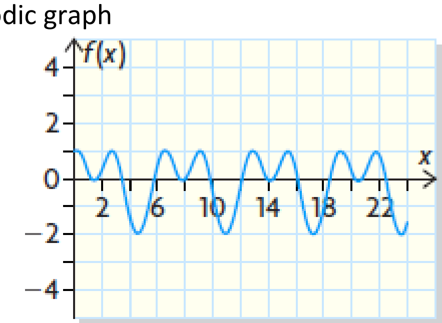

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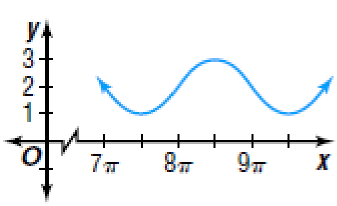

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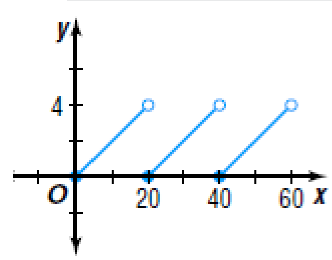
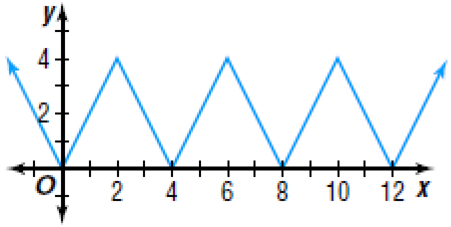


State whether the function is periodic or not

7.



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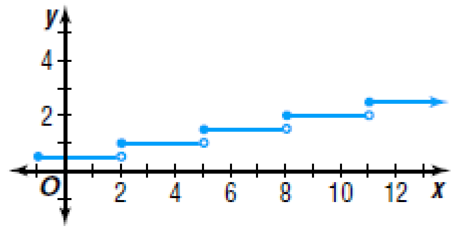


9.

x	y
-3	7
1	4
5	1
9	4
13	7
17	4
21	1

10.

x	y
0	-2
2	3
4	0
6	-2
8	-4
10	0
12	3

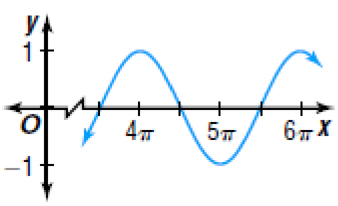
11. $y = |x + 5|$

12. $y = \frac{1}{x}$

13. The cost of riding in a taxi varies, depending on how far you travel.
- independent variable: distance travelled
- dependent variable: cost

14. Alex is doing jumping jacks.
- independent variable: time
- dependent variable: Alex's height above the ground

15.



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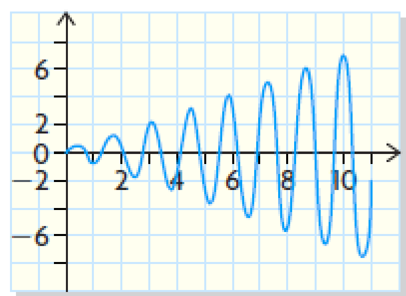


Sketch non-sinusoidal but periodic graph for #17, and sinusoidal for #18, from these descriptions. Start at x=4 not x=0.

17. Sketch the graph of a periodic function with a period of 20, an amplitude of 6, and whose equation of the axis is $y = 7$.

18. Sketch the graph of a periodic function whose period is 4 and whose range is $\{y \in \mathbf{R} \mid -2 \leq y \leq 5\}$.

19. Explain how the notation of Rotational Trig is switched to Sinusoidal Functions notation:

Graph each sinusoidal function on the given interval

20. $y = \cos x,\ 900^o \leq x \leq 1260^o$

21. $y = \sin x,\ -4\pi \leq x \leq -2\pi$

Use the parent graphs to figure out the solutions to the following:

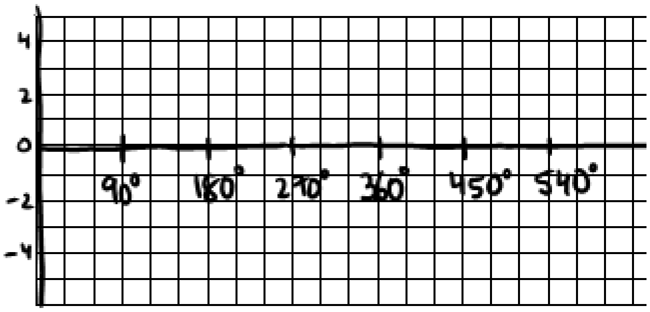
22. $\sin x = -\cos x$

23. $\sin x \leq \cos x$

ASSIGNMENT Transformations of Trig Functions (MCR + MHF)

Sketch by applying transformations (stretches/compressions/reflections must be done before shifts)

1. $y = -2\cos(0.75x - 45)^o + 3$



Follow these steps $y = a\cos(k(x-d)) + c$

a) Draw one parent cycle using pencil (make the 5 points of the cycle easily seen)

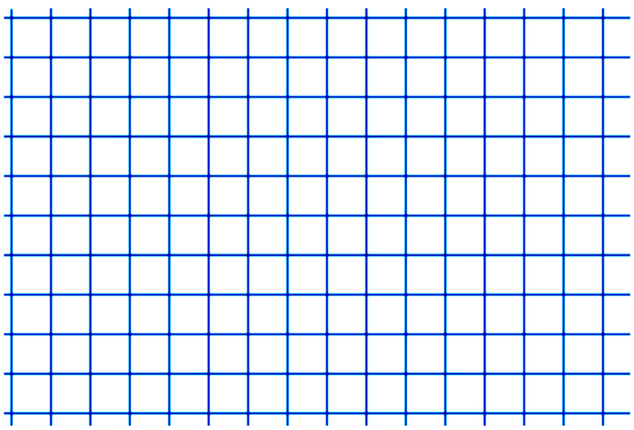
b) Apply vertical stretch/compress/reflect by multiplying $y \cdot a$ for all 5 points, draw new graph using red colour

c) To the red graph apply horizontal stretch/compress/reflect by dividing $x \div k$, draw the next graph using blue colour

d) Factor out k, (otherwise horizontal shift is not visible) and apply both shifts to the blue graph, draw the final graph in black pen. Label it with the word "FINAL"

Sketch from mapping 5 key points

2. $y = 10\sin\left(\frac{\theta}{4} - 4\pi\right) - 5$



Follow these steps

a) Factor k out, find d in degrees

b) Create a table of 5 points of the parent graph

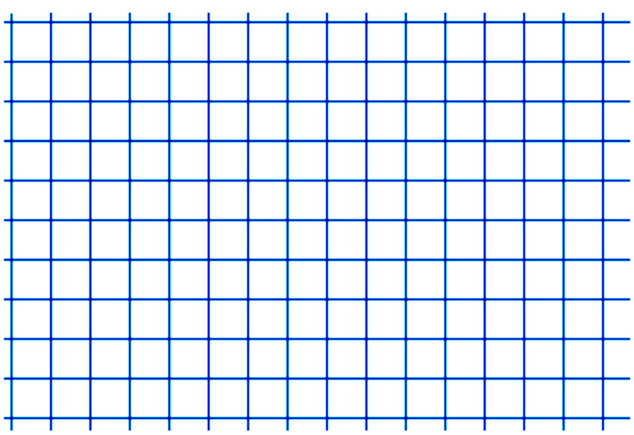
c) Create a new table of the image points by transforming each point with the following mapping rule

$$(x, y) \rightarrow (\frac{x}{k} + d, ay + c)$$

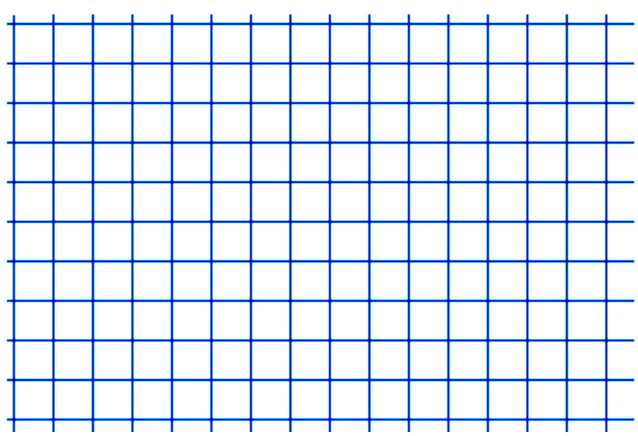
d) Sketch the result

Sketch between 3 horizontal lines like summarized in your journal (I like this method best since it is similar to what you'd do with word problems).

3. $y = -2\sin(3\theta + 120^o) - 4$



4. $y = -0.4\cos(90^o - 2x) - 2.5$



Sketch and label all five points of the cycle.

5. $y = -2\cos(0.75x)^{o} + 3$

6. $y = 2.5\sin(2x - 60^{o})$

7. $y = 235\sin(-8x - 288)^{o} + 300$

8. $y = 3\cos(2x - 120)^{o} + 5$

9. $y = 20 + 5\cos(3\theta + \pi)$

10. $y = -2 + \sin\left(\frac{\theta}{3} - \frac{\pi}{12}\right)$

Sketch and state the domain

11. $y = \csc(2\theta + \pi) - 3$

12. $y = \sec\left(\frac{\theta}{3} + \frac{\pi}{6}\right) - 2$

13. $y = 2\cot(\frac{\pi}{2}\theta + 3\pi) - 4$

14. $y = \tan\left(\frac{\theta}{2} - \frac{\pi}{4}\right) + 1$

15. $y = \csc(\pi x + 3\pi) + 1$

16. $y = \cot 2x - 1$

17. Given the trig function $f(x) = \cos x$, the statement which is true is:

a) $f(x) = -f(x)$

b) $f(x) = f(-x)$

c) $f(x) = f^{-1}(x)$

d) $f(x) = -f(-x)$

18. The function $f(x) = 2\sec x$ has a range of

a) $(-2, 2)$

b) $(-\infty, -2] \cup [2, \infty)$

c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

d) $(-\infty, \infty)$

19. A sine function has a range of [-6, 2] and a period of 4. A trig equation with these properties is:

a) $y = 8\sin\left(\frac{\pi}{2}\theta\right) + 2$

b) $y = 4\sin(4\theta) - 2$

c) $y = 4\sin\left(\frac{\pi}{2}\theta\right) - 2$

d) $y = 4\sin\left(\frac{2}{\pi}\theta\right) - 2$

20. The period of the function g(x) is 8. If $g(0) = 12$, $g(4) = 6$, and $g(8) = 12$, then the value of $g(12)$ is:

a) 0

b) 6

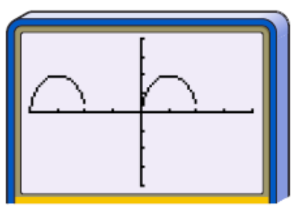
c) 12

d) 18

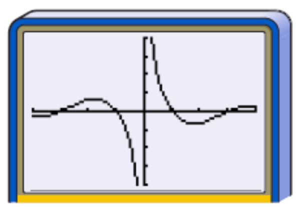
21. **Critical Thinking** The windows for the following calculator screens are set at $[-2\pi, 2\pi]$ scl: 0.5π by $[-2, 2]$ scl: 0.5. Without using a graphing calculator, use the equations below to identify the graph on each calculator screen.

$y = \cos x^2$ $\quad y = \sqrt{\sin x}$ $\quad y = \dfrac{\cos x}{x}$ $\quad y = \sin\sqrt{x}$

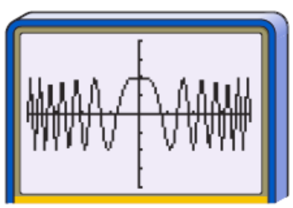
a.



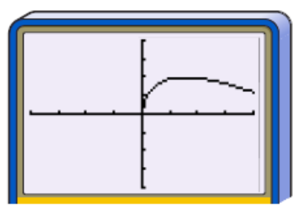
b.



c.



d.



22. a) Sketch the graph of $g(x) = -\frac{1}{2}\cos\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

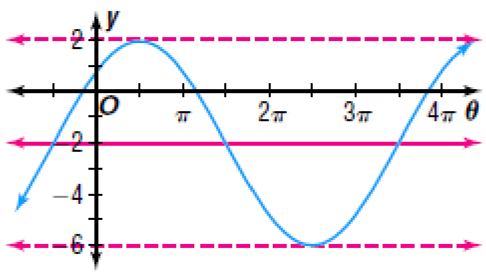
need x-axis drawn in to do b)

b) now sketch $|g(x)|$ over top a) with a different colour

ASSIGNMENT Finding Equations (MCR + MHF)

Find one sine and one cosine equation of the following (no rounding please) MCR

1.



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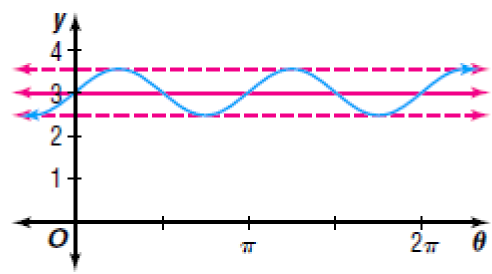

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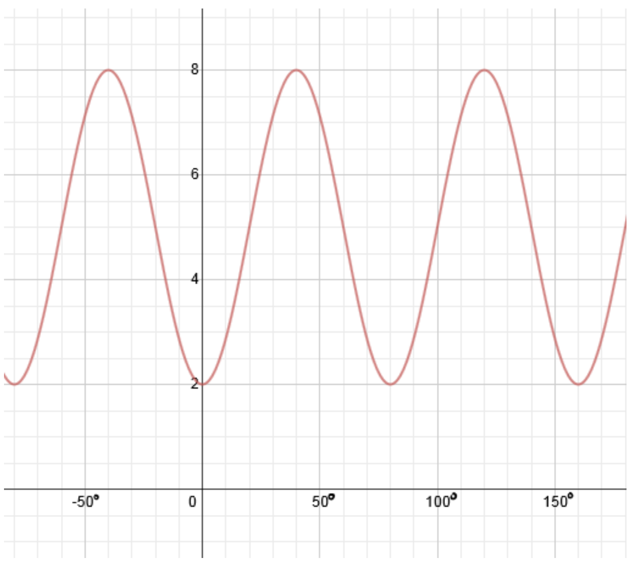

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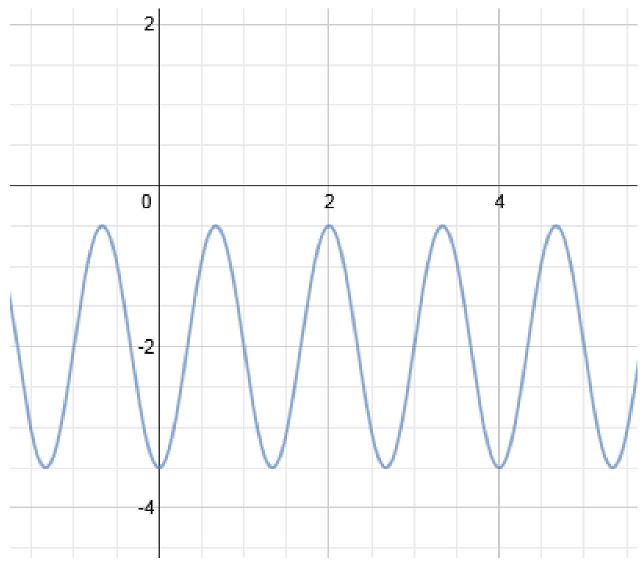
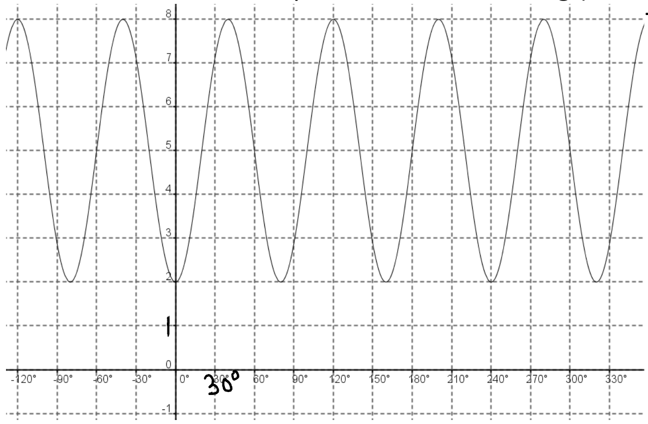


5.

Find one sine and one cosine equation of the following (no rounding please)

6.



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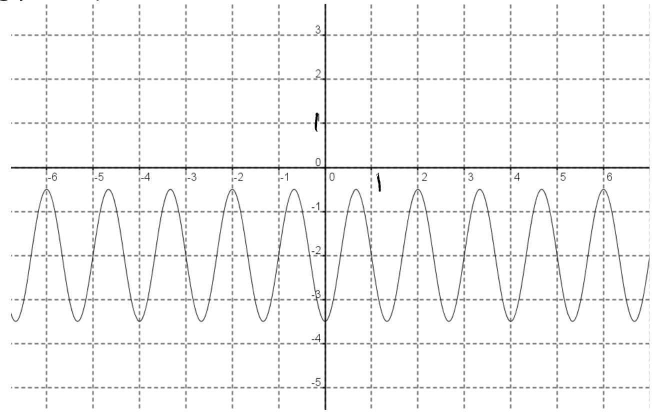
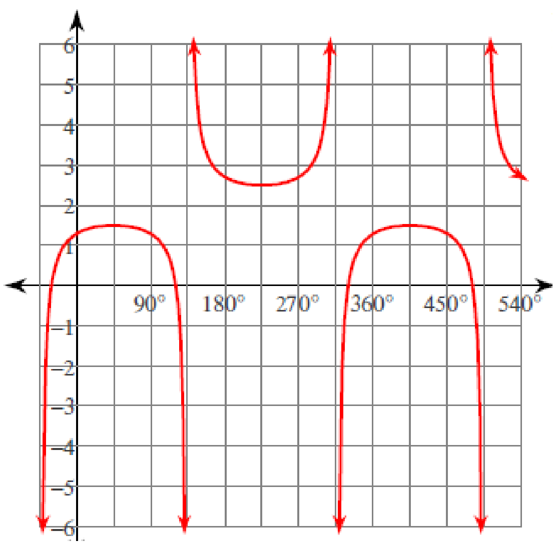


Find one cosecant and one secant equation of the following MHF

8.



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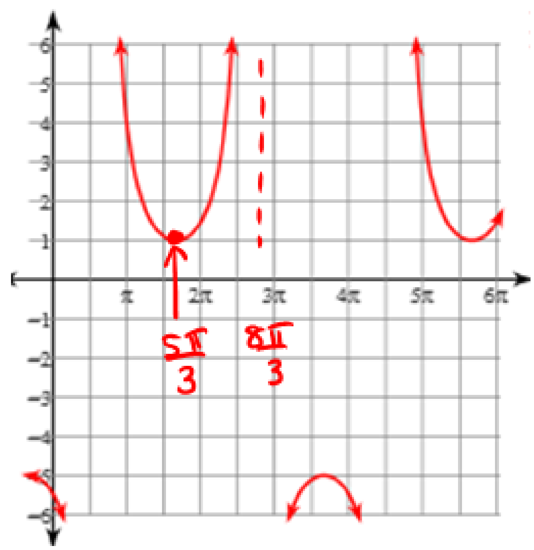

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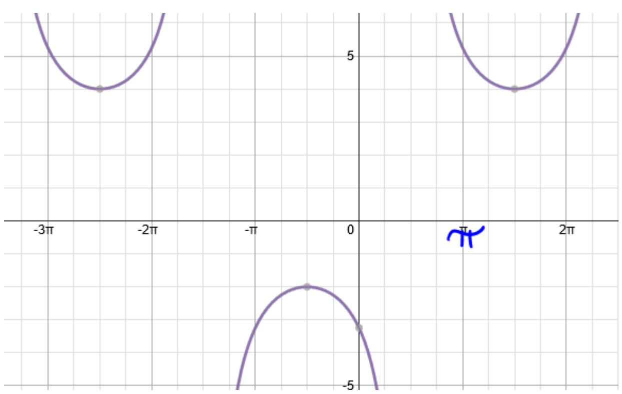
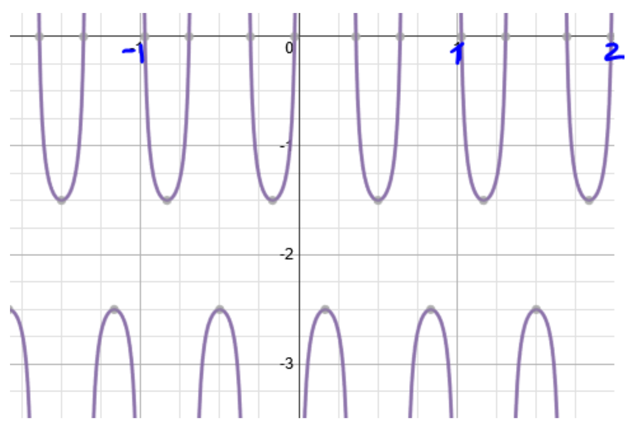


11.



Find one tangent and one cotangent equation of the following MHF

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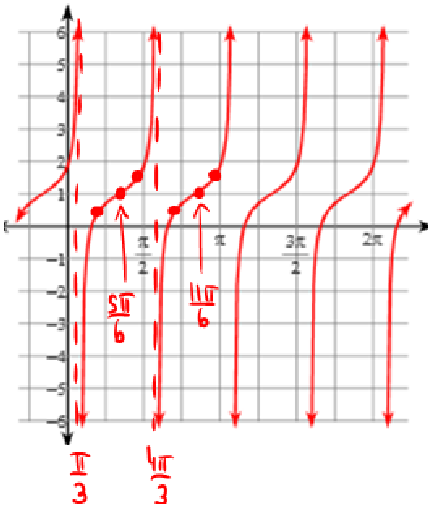

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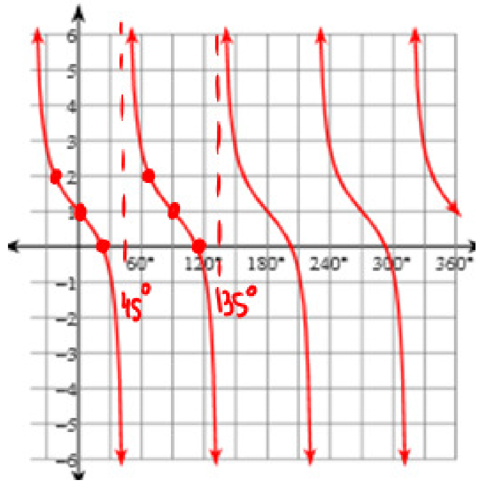

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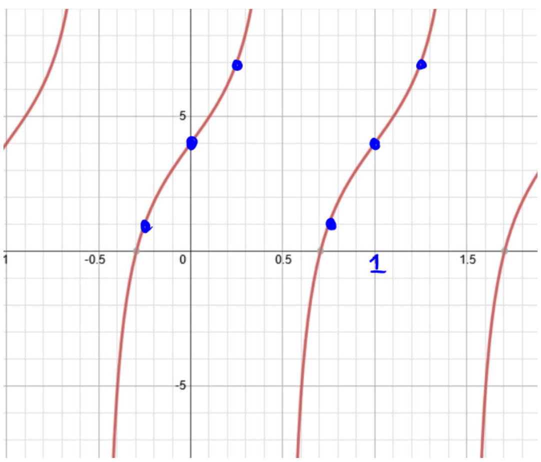

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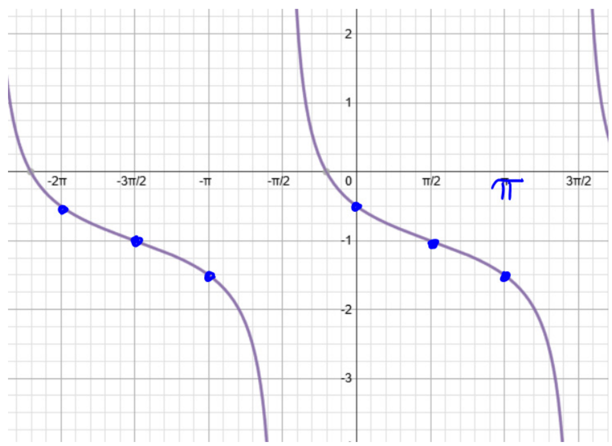
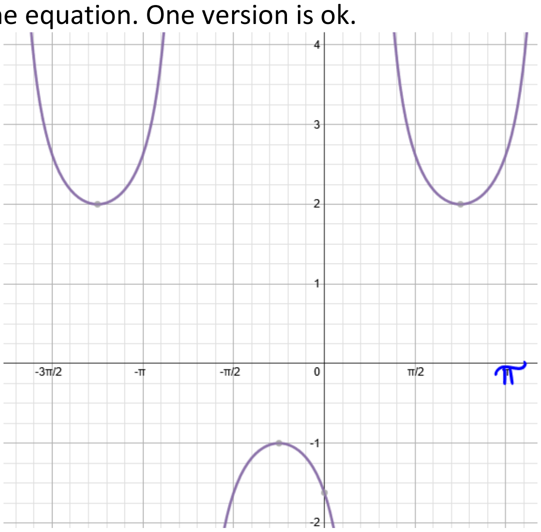


Find the equation. One version is ok.

16.



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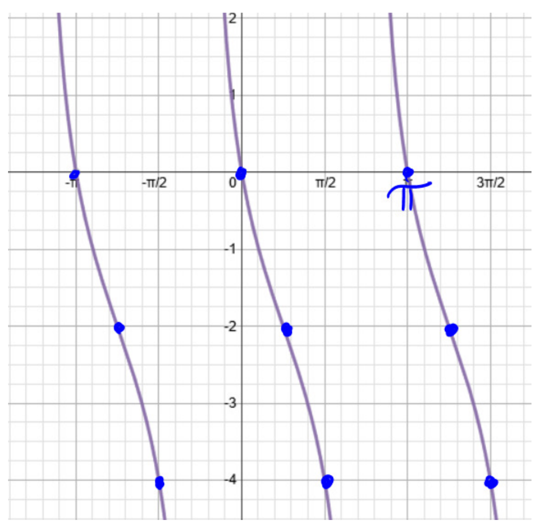


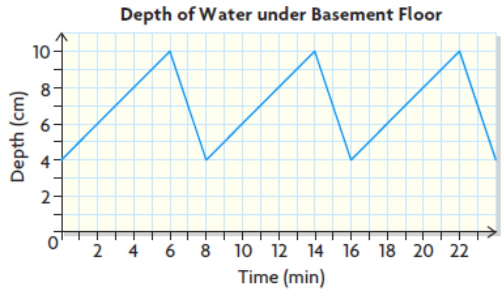
ASSIGNMENT Problem Solving (MCR + MHF)

1. **Music** Write a sine equation that represents the initial behavior of the vibrations of the note D above middle C having an amplitude of 0.25 and a frequency of 294 hertz. use radians MHF

2. A floating ball in a lake of depth 100m goes up and down with the tide. At 1 second, the ball has a minimum height of 5cm below surface level. At 3 seconds, the ball has a maximum height above surface level.
   a) Sketch the graph for the first 4 seconds of motion
   b) Write an equation for the function h(t), height in meters, time in seconds (use degrees MCR)

3. **Health** If a person has a blood pressure of 130 over 70, then the person's blood pressure oscillates between the maximum of 130 and a minimum of 70.
   a. Write the equation for the midline about which this person's blood pressure oscillates.
   b. If the person's pulse rate is 60 beats a minute, write a sine equation that models his or her blood pressure using *t* as time in seconds. use radians MHF
   c. Graph the equation. + label 5 pts of cycle

4. Chantelle has a submersible pump in her basement. During a heavy rain, the pump turned off and on to drain water collecting under her house's foundation. The graph models the depth of the water below her basement floor in terms of time. The depth of the water decreased when the pump was on and increased when the pump was off.



a) Is the function periodic?
b) At what depth does the pump turn on?
c) How long does the pump remain on?
d) What is the period of the function? Include the units of measure.
e) What is the range of the function?
f) What will the depth of the water be at 3 min?
g) When will the depth of the water be 10 cm?
h) What will the depth of the water be at 62 min?

5. The following equation models the average monthly temperatures in degrees Fahrenheit of a city. In this equation, t, is number of months with January t = 1.

$$y = 43 + 31\sin\left[\frac{\pi}{6}(t-4)\right]$$

a) Fill in the table

t	y
4	
7	
10	
13	
16	
19	

b) sketch using values of the table

c) Use the graph to approximate the month(s) for which the temperature is 58°F

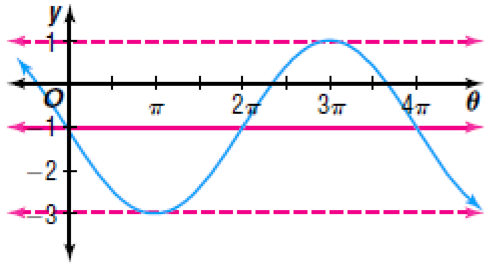
d) Use the equation to find out more accurately wether the temperature is 58°F in the beginning/middle/end of the months you approximated using the graph

6. Solve for x in [-250°,- 50°]

$$2 = -6\cos(90^o - 2x) - 2.5$$

Find equations (do one sine version and one cosine version) of the following

23.



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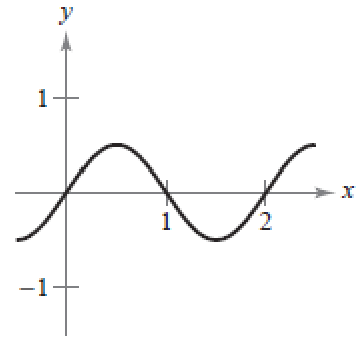
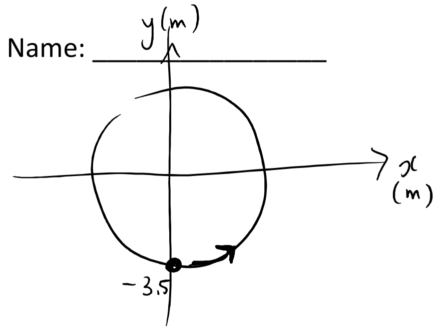


Solve MHF

25. In an industrial city, the amount of pollution in the air becomes greater during the working week when factories are operating, and lessens over the weekend. The number of milligrams of pollutants in a cubic metre of air is given by: $P(t) = 40 + 12\cos\left(\frac{2\pi}{7}\left(t - \frac{29}{6}\right)\right)$ where t is the number of days after midnight on Saturday night.

a) What is the minimum level of pollution?
b) Sketch and record at what times does the minimal level occur (record all possible answers as a sequence formula)
c) Use the equation to solve when will the pollution level reach 46mg? show steps in degrees
d) State a general interval for the solution of $P(t) \leq 46$, ie. The days for which the pollution is lower than 46mg.



Solve, set up the equations in radians MHF

7. **Entertainment** Several years ago, an amusement park in Sandusky, Ohio, had a ride called the Rotor in which riders stood against the walls of a spinning cylinder. As the cylinder spun, the floor of the ride dropped out, and the riders were held against the wall by the force of friction. The cylinder of the Rotor had a radius of 3.5 meters and rotated counterclockwise at a rate of 14 revolutions per minute. Suppose the center of rotation of the Rotor was at the origin of a rectangular coordinate system.
   a) Sketch the function that models the position of the door at t seconds, if the initial coordinates of the hinges on the door are (0, -3.5) where (x,y) are both in meters.
   b) Find the equation p(t) to model this, t is still seconds
   c) Find the (x,y) coordinates in meters of the hinges on the door at 4sec.

Solve, set up the equations in degrees MCR

8. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds. (Don't assume that at time zero the weight is at the minimum!)
   a. Sketch a graph of this sinusoidal function.
   b. Write an equation expressing distance from the floor in terms of the number of seconds the stopwatch reads. use cosine
   c. What was the distance from the floor when you started the watch?
   d. Predict the time at which the weight was 59 cm above the floor for the first time and for the second time. + general sol for all times

Solve, set up the equations in degrees MCR

9. You are on an 8-seat Ferris wheel at an unknown height when the ride starts. It takes you 4 seconds to reach the top of the wheel 26m above the ground. The loading platform is 4m high. Your seat revolves at a speed of 4km/h.

   a. Sketch height versus time + label period in seconds
   b. Find the sinusoidal equation that models height versus time
   c. Sketch height versus distance traveled by the seat + label period in meters
   d. Find the sinusoidal equation that models your height versus distance traveled

   e. How high above the ground was your seat when the ride starts?

   f. Find the two times within one cycle when the height is at 24m.

   g. What are the intervals of time, in general, when you are above 24m?

Solve, set up the equations in radians MHF

10. **Tides** Burntcoat Head in Nova Scotia, Canada, is known for its extreme fluctuations in tides. One day in April, the first high tide rose to 13.25 feet at 4:30 A.M. The first low tide at 1.88 feet occurred at 10:41 A.M. The second high tide was recorded at 4:53 P.M.

a) Sketch the depth in terms of number of hours that have elapsed since 12:00 midnight on April 1st. (hint label x-axis starting from t=0 representing midnight, and convert clock time to be strictly in terms of hours, for example 6:31pm will be $18\frac{31}{60}$)

b) Derive an equation

c) Use your mathematical model to predict the depth of water at 7:30 pm on April 2nd.

d) At what time will the first low tide occur on April 4th ?

e) What is the earliest time on April 1st that the water will be at 9.25 foot level?

Solve, set up the equations in radians MHF

11. The piston in the engine of a small aircraft moves horizontally relative to the crankshaft, from a minimum distance of 25cm to a maximum distance of 75cm. During normal cruise power settings, the piston completes 2100 rpm (revolutions per minute).
    a. Sketch the horizontal distance position, h, in centimeters, of the piston as a function of time, t, in seconds.
    b. What is the equation that models this?

12. A salesperson selling a car alarm reports that the sound has a minimum frequency of 250 Hz, the maximum being 1150 Hz, and the frequency at t=0 being 700 Hz. The salesperson reports that the car alarm reaches its maximum frequency after 1 second and that the frequency increases before it decreases.
    a. Sketch the graph of the frequency, f, in Hertz, as a function of time, t, in seconds.
    b. What is the equation that models this?
    c. What is the frequency of the alarm at 1.2 seconds?
    d. At what times in the first 7 seconds, is the frequency at 1000 Hz?

ASSIGNMENT Inverse Trig

Find each value (use radians)

1. $\text{Arccos } 0$

2. $\text{Tan}^{-1} \frac{\sqrt{3}}{3}$

3. $\text{Sin}^{-1}\left(\tan \frac{\pi}{4}\right)$

4. $\sin\left(2 \text{ Cos}^{-1} \frac{\sqrt{2}}{2}\right)$

5. $\cos\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{2}\right]$

6. $\sin\left(\text{Sin}^{-1} 1 - \text{Cos}^{-1} \frac{1}{2}\right)$

7. $\sec(\arcsin 2x)$

8. $\cot[\arctan(x - 2)]$

9. $\tan(\arccos 3x)$

10. $\cos\left(\arcsin \frac{x - 1}{2}\right)$

Determine if each of the following is true

11. $\operatorname{Cos}^{-1}(\cos x) = x$ for all values of x

12. $\tan(\operatorname{Tan}^{-1} x) = x$ for all values of x

13. $\operatorname{Sin}^{-1} x = -\operatorname{Sin}^{-1}(-x)$ for $-1 \leq x \leq 1$

14. $\operatorname{Sin}^{-1} x + \operatorname{Cos}^{-1} x = \frac{\pi}{2}$ for $-1 \leq x \leq 1$

15. Jeremy and Vanessa are trying to find the value of the following expression:

$$\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$$

Jeremy thinks that the value is $\frac{4\pi}{3}$

(a) Explain (using full sentences) why Jeremy is wrong.

(b) Vanessa found the correct value. Find it.

16. On New Year's Eve, Megan watches the ball drop in Times Square from a standstill. Before the ball begins its drop, she measures the angle of elevation of the ball to be 22° from her location. After the ball drops 100 feet, she measures the angle of elevation of the ball to be 16.8°. How tall is the building?