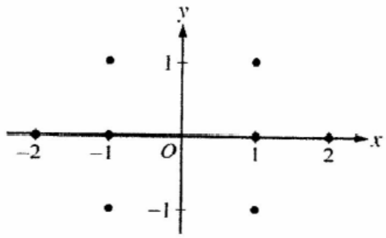
Review

January 7, 2015 10:24 AM

AB

1. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

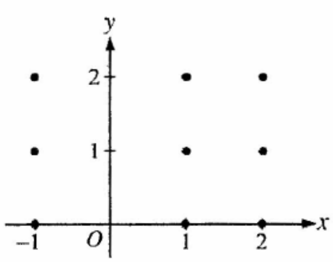
(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

2. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



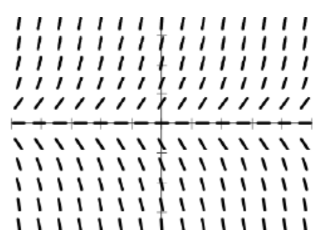
(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

3. The slope field for a differential equation is shown at right. Which statement is true for all solutions of the differential equation?

I. For $x < 0$, all solutions are decreasing.
II. All solutions level off near the x-axis.
III. For $y > 0$, all solutions are increasing

(A) I only (B) II only (C) III only
(D) II and III only (E) I, II, and III



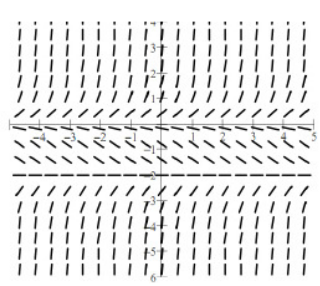
4. The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y+y^2}{4x+2y}$ will have vertical segments when

(A) $y = 2x$ (B) $y = -2x$ (C) $y = -x^2$ only (D) $y = 0$ only (E) $y = 0$ or $y = -x^2$

5. Which statement is true about the solutions, $y(x)$, of a differential equation whose slope field is shown at right?

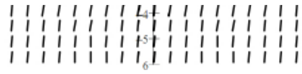
I. If $y(0) > 0$, then $\lim_{x\to\infty} y(x) \approx 0$
II. If $-2 < y(0) < 0$, then $\lim_{x\to\infty} y(x) \approx -2$
III. If $y(0) < -2$, then $\lim_{x\to\infty} y(x) \approx 0$

(A) I only (B) II only (C) III only
(D) II and III only (E) I, II, and III



$x \to \infty$

(A) I only (B) II only (C) III only
(D) II and III only (E) I, II, and III



6) A company's production is increasing at a rate proportional to the number N of units produced and inversely proportional to the square of the time t in years. The maximum production is 5000 units. In the first year, 1000 units are produced. What is the production after 5 years?

7) A rumor starts in a population of 10,000. The rumor spreads at a rate proportional to the number of people who at time t have not heard the rumor. Initially, 25 people have heard the rumor; at the end of 3 weeks, 6675 people have heard it. How many people will have heard the rumor after 6 weeks?

8) A piece of furniture is worth \$2500. Its value V is depreciating at a rate proportional to the value and inversely proportional to the square root of the time t in years. The piece of furniture will be worth \$1600 in four years. How long will it take for it to be worth \$1280?

9) A car going 40m/s brakes to a stop in seven seconds. Assume the deceleration is constant.

a) Graph the velocity against time.
b) Represent, as an area on the graph, the total distance traveled from the time the brakes are applied until the car comes to a stop.
c) Find this area and explain what it represents.
d) Now find an equation for the distance travelled in terms of time. since brakes were applied
e) Use the equation from part d) to determine the distance the car travelled before stopping.

10) When Apollo 15 landed on the moon, astronaut David Scott dropped a hammer and a feather to demonstrate that all objects, in a vacuum, fall at the same rate. Mr. Scott dropped the hammer and feather at a height of 4ft. How long did it take the objects to fall those 4ft, when the acceleration due to gravity on the moon is $5.2\frac{ft}{\sec^2}$.

BC

11) Evaluate the integral $\int \ln x^2 dx$

12) Evaluate the integral $\int x^2 e^{-3x} dx$

13) Evaluate the integral $\int \theta \cos \pi\theta \, d\theta$

14) Evaluate the integral $\int x^2 \sin x dx$

15) Evaluate the integral $\int_1^e x^3 \ln x dx$

16) Express the integrand as a sum of partial fractions and evaluate the integrals $\int_4^8 \frac{y dy}{y^2 - 2y - 3}$

17) Express the integrand as a sum of partial fractions and evaluate the integrals $\int_1^{\sqrt{3}} \frac{3x^2 + x + 4}{x^3 + x} dx$

18) Express the integrand as a sum of partial fractions and evaluate the integrals $\int \frac{8x^2 + 8x + 2}{\left(4x^2 + 1\right)^2} dx$

19) Express the integrand as a sum of partial fractions and evaluate the integrals $\int \frac{x^2 + x}{x^4 - 3x^2 - 4} dx$

(19) Express the integrand as a sum of partial fractions and evaluate the integrals $\int \frac{x^2+x}{x^4-3x^2-4}dx$

(20) Express the integrand as a sum of partial fractions and evaluate the integrals $\int \frac{x^4}{x^2-1}dx$

(21) Use Euler's method to calculate the first three approximations to the given initial value problem for the specified increment size. Calculate the exact solution and investigate the accuracy of your approximations. Round the results to four decimals

$$y' = x(1-y), \quad y(1) = 0,$$

$\Delta x = 0.5$

x_n	y_n	$m = \left.\frac{dy}{dx}\right\|_{(x_n, y_n)}$	$\Delta y = m\Delta x$	$y_{n+1} = y_n + \Delta y$

(22) Use the Euler method with $dx = 0.2$ to estimate $y(2)$ if $y' = \frac{y}{x}$ and $y(1) = 2$. What is the exact value of $y(2)$?

x_n	y_n	$m = \left.\frac{dy}{dx}\right\|_{(x_n, y_n)}$	$\Delta y = m\Delta x$	$y_{n+1} = y_n + \Delta y$

(23)

Use trig. sub.

(a) Evaluate the integrals $\int \frac{dx}{x^2\sqrt{x^2+1}}$

(b) Evaluate the integrals $\int \frac{2dx}{x^3\sqrt{x^2-1}}, \quad x > 1$

(c) Evaluate the integrals $\int \frac{5dx}{\sqrt{25x^2-9}}, \quad x > \frac{3}{5}$

(d) Evaluate the integrals $\int_0^{3/2} \frac{dx}{\sqrt{9-x^2}}$