ReviewSol

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MCR

used 2013 version

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

1. Expand
$(x-4)^6$

$$x^6 + 6(x)^5(-4) + 15(x)^4(-4)^2 + 20(x)^3(-4)^3 + 15(x)^2(-4)^4 + 6(x)(-4)^5 + (-4)^6$$
$$= x^6 - 24x^5 + 240x^4 - 1280x^3 + 3840x^2 - 6144x + 4096$$

2. How long will it take \$4,000 to grow to \$9,000 if it is invested at 7% compounded monthly?

$A = P(1+i)^n$
$9000 = 4000\left(1 + \frac{0.07}{12}\right)^n$
$2.25 = (1.0058\bar{3})^n$
$\updownarrow$
$\log_{1.00583}(2.25) = n$
$139.42 = 12t$
$11.6\text{yrs} = t$

3. Recently, Guaranty Income Life offered an annuity that pays 6.65% compounded monthly. If \$500 is deposited into this annuity every month, how much is in the account after 10 years? How much of this is interest?

$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$

$FV = \frac{500\left[\left(1+\frac{0.0665}{12}\right)^{120} - 1\right]}{\left(\frac{0.0665}{12}\right)}$

$FV = \$84\,895.40$
subtract 500×120
$\therefore I = 24\,895.40$

4. A loan of \$4,000 was repaid at the end of 10 months with a check for \$4,270. What annual rate of interest was charged?

$A = P + Prt$
$4270 = 4000 + 4000(r)\left(\frac{10}{12}\right)$
$270 = 3333.\bar{3}r$
$0.081 = r$ or 8.1%

5. Sharon has found the perfect car for her family (anew mini-van) at a price of \$24,500. She will receive a \$3500 credit toward the purchase by trading in her old Gremlin, and will finance the balance at an annual rate of 4.8% compounded monthly.
   a) How much are her payments if she pays monthly for 5 years?
   b) How much interest did she pay?

$PV = \frac{R\left[1 - (1+i)^{-n}\right]}{i}$

$21000 = \frac{R\left[1 - \left(1 + \frac{0.048}{12}\right)^{-60}\right]}{\left(\frac{0.048}{12}\right)}$

$21000 = R(53.2488\ldots)$

a) $\$394.37 = R$

b) $I = FV - PV$
$= 394.37 \times 60 - 21000$
$= \$2662.20$

6. For the geometric sequence with
$t_2 = 6$
and
$t_5 = \frac{3}{4}$
determine
   a. the general term (explicit) formula for tn (use fractions)
   b. the sum of first seven terms (use fractions)

$6 = ar^1$
$\frac{3}{4} = ar^4$ divide

$\frac{3}{4} \div 6 = \frac{ar^4}{ar^1}$
$\frac{1}{8} = r^3$
$\frac{1}{2} = r$
$\therefore 6 = a\left(\frac{1}{2}\right)$
$12 = a$

a) $t_n = 12\left(\frac{1}{2}\right)^{n-1}$

b) $S_7 = \frac{12\left(\left(\frac{1}{2}\right)^7 - 1\right)}{\frac{1}{2} - 1}$

$= \frac{12\left(\frac{1}{128} - \frac{1 \times 128}{1 \times 128}\right)}{-\frac{1}{2}}$ $\quad \frac{-2\times}{1}$

$= -24\left(\frac{-127}{128}\right)$

$= \frac{381}{16}$

7. An auditorium contains 10 seats in the first row, 12 seats in the second, 14 in the third, and so on.
   a. How many seats are in the back row if there are 50 rows in the auditorium?
   b. How many total seats are in the auditorium?
   c. What is the recursive formula for the number of seats in row n?

$a = 10$
$d = 2$

$t_n = a + d(n-1)$
a) $t_{50} = 10 + 2(49) = 108$ seats
b) $S_{50} = \frac{50}{2}\left[2(10) + 2(49)\right]$
$= 2950$ seats in total

c) $t_n = (t_{n-1}) + 2, \; t_1 = 10$

8. Find the first six terms of a sequence defined by
$t_1 = -1$
$t_n = \begin{cases} -2t_{n-1} & \text{if } t_{n-1} < 0 \\ (t_{n-1} - 3) & \text{if } t_{n-1} > 0 \end{cases}$

$t_1 = -1$
$t_2 = -2(-1) = 2$ use 1st piece since t_1 was neg

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$t_1 = -1$

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AP

8.) $t_1 = -1$

$t_2 = -2(-1) = 2$ use 1st piece since t_1 was neg

$t_3 = 2 - 3 = -1$ use 2nd piece

$t_4 = 2$

$t_5 = -1$ will repeat

$t_6 = 2$

9. Find the sum: $\sum_{k=253}^{571} \left(\frac{1}{3}\right)$

$\boxed{\text{\# of terms = top index − bottom index + 1}}$

9.) $571 - 253 + 1 = 319$ terms all together

all of them $\frac{1}{3}$ in value

$\therefore$ total $= 319\left(\frac{1}{3}\right)$

$= \frac{319}{3}$

10. Record in Sigma notation:

a) 2.1 + 2.01 + 2.001 + 2.0001 + … + 2.000000001

b) $\sqrt{3} + 2\sqrt{5} + 3\sqrt{7} + 4\sqrt{9} + 5\sqrt{11} + \ldots$

10 a) $t_n = 2 +$ geo seq.

$a = 0.1$

$r = 0.1$

$t_n = 2 + 0.1(0.1)^{n-1}$

$t_n = 2 + (0.1)^n$ and there are 9 terms

$\therefore \sum_{n=1}^{9} 2 + (0.1)^n$

10 b) $t_n = a_n\sqrt{b_n}$

$a_n: 1, 2, 3, 4, \ldots \quad a_n = n$

$b_n: 3, 5, 7, \ldots \quad b_n = 3 + 2(n-1)$

$= 2n + 1$

there are ∞ many terms

$\therefore \sum_{n=1}^{\infty} n\sqrt{2n+1}$

11. Find the explicit equation of the following pattern:

8	16	0	−64	−200	−432	−784

+8, −16, −64, −136, −232

−24, −48, −72, −96

−24, −24, −24

$\therefore$ cubic

$t_n = an^3 + bn^2 + cn + d$

11.)

n	t_n	Δt_n	$\Delta\Delta t_n$	$\Delta\Delta\Delta t_n$
1	$a+b+c+d$	$7a+3b+c$	$12a+2b$	$6a$
2	$8a+4b+2c+d$	$19a+5b+c$	$18a+2b$	
3	$27a+9b+3c+d$	$37a+7b+c$		
4	$64a+16a+4c+d$			

$\therefore 6a = -24$

$a = -4$

$12a + 2b = -24$

$12(-4) + 2b = -24$

$2b = 24$

$b = 12$

$7a + 3b + c = 8$

$7(-4) + 3(12) + c = 8$

$c = 0$

$a + b + c + d = 8$

$-4 + 12 + 0 + d = 8$

$d = 0$

$$\boxed{\therefore t_n = -4n^3 + 12n^2}$$

12. Evaluate the sum in terms of n

12.)

$$\sum_{i=1}^{n} (3+2i)^2 = \sum_{i=1}^{n} 9 + 12i + 4i^2 = \sum_{i=1}^{n} 9 + 12\sum_{i=1}^{n} i + 4\sum_{i=1}^{n} i^2$$

$$= 9n + \cancel{12}^{6}\left[\frac{n^2+n}{\cancel{2}}\right] + \cancel{4}^{2}\left[\frac{2n^3+3n^2+n}{\cancel{6}_{3}}\right]$$

$$= 9n + 6n^2 + 6n + \frac{4}{3}n^3 + 2n^2 + \frac{2}{3}n$$

$$= \frac{4}{3}n^3 + 8n^2 + \frac{47}{3}n$$

13. How many years will it take for an initial investment of \$25,000 to grow to \$80,000. Assume the interest rate of interest of 6% compounded continuously.

13) $A = Pe^{rt}$

$80\,000 = 25000\,e^{0.06t}$

$3.2 = e^{0.06t}$

$\ln(3.2) = 0.06t$

$19.4\text{yrs} \doteq t$