



### Big idea

This unit reviews what you learned last year, concepts like: what is a function, how to use function notation, domain and range, as well as how to find an inverse of a function. Then you will learn functions of more complex nature, like functions composed from different operations and pieces. The key characteristics you will study here will help you in the next units: exponential, logarithmic, discrete, trigonometric, and polynomial functions.



### Feedback & Assessment of Your Success

			Finished assignment pages?	Summarized notes in a journal?	How many extra practice questions did you try in each topic?	Tentative TEST date:
			Made corrections?	Added your own explanations?		_____
Date	Pages	Topics				Questions to ask the teacher:
	2-4	Review Functions (MCR) NO JOURNAL				
1.5days	5-8	Function Operations and Compositions (MHF) Journal #1				
2.5days	9-14	Piecewise Functions (MHF) Journal #2				
2days	15-20	Inequalities & Absolute Value (MHF) Journal #3				
	21-23	Function Properties (MHF) Journal #4				

**ASSIGNMENT Review Functions (MCR)**

Identify if each of the following is a function or not.

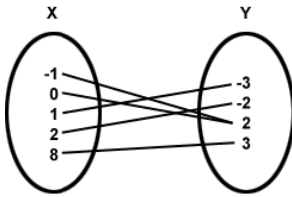
1.  $\{(-3,5), (2,8), (-3,9), (0,-4)\}$

2.  $\{(-1,4), (0,7), (5,-2), (10,7)\}$

3.

4.

You are investigating the average height of a grade 9 student in ajax



Find domain and range, record in any notation. For #7 & 8 just do domain.

5.  $y = \sqrt{3-6x}$

6.  $x^2 + y^2 = 25$

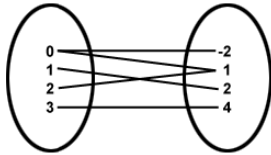
7.

$$f(x) = \frac{2x-6}{x^2-2x-3}$$

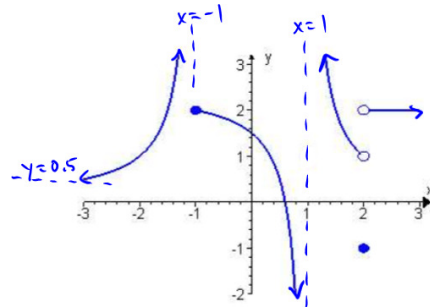
8.

$$f(x) = \frac{\sqrt{x+5}}{x-1}$$

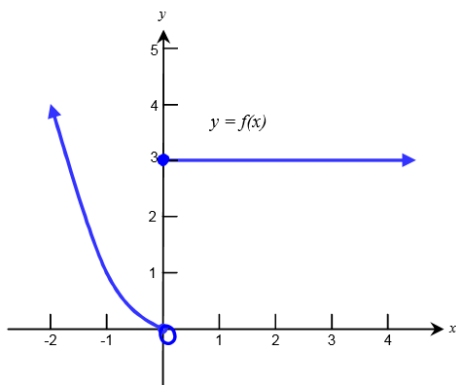
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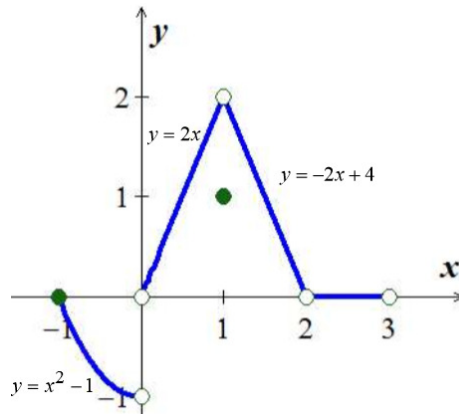
10.



11.



12.

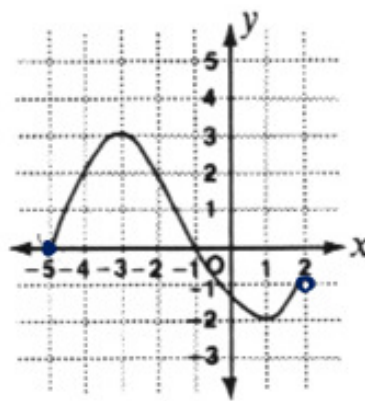


13. Let  $f(x) = x^2 - 3x - 10$ . Determine the values of  $x$  for which  $f(x) = -12$
14. You work in a store where you earn \$11 per hour plus a commission of 2% on everything you see. Write a function that describes how much you earn by working 40 hours and having sales of \$ $x$ .

15. State transformations and do a sketch

$$h(x) = \frac{4}{6 - 0.5x} + 2$$

16. Find the graphical inverse



Find the inverse FUNCTION for each of the following.

17.  $f(x) = \frac{x}{x-3}$

18.  $f(x) = -x^2 + 2x - 9$

**ASSIGNMENT Function Operations & Compositions (MHF)**

Perform the indicated operation

1.  $g(n) = n^2 + 4 + 2n$   
 $h(n) = -3n + 2$   
 Find  $(g \cdot h)(1)$

2.  $f(x) = 4x - 3$   
 $g(x) = x^3 + 2x$   
 Find  $(f - g)(4)$

3.  $h(x) = x^2 - 2$   
 $g(x) = 4x + 1$   
 Find  $(h \circ g)(x)$

4.  $g(n) = 2n + 3$   
 $h(n) = n - 1$   
 Find  $(g \circ h)(n)$

5.  $g(x) = -x^2 - 1 - 2x$   
 $f(x) = x + 5$   
 Find  $(g - f)(x)$

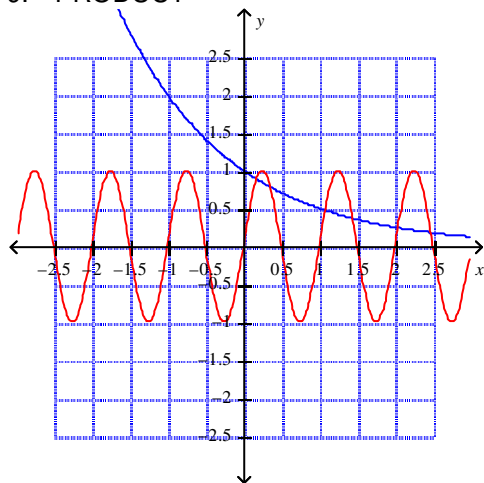
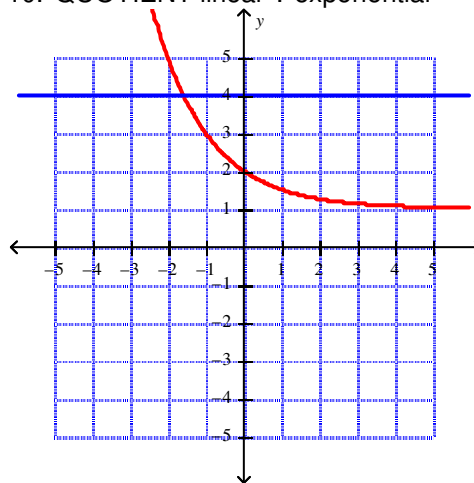
6.  $g(a) = -3a^2 - a$   
 $h(a) = -2a - 4$   
 Find  $\left(\frac{g}{h}\right)(a)$

7.  $f(x) = \sin x, g(x) = 4x$   
 find  $f(g(x))$  and  $g(f(x))$

8.  $f(x) = |x| - 2, g(x) = x + 5$   
 find  $f(g(x))$  and  $g(f(x))$

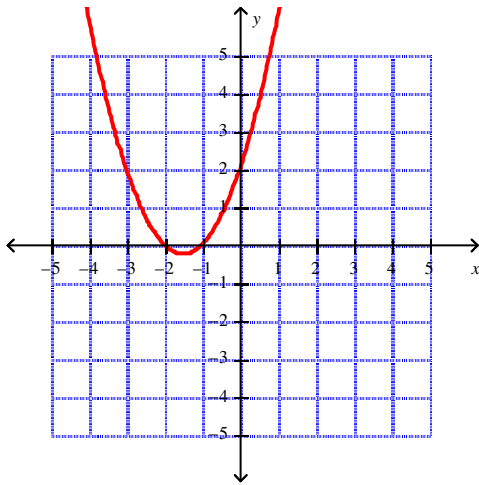
Sketch the COMBINED version of the following graphs under the given operation

9. PRODUCT

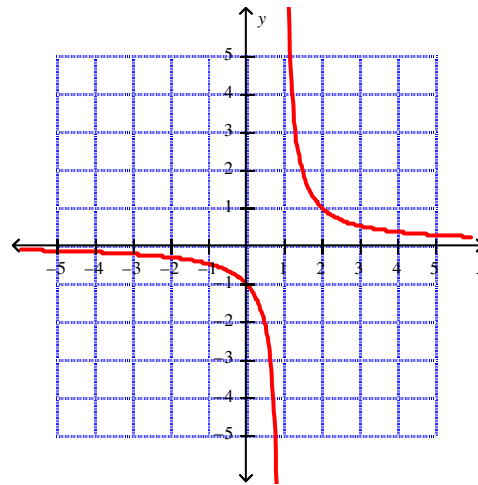
10. QUOTIENT linear  $\div$  exponential

11. Come up with the equations for each of the following

$f(x) =$



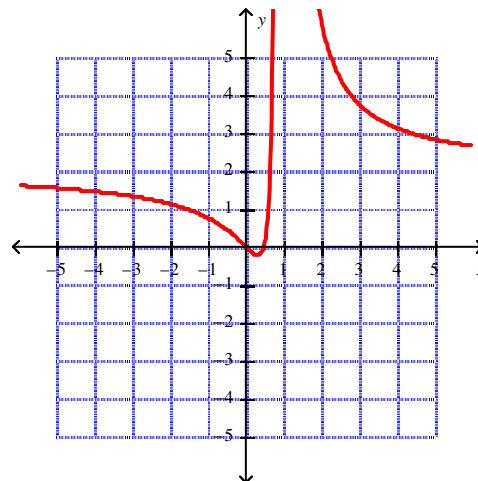
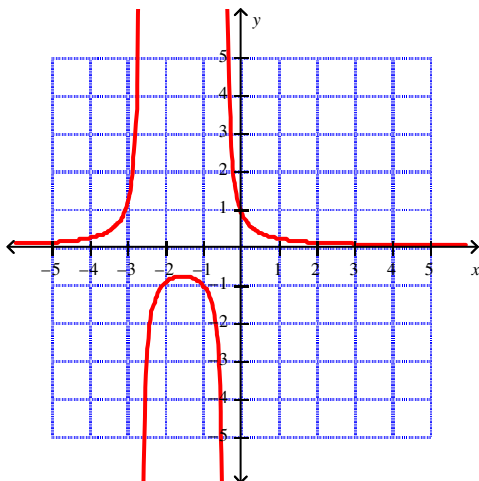
$g(x) =$



12. Write down and simplify the algebraic models of the different COMPOSITIONS

$h(x) = (f \circ g)(x) = f(g(x))$  and  $i(x) = (g \circ f)(x) = g(f(x))$

13. Match these graphs to the composition equations above  
(Use what you learned of rational functions to help you decide – zeros, y-int, VA, HA)



14.  $f(v) = \frac{1+v}{1-v}$ ; Find  $f(1-a)$

15.  $h(x) = -7$ ; Find  $h(x^2 - 3x + 1)$

16.  $g(x) = x^3 + 3$   
 $h(x) = 3x + 2$   
Find  $(3g + 3h)(-x)$

17.  $f(x) = -x - 4$   
 $g(x) = 2x^2 - 2$   
Find  $f(-2x) - g(-2x)$

18. For  $f(x) = 3x - 2$   
 $g(x) = x^2 - 6x$   
 $h(x) = \{(3, 2), (5, 1), (7, 4), (9, 3), (11, 5)\}$   
 $i(x) = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$   
Find

- $(f - g)(x)$
- $(f \circ g)(x)$
- $(h + i)(x)$
- $(h \circ i)(x)$

19.  $f(x) = \{(0, 2), (1, 6), (2, 4), (3, 8), (4, 10)\}$   
 $g(x) = \{(2, 4), (4, 2), (6, 3), (8, 0), (10, 1)\}$   
Find

- $(f - g)(x)$
- $(f \circ g)(x)$
- $(f \times g)(x)$
- $(g \circ f)(x)$

20. Answer the following questions about inverses

a. For  $g(x) = (x+5)^2 - 2$ ,  $f(x) = \frac{1}{x+3}$ ,

what is  $f^{-1}(x)$ ?

b. Compose  $f \circ f^{-1}$ . What do you notice?

c. Evaluate the following:

i.  $(f \circ g)(1)$

ii.  $(f^{-1} \circ g)(0)$

iii.  $(f \circ f^{-1})(-5)$

21. Express the area of a circle,  $A$ , in terms of its circumference,  $C$ .

22. Write a formula for the function  $V(h)$  if  $V = \frac{1}{3}\pi r^2 h$  and  $\frac{r}{6} = \frac{h}{10}$

23. Write a formula for  $A$  in terms of  $l$  if  $A=lw$  and  $2l+2w=100$

24. Write down the formula for the area of a triangle. Make  $h$  your output, ie isolate it. If this was  $h$ , a function of  $A$  what type of function is it? If this was  $h$ , a function of  $b$ , what type of function is it?

25. Let  $f(x) = \frac{1}{x^2}$ , find and simplify

- |                  |                  |
|------------------|------------------|
| a. $f(2x)$       | b. $2f(x)$       |
| c. $f(\sqrt{x})$ | d. $\sqrt{f(x)}$ |
| e. $f(x^2)$      | f. $(f(x))^2$    |

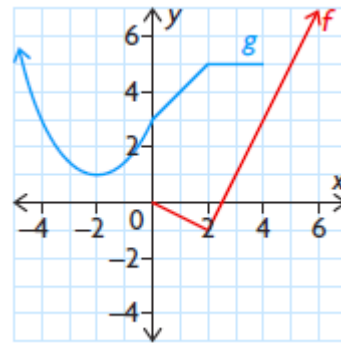
26.

Let  $h(x) = \tan x$ , find

- |                |                       |
|----------------|-----------------------|
| a. $h(\alpha)$ | b. $h(\ominus)$       |
| c. $h(x^2)$    | d. $[h(x)]^2$         |
| e. $xh(x)$     | f. $\frac{h(2x)}{2x}$ |

27. Let  $f(x) = x^2 + 3x - 5$ .  
Find and simplify  $\frac{f(1+h) - f(1)}{h}$

28.



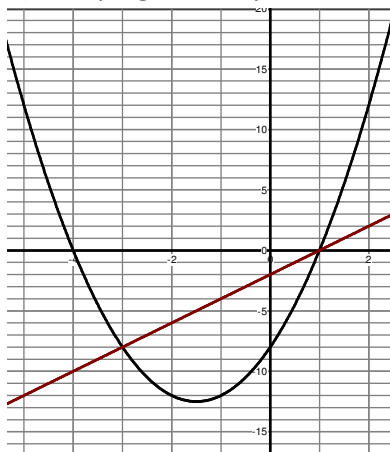
Use the graphs of  $f$  and  $g$  to evaluate each expression.

- |              |                      |
|--------------|----------------------|
| a) $f(g(2))$ | c) $(g \circ g)(-2)$ |
| b) $g(f(4))$ | d) $(f \circ f)(2)$  |

29. Below are the graphs of the functions  $f(x)$  parabola and  $g(x)$  line.

a) Sketch the graph of  $f - g$ .

b) Verify algebraically this result by finding the zeros of  $f - g$





## ASSIGNMENT Piecewise Functions (MHF)

Some relationships in real life cannot be represented by one single function. You can define a relationship using different pieces/functions. Set up equations and explain the main thing to remember

- Buying in bulk often saves money. A teacher can order individual textbooks for \$120 each, or if the order is over 50 books, the price is \$100 each, for books over 50.
- A certain income tax system takes 10% of the first \$5000 made, 20% of the \$30,000, and 40% of the rest. What is a piecewise function modelling the amount of taxes paid according to income?

Sketch

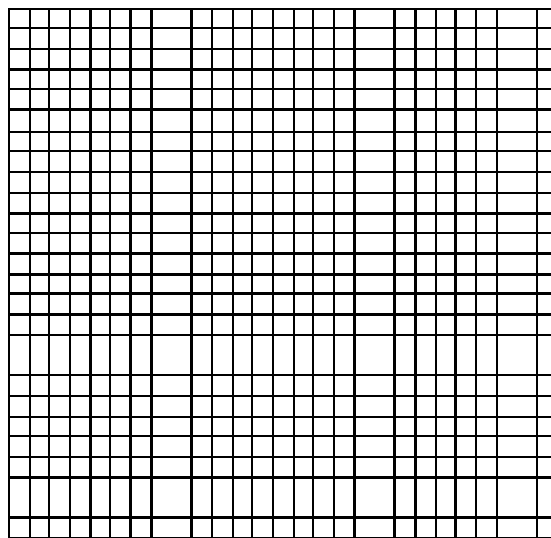
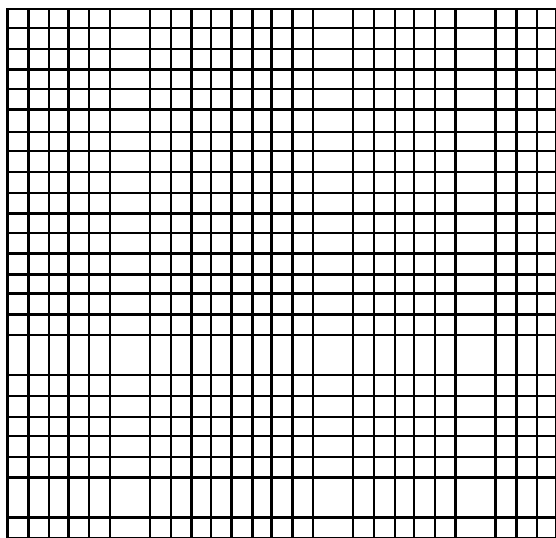
3.

$$f(x) = \begin{cases} -3 + \sqrt{-x} & -9 < x < -4 \\ -2 & x = -4 \\ x^2 + 4x + 3 & -4 < x \leq -1 \\ 2^x & 0 < x \end{cases}$$

4. Let a function be 'defined' as follows

$$f(x) = \begin{cases} -x^2 - 1, & x \leq 0 \\ 2, & 0 < x < 4 \\ \sqrt{x}, & x \geq 4. \end{cases}$$

- Find  $f(-2)$ ,  $f(0)$ ,  $f(\pi)$
- Sketch a graph of  $f$ .



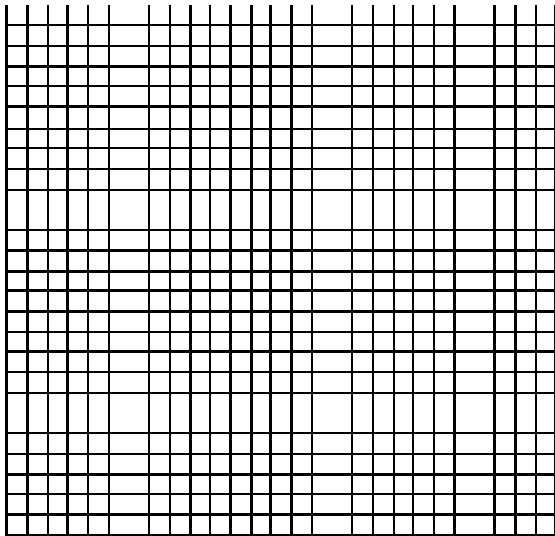
5.

$$f(x) = \begin{cases} x+1 & x > 3 \\ -x+3 & -2 < x \leq 3 \\ 3+\sqrt{x+6} & x < -2 \end{cases}$$

- Sketch
- Find  $f(2)$
- Find  $f(1)$

6.

Find  $f(x^2-6)$  for previous question, and notice that you need to learn how to solve non-linear inequality to finish this question...



7. Set up the model for one month:

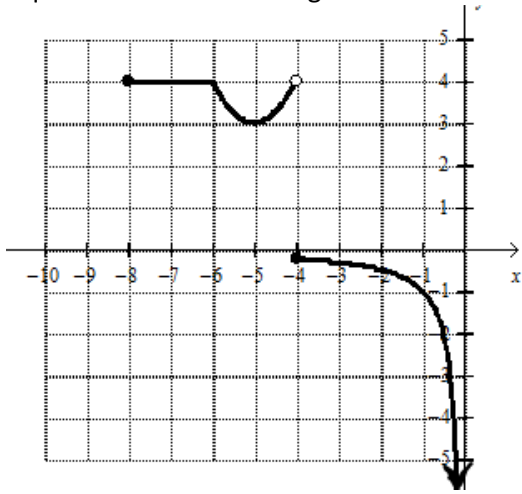
A cell phone company charges \$25 per month for 300 minutes. For any minutes over 300 it costs 0.10 per minute.

8. Set up the model for:

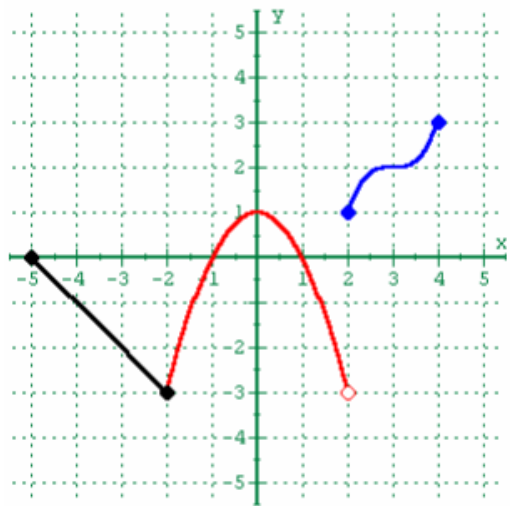
- An initial charge of \$20 to create the silk screen
- \$17.00 per shirt for orders of 50 or fewer shirts
- \$15.80 per shirt for orders of more than 50 shirts

Find the equations of the following

9.



10.



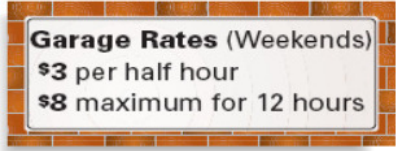
11. a. Sketch  $f(x) = -x + 4$

- b. Sketch  $|f(x)|$  hint: reflect the negative y-values into positive y-values
- c. Record the result of  $|f(x)|$  in piecewise notation.

12. a. Sketch  $f(x) = -2(x+3)(x-1)$

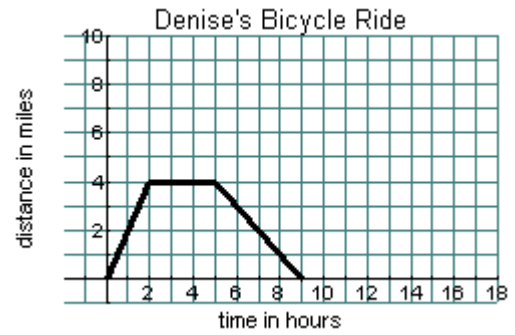
- b. Sketch  $|f(x)|$
- c. Record the result of  $|f(x)|$  in piecewise notation.

Set up the model for the following:

13. You have a summer job that pays time and half for overtime. That means, if you work more than 40 hours in a week, your hourly wage for the extra hours is 1.5 times your normal rate of \$10 per hour.
14. An airport parking garage costs \$20 per day for the first week. After that, the cost decreases to \$17 per day.
15. Buy 5, get each one after that at half price. Original price is \$20.
16.  A sign with a brick border containing the text: **Garage Rates (Weekends)**  
\$3 per half hour  
\$8 maximum for 12 hours
17. During a nine hour snowstorm it snows at a rate of 1 inch per hour for the first two hours, at a rate of 2 inches per hour for the next six hours, and at a rate of 1 inch per hour for the final hour
18. A city parking lot uses the following rules to calculate parking fees:
  - A flat rate of \$5.00 for any amount of time up to and including the first hour
  - A flat rate of \$12.50 for any amount of time over 1 h and up to and including 2 h
  - A flat rate of \$13 plus \$3 per hour for each hour after 2 h
19. **Provincial tax rates for 2011 are:**  
5.05% on the first \$37,776 of taxable income, +  
9.15% on the next \$37,776 (on the portion of taxable income between \$37,776 and \$75,552), +  
11.16% on the amount over \$75,552

20. Graham's long-distance telephone plan includes the first 500 min per month in the \$15.00 monthly charge. For each minute after 500 min, Graham is charged \$0.02.

21. Denise took a bicycle ride away from her home today. She left home at 12 p.m. and arrived back at home at 9 pm. The graph shown represents Denise's distance from home during her ride. Let the function  $d(t)$  represent Denise's distance in miles from home during her trip in terms of the time in hours. Create an algebraic model for this graph.



22. Many income tax systems are calculated using a tiered method. Under a certain tax law, the first \$100 000 of earnings are subject to a 35 % tax; earnings greater than \$100 000 and up to \$500 000 are subject to a 45% tax. Any earnings greater than \$500 000 are taxed at 55 %.

23. The fish population, in thousands, in a lake at any time,  $x$ , in years is modelled by the following function:

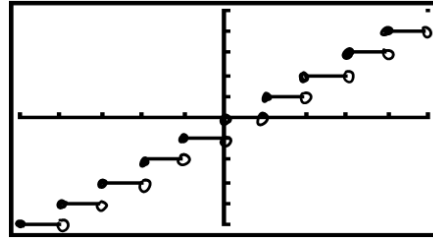
$$f(x) = \begin{cases} 2^x, & \text{if } 0 \leq x \leq 6 \\ 4x + 8, & \text{if } x > 6 \end{cases}$$

This function describes a sudden change in the population at time  $x = 6$ , due to a chemical spill.

- Sketch the graph of the piecewise function.
- Describe the continuity of the function.
- How many fish were killed by the chemical spill?
- At what time did the population recover to the level it was before the chemical spill?

### Greatest Integer Function $f(x) = \llbracket x \rrbracket$ ,

- the greatest integer less than or equal to  $x$
- the round down function



24. Record as piecewise for the domain  $-3 \leq x \leq 3$

25. What is its range?

Evaluate the following

26.  $\llbracket -6.8 \rrbracket$

27.  $\llbracket \pi \rrbracket$

28.  $\llbracket 7.1 \rrbracket$

29.  $\llbracket -2.1 \rrbracket$

Graph the following

30.  $f(x) = \llbracket x \rrbracket + 2$

31.  $g(x) = \llbracket -x \rrbracket$

32. Prior to September, 2000, taxi fares from Washington DC to Maryland were described as follows: \$2.00 up to and including  $\frac{1}{2}$  mile, \$0.70 for each additional  $\frac{1}{2}$  mile increment
- a) Describe the independent and dependent variables and explain your choices
  - b) Graph the fares for the first 2 miles: (*Make sure to label the axes.*)
  - c) Write the piecewise function for 0 to 2 miles.

## ASSIGNMENT Inequalities & Absolute Values (MHF)

### Conjunctions & Disjunctions

Graph the following

1.  $x \geq -3$  AND  $x < 2$

2.  $x \leq -3$  OR  $x > 2$

3.  $-2 \leq x < 1$

4.  $x + 3 > 7$  OR  $3x \geq 18$

*Dis-* means "apart."  
Disjunctions have two separate pieces.  
*Con-* means "together."  
Conjunctions represent one piece.

### Absolute Value Evaluating

5. Evaluate if  $x = -5$

$$|2x + 5|$$

6. Evaluate if  $x = -6$

$$|x + 4| + |2x|$$

### Absolute Value Solving

Solve each absolute value, graph the solution set.

7.  $|m| + 2 = 11$

8.  $-10|v + 2| = -70$

9.  $|x + 3| - 2 = 6$

10.  $-2\left|\frac{x}{4} + 3\right| = 8$

### Linear Inequalities

Solve each inequality, graph the solution set and state the solution in both set notation and interval notation

11.  $-11x - 4 > -15$

12.  $\frac{-9 + a}{15} \geq 1$

Solve each inequality, graph the solution set and state the solution in interval notation

13.  $2x < 10$  or  $\frac{x}{2} \geq 3$

14.  $x + 2 < 1$  or  $2x + 6 \leq -4$

15.  $2x > 3 - x$  or  $2x < x - 3$

16.  $-2x + 1 > 7$  or  $x + 4 < 5$

17.  $x + 8 \geq 9$  and  $\frac{x}{7} \leq 1$

18.  $-2x < 8$  AND  $x - 3 \leq 2$

19.  $-2 \leq 3t - 8 \leq 10$

20.  $-36 < 3p - 6 < -15$

**Linear Inequalities with Absolute Values**

Solve. State solution in interval notation

21.  $4 - 4|8x - 4| > -76$

22.  $-4|-3 + 7v| + 9 \leq -59$

Think: Greater inequalities involving  $>$  or  $\geq$  symbols are disjunctions.  
Think: Less than inequalities involving  $<$  or  $\leq$  symbols are conjunctions.



Solve. State solution in interval notation

23.  $3 + 2|9 + n| \leq -1$

24.  $-1 + 4|6r| > -97$

25.  $3\left|x + \frac{1}{5}\right| \leq \frac{18}{5}$

26.  $5|18 - 3x| > 55$

27.  $-3|2x + 5| < 69$

28.  $|5x - 15| - 3 \leq -42$

**Sketching with Absolute Values**

29.  $4|n + 8| = 56$

30.  $y = |x - 5| - 3$

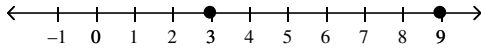
31.  $p(x) = |x^2 - 9| - 5$

32.  $f(x) = -2|(x + 1)^2 - 4| + 1$

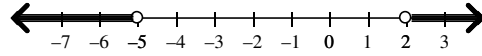
**Recording Solutions Sets**

Write the following in a) set notation b) interval notation c) absolute value equation, if possible

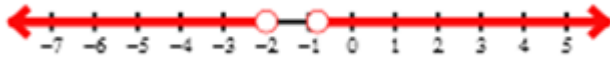
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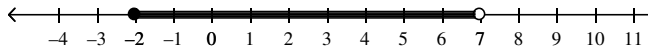
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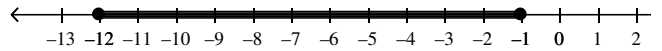
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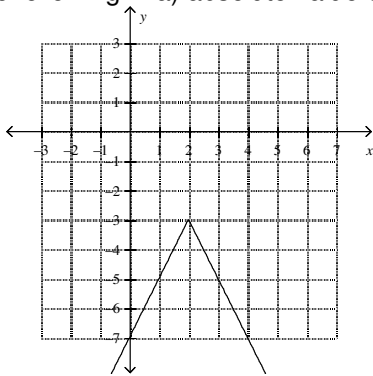


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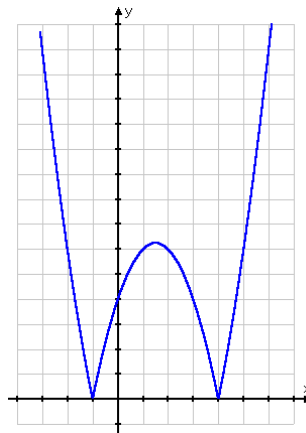


Write the following in a) absolute value equation b) piecewise equation

39.



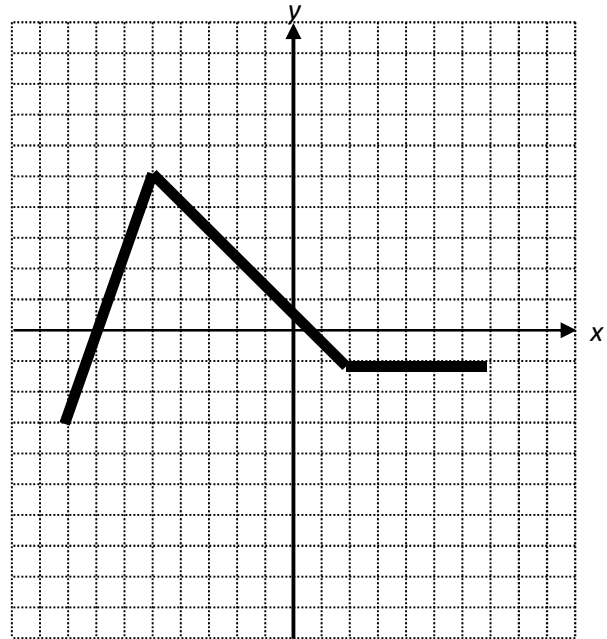
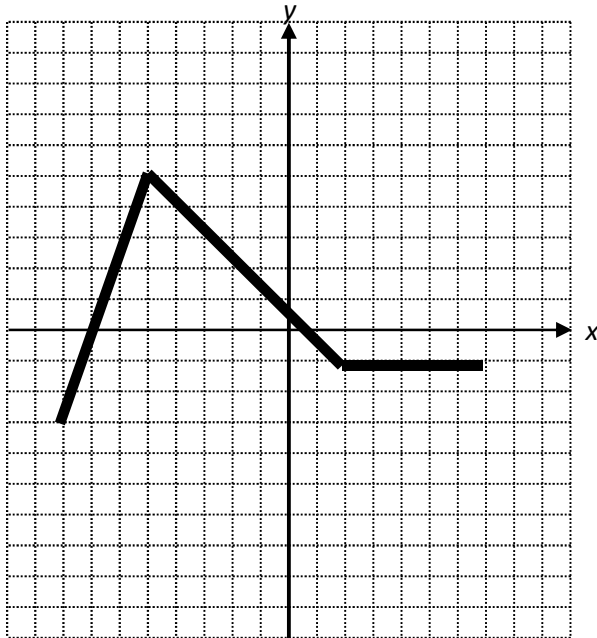
40.



41. Below is the function  $f(x)$ .

a) Draw the transformation  $g(x) = |f(2(x-4))| + 3$ .

b) Draw  $g(x) = 2f^{-1}(x-4) + 3$



42. The domain and range of the function  $f(x)$  are  $\{x \in R, 5 < x\}$  and  $\{y \in R, y \geq 2\}$  respectively. Write the domain and range of  $h(x)$  such that  $h(x) = 2|f(x-4) + 3| - 4$ .

Solve the following Greatest Integer Functions

33.  $\left\lceil \frac{2x}{7} \right\rceil = 1$

34.  $\lceil 3x \rceil = 12$

**NON-Linear Inequalities**

Solve. State solution in interval notation

43.  $-4 < x^2 - 8$

44.  $3x(x + 3) < 0$

45.  $2x^2 + 9x - 3 \geq 2$

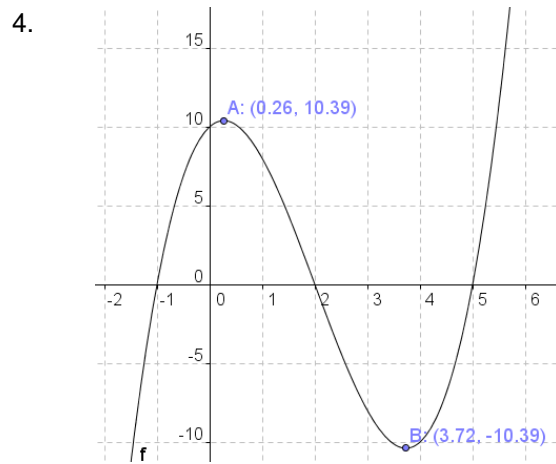
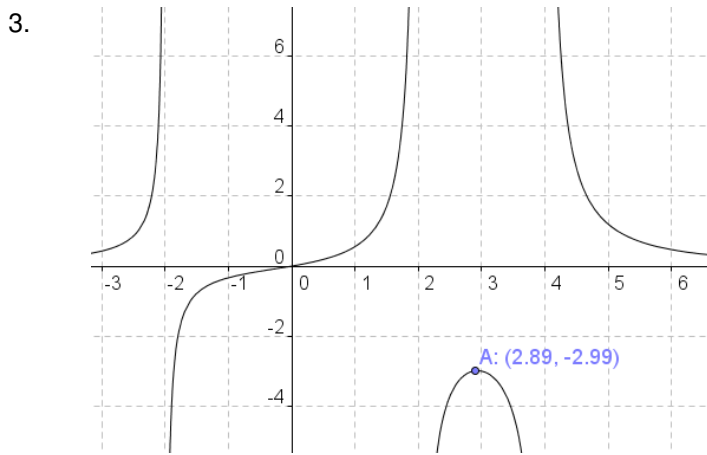
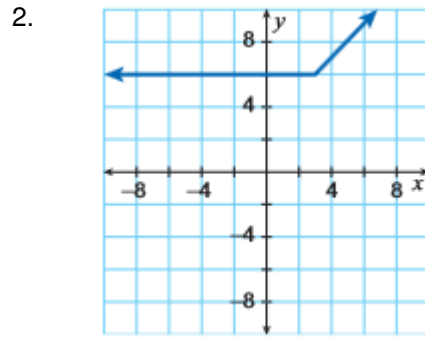
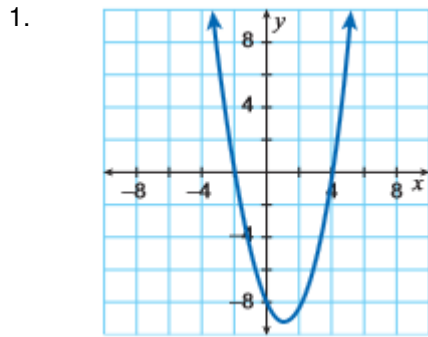
46.  $c^2 + 13c - 25 \leq 5$

47.  $4x^2 \leq 12x - 9$

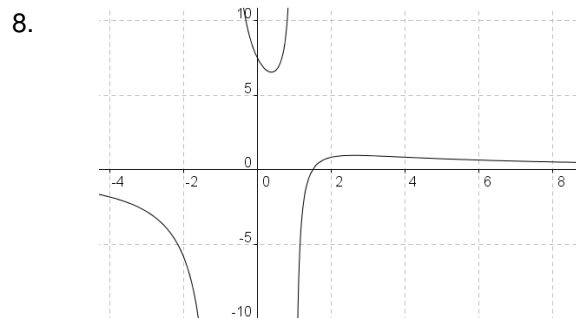
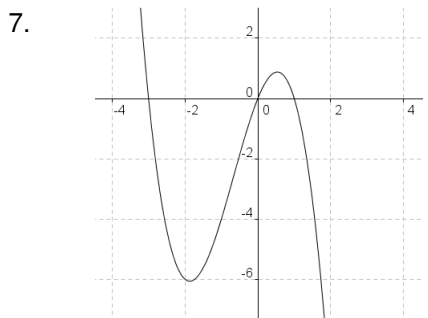
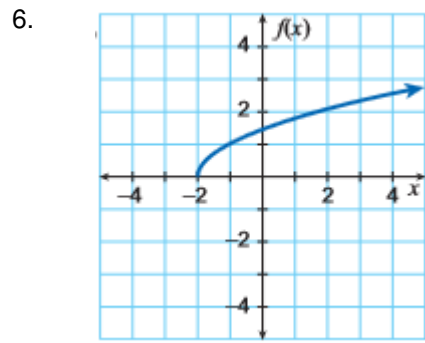
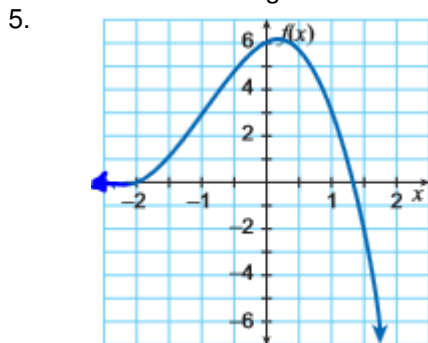
48.  $2x^2 - 3x > 5$

**ASSIGNMENT Properties of Functions (MHF)**

State the constant intervals, decreasing intervals (left column) and increasing intervals(right column),



Functions can also be described using **end behaviour**. This helps you recognize what the output values are approaching on the left-most and right-most sides of the graph. For the functions below state the end behaviour in proper notation.

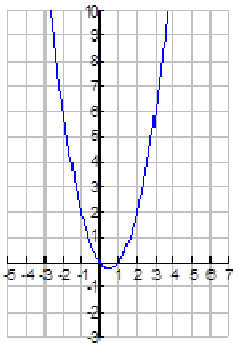




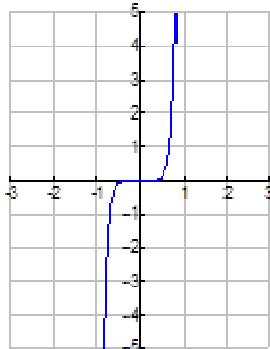
9. Functions can be described using **symmetry**. Describe what is meant by **even**, **odd**, and **neither** symmetry. Show graphical representations and algebraic.

Determine if the functions have even, odd or neither symmetry

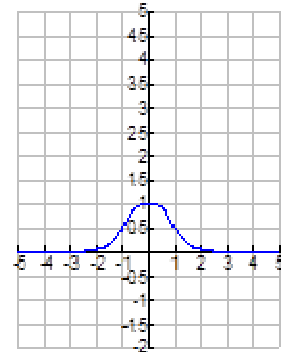
10.



11.



12.



13.  $f(x) = \frac{1}{x^4 + 1}$

14.  $f(x) = x^2 - x$

15.  $f(x) = (x + x^3)^5$

Determine if the functions have even, odd or neither symmetry

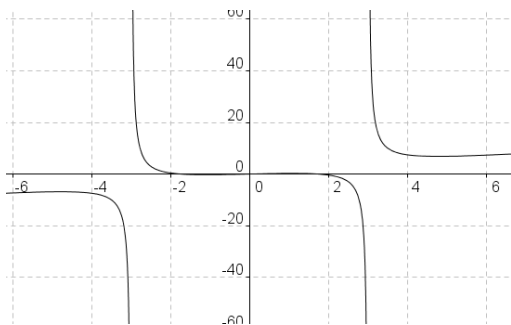
16.  $y = x^6 - 5x^4 + 2$

17.  $y = \frac{2}{x^2 - 1}$

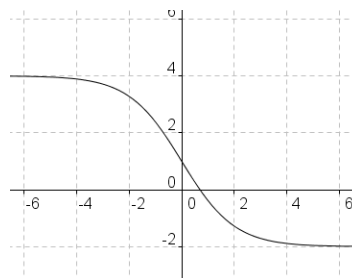
18.  $y = 2x^3 - 5x$

19.  $y = 4x^5 - x^3 + 10$

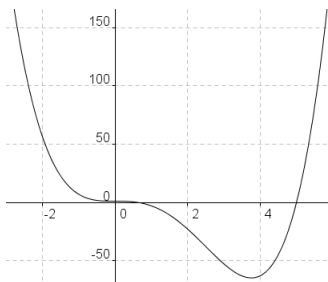
20.



21.



22.



23.

