Big idea

This unit introduces you to a new type of function – the exponential function. There are many relationships in real life that either grow or decay at a constant rate, for example: population growth, radioactive decay, inflation, spread of viruses, or processing power of technological gadgets. These types of relationships can be modelled with an exponential function. Before you learn about properties of this function, you must review exponent laws. You will then learn about the inverse of exponential functions – which is a logarithmic function of the same base. Just like there are several exponent laws there will be several logarithmic laws that you will have to know. These laws will help you in solving more complicated exponential equations to avoid the use of trial & error method. The laws will also help you solve logarithmic equations. You will learn how to graph exponential and logarithmic functions and finally you will study some real life situations that involve exponentials and logarithms.

Feedback & Assessment of Your Success

Date	Pages	Topics	**Finished assignment pages?** **Made corrections?**	**Summarized notes in a journal?** **Added your own explanations?**	**How many extra practice questions did you try in each topic?**	**Tentative TEST date:** ______________ **Questions to ask the teacher:**
0.5day	2-3	Recap of Exponent Laws (MCR) Journal #1				
1.5days	4-6	Exponential Functions (MCR) Journal #2				
	7-9	Logarithmic Functions (MHF) Journal #3				
2days	10-14	Logarithmic Laws (MHF) Journal #4				
2days	15-19	Solve Exponential (MCR) + Log Equations (MHF) Journal #5				
2days	19-24	Real Life Applications with Exponentials (MCR) + Logarithms (MHF) Journal #6				

ASSIGNMENT Recap of Exponent Laws (MCR)

Simplify the following without using a calculator, insert domain or absolute values where needed for questions with variables.

1. $\frac{3\sqrt{2}}{\sqrt{5}}$

2. $\frac{4}{1+\sqrt{x}}$

3. $\left(\frac{8}{27}\right)^{\frac{-2}{3}}$

4. $(-16)^{\frac{5}{4}}$

5. $\left(-2\sqrt{2}\right)^{\frac{2}{3}}$

6. $\left(\left(2x^{2}\right)^{4}\left(3x^{-3}\right)^{2}\right)^{\frac{1}{2}}$

7. $\left(3x^{\frac{-2}{3}}y^{\frac{1}{2}}\right)^{3}\left(2x^{\frac{1}{2}}y^{\frac{3}{4}}\right)^{2}$

8. $\left(\frac{4x^{2}y^{-1}}{z^{2}}\right)^{\frac{1}{-2}}\left(\frac{3xz^{2}}{y}\right)^{3}$

9. $(x+y)^{5}$ you will learn a short cut for this soon

10. Are the following statements True or False for all $x \in \mathbb{R}$?

a) $\sqrt[4]{x^4} = x$ b) $\sqrt[5]{x^5} = x$ c) $\sqrt{x^4} = x^2$

d) $\left(x^2\right)^5 = x^7$ e) $3^{2x}3^{4x} = 9^{6x}$ f) $\sqrt[3]{x^3+8} = x+2$

Rewrite an approximate equation so that exponent is only x. ie. In the form $y = a(b)^x + c$ with no 'k' and no 'd'

11. $f(x) = (0.8)^{3x} - 3$

12. $q(x) = -400(3.1)^{x-5} + 2$

13. $y = 2(3.5)^{-x+6} - 3$

14. $y = 0.5(2.5)^{\frac{2-x}{3}} - 4$

Rewrite the equations so x appears once. Do not solve, will do that later.

15. $2^x = 3^{x+1}$

16. $2^{2x-1} = 5^{3x+2}$

17. $4^{3x} = 2(0.5)^{2-x}$

18. $-2^x = 3^{4x+1}$

ASSIGNMENT Exponential Functions (MCR)

1. For the infinitely many parent exponential graphs $y = b^x$ Discuss and show what kind of graph you will have if
i) $b = 1$, ii) $b \in (0,1)$, iii) $b \in (1,\infty)$, iv) $b \in (-\infty,0)$

2. Determine if the following is Linear Quadratic or Exponential

a.

x	y
1	3
2	9
3	27
4	81
5	243
6	729

b.

x	y
10	7
12	9
14	11
16	13
18	15
20	17

c.

x	y
1	2
2	5
3	10
4	17
5	26
6	37

d.

x	y
1	16
2	8
3	4
4	2
5	1
6	0.5

e.

X	y
1	2.2
3	-0.5
5	-5.8
6	-10.1
7	-16.1
9	-36.3
11	-76.0

f.
same as table e, just
subtract 5 from each y value

3. For the parent graph $y = b^x$ write down the full transformed form and then the simplified version.

4. Identify the parent function and transformation constants for each. Discuss ways to sketch these versions

$y = -4(2)^{3(x-1)} + 5$ → $y = -\frac{1}{2}(8)^x + 5$

Parent y= y=

a= a=

k= k=

d= d=

c= c=

Sketch and label the horizontal asymptote and the y-intercept.

5. $f(x) = 1 - 2^{x}$

6. $f(x) = 4^{-x} + 3$

7. $f(x) = \left(\frac{1}{2}\right)^{x+2} - 2$

8. $f(x) = -e^{-x} + 2$

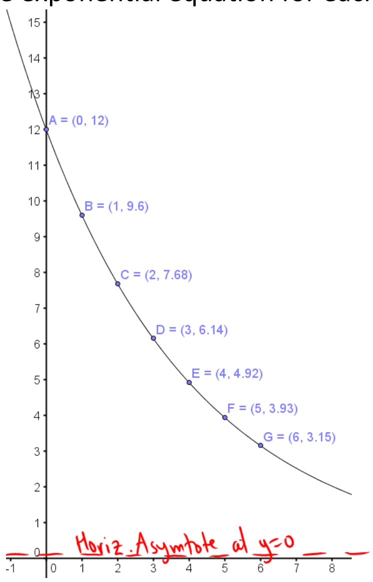
you will learn later that e=2.718…

9. $y = 0.5(2.5)^{\frac{2-x}{3}} - 4$

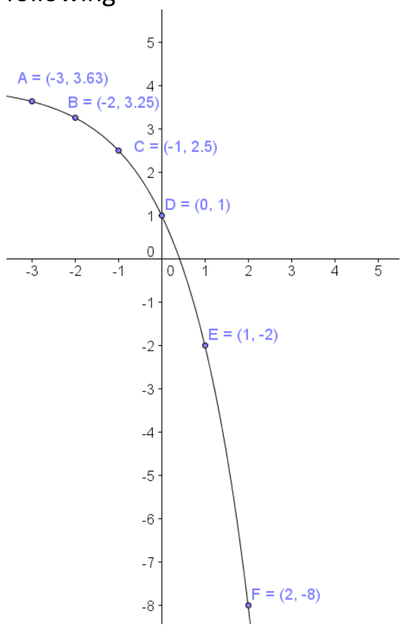
10. $y = 2(3.5)^{-x+6} - 3$

Find the exponential equation for each of the following

11.



12.

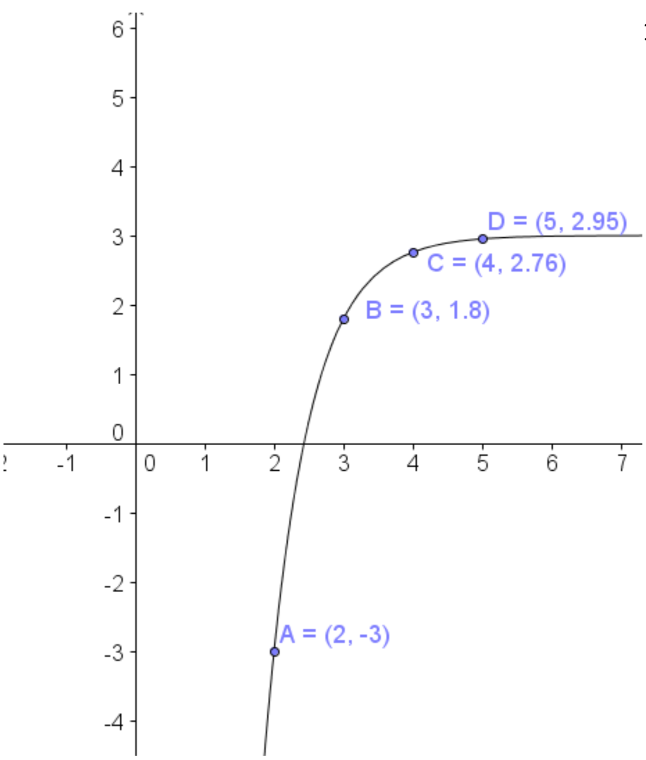


13.

x	y
-1	168
0	84
1	42
2	21
3	10.5
4	5.25
5	2.625

Horizontal asymptote at y=0.

14.



15.

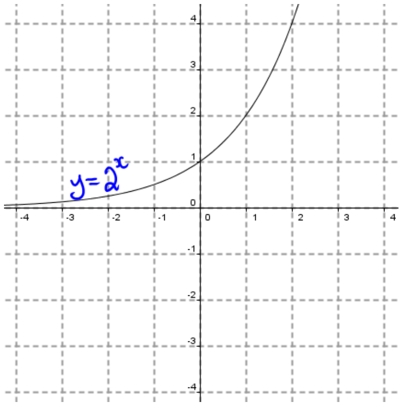
x	y
3	17.49783
6	71.6182
9	314.7336
12	1406.839
15	6312.711
18	28350.5

Horizontal asymptote at y=2.

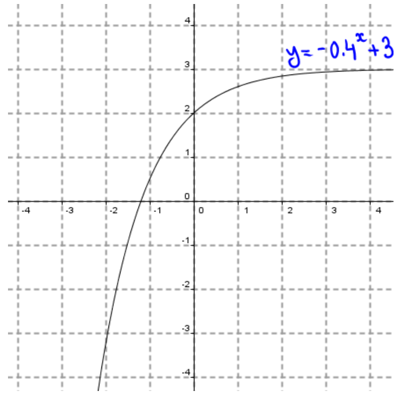
ASSIGNMENT Logarithmic Functions (MHF)

Sketch the inverse function for the following exponentials. State the implicit (y doesn't have to be isolated) inverse equations.

1.



2.



3. The inverse equations you've found above do not have the output isolated and hence cannot be written in function notation. This is one of the reasons that logarithms were invented. Summarize the rule of switching exponential form to logarithmic form or vise versa, then finish finding the inverse functions for the above questions.

4. Use the fact that exponentials and logs of the same base undo each other to develop two cool laws:

5. Another reason logs are used is to avoid working with really tiny or really huge numbers. Switch the following number to another form
$x = 1\ 000\ 000\ 000$

6. For the infinitely many parent logarithmic graphs $y=\log_b x$ Discuss and show what kind of graph you will have if i) $b=1$, ii) $b\in(0,1)$, iii) $b\in(1,\infty)$, iv) $b\in(-\infty,0)$

Express each log in exponential form or vice versa.

7. $\log \frac{1}{100} = -2$

8. $4^{\frac{-3}{2}} = \frac{1}{8}$

9. $\left(\frac{1}{2}\right)^{-4} = 16$

10. $\log_{27} 9 = \frac{2}{3}$

Evaluate without a calculator.

11. $\log_5 125$

12. $\log_4 \frac{1}{2}$

13. $\log 1000000$

14. $\log_5 1$

Sketch and label the asymptotes and x and y-intercepts (if possible)

15. $f(x) = \log_2(x-1) + 3$

16. $f(x) = \log_{\frac{1}{2}}(x+1)$

17. $f(x) = -\log_4(-x)$

18. $y = \ln(2-x)$

Determine the domain of the following

19. $y = \log_{12}(5 - 3b)$

20. $y = \log_4 x(x+2)$

21. $f(x) = \ln(3 - x^2)$

Find the inverse equations of the following

22. $f(x) = 5 \cdot 2^{x+2} - 1$

23. $y = \ln(x+5) - 2$

Find y as a function of x, (C is a constant)

24. $\ln(y-5) = \ln 3x^2 + \ln C$

25. $\frac{1}{2}\ln y = \ln 4x^3 + C$

ASSIGNMENT Logarithmic Laws (MHF)

Read and understand the following proofs to the new logarithm laws.

1. Proof for

$$\boxed{\log_a(xy) = \log_a x + \log_a y}$$

Use the exponent multiplication law:

$a^m \cdot a^n = a^{m+n}$

$$\begin{cases} \text{let } x = a^m & \Leftrightarrow \log_a x = m \\ \text{and } y = a^n & \Leftrightarrow \log_a y = n \end{cases}$$

$\log_a(x \cdot y)$

$= \log_a(a^m \cdot a^n)$

$= \log_a a^{m+n}$

$= m + n$

$= \log_a x + \log_a y$

2. Proof is similar for

$$\boxed{\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y}$$

Notice bases have to match and coefficients as well.

$$\boxed{c\log_a(x \cdot y) = c\log_a x + c\log_a y}$$

3. Proof for

$$\boxed{\log_a(x)^n = n\log_a x}$$

4. Proof for CHANGE of BASE

$$\boxed{\log_b a = \frac{\log_{\#} a}{\log_{\#} b}}$$

5. Proof for

$$\boxed{\log_a 1 = 0}$$

Express each log in exponential form or vice versa

1. $x^{\sqrt{2}} = \pi$

2. $\ln x = e$

Evaluate without a calculator

3. $\log_{\sqrt{3}} 9$

4. $\ln \sqrt{e}$

5. $9^{\log_3 5}$

6. $1000^{\log 5}$

7. $\log(\sqrt{10}\,\sqrt[3]{10}\,\sqrt[5]{10})$

8. $\sqrt{\log(10000)} - \log \sqrt{100}$

9. $2^{\log_2 3 - \log_8 9}$

10. $e^{3\log_e(2)}$

11. $\log_2 6 \times \log_6 4$

12. $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8$

Simplify

13. $10^{2\log x+3\log(y)}$

14. $(4^{3t}8^{2t})^t$

15. $\ln e^{5x+\ln x}$

16. $e^{4\ln 5-5\ln 4}$

17. $x\ln y-y\ln x$

18. $\dfrac{\ln x^2-\ln^2 x}{\ln x}$

19. $\dfrac{e^{2y}-e^{2x}}{e^y-e^x}$

20. $\log_2 x+\log_4(x+1)-\log_8(x^2-1)$

21. $(\log_7 11)(\log_{11} 5)$

22. $\frac{1}{2}\left[\ln x-3\ln\left(x^2-1\right)+2\ln\left(x^2+1\right)\right]$

23. $21\log_3\sqrt[3]{x}+\log_3(9x^2)-\log_5 25$

Condense using Log Laws and evaluate ***without*** using a calculator

24. $\log_5 375 - \log_5 3$

25. $2\log_{27} 3 + 2\log_{27} 9$

26. $-2\log 25 - 4\log 2$

27. $\log_6 3 + \log_{36} 4$

Expand using Log Laws (record without exponents on the input, if possible)

28. $\log[(x)(x+2)^3(x+3)]$

29. $\log \frac{10\sqrt[3]{x^2+y^2}}{x^2y^4}$

30. $\ln \frac{t^2}{(t+1)^3}$

31. $\ln \sqrt{x\sqrt{y\sqrt{z}}}$

32. Suppose $x = \log(A)$ and $y = \log(B)$, write the following expressions in terms of x and y.

(a) $\log(AB) =$

(b) $\log(A)\log(B) =$

(c) $\log\left(\frac{A}{B^2}\right) =$

Prove

33. $\ln\left(1+e^{2x}\right) = 2x + \ln\left(1+e^{-2x}\right)$

34. $\log_{\frac{1}{a}} x = -\log_a x$

Change into an exponential form specified

35. $y = 4^x$ change to exponential of base e

36. $y = \left(\frac{1}{2}\right)^x$ change to exponential of base 5

Change into a log form specified

37. $y = \log_{16} x$ change to log of base 2

38. $y = \log_{\frac{1}{3}} x$ change to log of base 27

ASSIGNMENT Solve Exponential (MCR) + Logarithmic Equations (MHF)

Part A (MCR) Factor the following

1. $3^{2x} - 12(3^x) + 27$

2. $4(4^{2x-1} + 1) - 5(4^x)$

3. $4e^{2x} - 64$

4. $\ln^2 x - 9\ln x + 20$

5. $\ln^3 x - 27$

6. $e^{2x} - 16y^2$

Solve

7. $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

8. $3^{x+2} + 3^x = 30$

9. $8(5^{2x}) + 8(5^x) = 6$

Part B (MCR) Solve, give exact answers and rounded versions beside them.

1. $3^{2x} = 243$

2. $\left(\frac{1}{25}\right)^{x-1} = 125(5^{x^2})$

3. $625^{k+1} \cdot 125^{-2k} = 25$

4. $\dfrac{36^{3-3a}}{\left(\frac{1}{216}\right)^{-3a}} = 216$

5. $3(3^x) + \frac{81}{3^{x-1}} = 90$

6. $4b^{-\frac{3}{4}} + 10 = \frac{21}{2}$

Part C (MHF) Solve, give exact answers and rounded versions beside them.

1. $7(2^{x-3})-11=20$

2. $(2^{x})^{2}+20=18(2^{x})-12$

3. $3^{x}=7^{x+1}$

4. $\log_{3}\sqrt{4x+1}=2$

5. $\log_{2}x^{\log_{2}x}=9$

6. $\log(x+1)-2=\log(x^{2}-4)-\log(x+2)$

Part D (MHF) Solve, give exact answers and rounded versions beside them.

1. $2e^{2x-6}-17e^{x-3}+40=5$

2. $-2e^{-x^2}+4x^2e^{-x^2}=0$

3. $2e^{x+3}=\pi^x$

4. $14=\dfrac{20}{1-3e^{0.5x}}$

5. $750=500\ln(12x+2)-50$

6. $\log_2(x^2+1)-\log_4 x^2=1$

ASSIGNMENT Real Life Applications with Exponentials (MCR) + Logarithms (MHF)

Part A (MCR)

1. Most real life word problems of growth or decay have a horizontal asymptote at y=0. What is the equation usually used for **word problems?** Explain the significance of EACH letter in the context of a word problems.

2. Summarize how to find the 'b' in the equation.

3. Clarify the differences between growth **facto**r and growth **rate** for $f(x)=125(1.32)^x$

4. Assign variables and set up the models for the following word problems.

a. The value of the $250 thousand cottage increases by 0.1% every 3 weeks.

b. The 40 grams of radioactive matter within a mass decays at 2% every 30 seconds.

c. The 200 fruit fly population doubles every 5 days.

5. Assign variables and set up the models for the following word problems. Then solve the question.

a. A certain strain of yeast cell doubles under certain conditions every 20 minutes. If there were 350 initially, how many will there be in 3 hours?

b. How long will it take for a 1 gram sample of polonium-210 to lose $\frac{3}{4}$ of its radioactivity if its half life is 140 days?

c. For a biology experiment, the number of cells present is 1000. After 4 hours the count is estimated to be 256 000. What is the doubling period of the cells?

For question #6 and #7:

a.) Identify whether it is a growth or decay (ensure that the exponent is just x before you classify!)
b.) Find growth/decay factor
c.) Find growth/decay rate as %

6. $y = 0.5^{-x} + 6$

7. $g(x) = 5(3.5)^{\frac{8-x}{2}} - 10$

Find the equation that describes:

8. A population of cells that start with 5 cells and double every hour.

9. The behavior of cell duplication in a lab. In one experiment, you started with 1,000,000 cell and the cell population decreased by ten percent every 15 minutes.

Solve

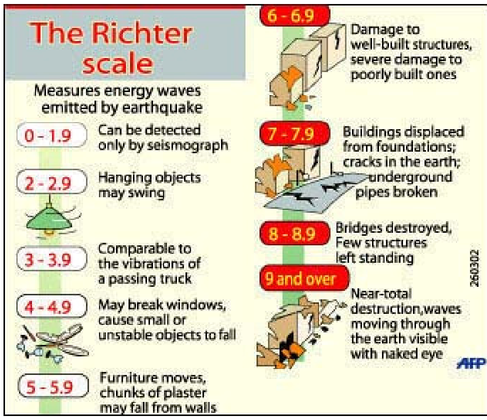
10. A $1,000 deposit is made at a bank that pays 12% compounded annually.
a. What is the value of the deposit after 5 years?
b. After how many years, months, days does the deposit reach double it's value?

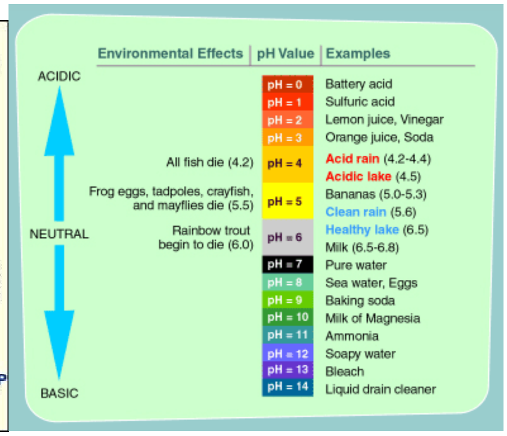
11. Hospitals utilize the radioactive substance iodine-131 in the diagnosis of conditions of the thyroid gland. The half-life of iodine-131 is eight days
    a. If a hospital acquires 2 g of iodine-131, how much of this sample will remain after 20 days ?
    b. How long will it be until only 0.01 g remains?

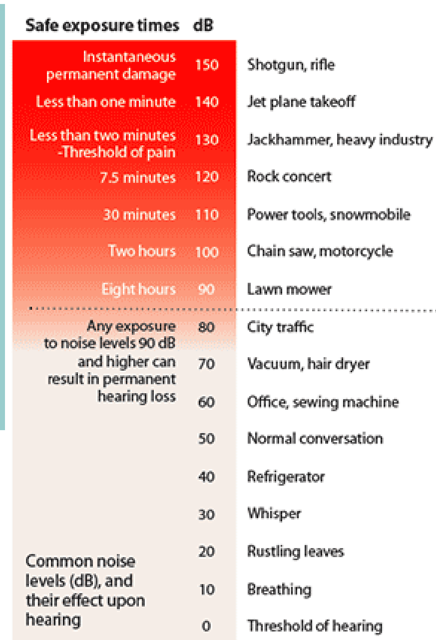
12. On their last fishing trip, Louis and Carmen took a block of dry ice with them. When they started out, the block had a mass of 25 kg. During the trip, the block sublimated at a constant rate such that every 12 hours its mass decreased by 9 %. At the end of the trip, the block had a mass of 10 kg. To the nearest tenth of an hour, how long did their trip last?

13. A city in Texas had a population of 75,000 in 1970 and a population of 200,000 in 1995. The growth between the years 1970 and 1995 followed an exponential pattern.
    a. Find a model (equation) that **estimates** the population at any time during those years.
    b. From this model, estimate the population for the year 2010.
    c. What is the approximate annual growth rate?

Part B (MHF)







1.

$$M = \log\left(\frac{I}{I_0}\right)$$

___ = the magnitude of the earthquake on the Richter scale
___ = the intensity/amplitude of the reference earthquake
___ = the intensity/amplitude of the wave detected by the **seismograph** of the earthquake being measured

2.

$$L = 10\log\left(\frac{I}{I_0}\right)$$

___ = the loudness of sound in decibels
___ = the intensity of sound power per unit area (watts/m^2) of the threshold of hearing
___ = the intensity of sound power per unit area (watts/m^2) being measured

3.

$$pH = -\log\left(H^+\right)$$

___ = the acidity of the substance
___ = the concetration of hydrogen ions (mol/L)

4.
In October of 2005, Pakistan experienced an earthquate of magnitude 7.6 resulting in the death of over 73 000 people. Later on that month, Owen Sound experienced an earthquake of 4.2 in magnitude. How many times more intense was the Pakistan earthquake to the quake in Owen Sound?

5.
How many times more loud is a rock concert with a sound intensity of 123 dB than the threshold of sound?

6.
A fish tank's water was recently changed with distilled water of pH 7. The day after it was changed, apple juice was spilled into it which caused the pH to drop to 5.8. By what factor has [H$^+$] changed?

7. Early in the century the earthquake in San Francisco registered 8.3 on the Richter scale. In the same year, another earthquake was recorded in South America that was four times stronger. What was the magnitude of the earthquake in South America?

8. A recent earthquake in San Francisco measured 7.1 on the Richter scale. How many times more intense was the San Francisco earthquake described in question #7

9. A bacterial population's mass in grams at time t minutes is given by the logistic equation: $P(t) = \frac{10^7}{1+10^7 e^{-0.3t}}$

a) What is the initial mass of the bacteria population?

b) Determine the number of minutes that have elapsed when the population reaches 780g.

10. You test some ammonia and determine the hydrogen ion concentration to be [H+] = 1.3×10^{-9}. Find the pH value

11. Find the hydrogen ion concentration, [H+], of a solution with pH 9.3

12. The intensity of a sound wave is interpreted by our ears as its loudness. The weakest sound wave that a human ear can hear has a value of 1×10^{-12} watts/m^2 and is called the threshold of human hearing, Io. What is the intensity in watt/m^2 of a sound wave that has a sound level reading of 125 dB, the loudness of an average fire alarm

13. Considering that prolonged exposure to sounds above 85 decibels can cause hearing damage or loss, and considering that a gunshot from a .22 rim fire rifle has an intensity of about $I = (2.5 \times 10^{13})I_0$, should you follow the rules and wear ear protection when at the rifle range? Show work.