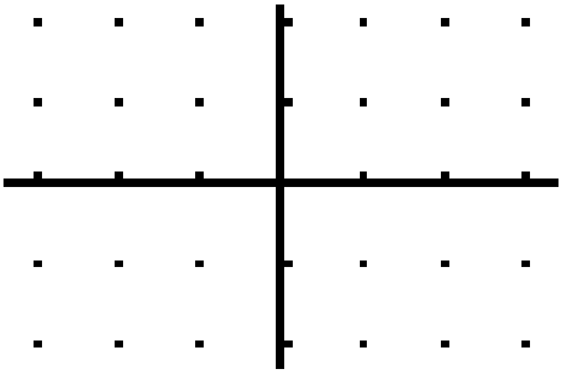
Calculus UNIT 7

DIFFERENTIAL EQUATIONS (AB + BC) – journal questions

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. SLOPE FIELDS (AB)
   a. What is a slope field? Why is it important to learn? What are key things to keep in mind when sketching these?
   b. Draw the slope field $y' = (x-1)^2 y^2$, what type of solution does the graph represent? (general/unique?) Beside the graph outline your steps of how you decided each part.



   c. Draw a curve at point (2,1), what type of solution does this represent? (general/unique?)

2. SEPARABALE DIFFERENTIAL EQUATIONS (AB)
   a. Solve for the general solution of $y' = x\sqrt{y}\cos^2(\sqrt{y})$, Explain your steps.
   b. Solve for the unique solution of $y' = e^{x-y}$ given point y(1)=2, Explain your steps

3. EULER's METHOD for SOLVING DE (BC)

Explain how Euler's Method works and then do the following:

If $\frac{dy}{dx} = 2x - y$ and if $y = 3$ when $x = 2$, use Euler's method with five equal steps to approximate y when $x = 1.5$.

4. INTEGRATION by PARTS (BC)

Make a list of how to decide to choose the u and explain how to do the following and do it:

   a. $\int \theta \sec^{-1}\theta \, d\theta$ use "backwards Zorro"
   b. $\int x^5 e^x \, dx$ **and** $\int x^7 \ln x \, dx$ use tabular method

5. INTEGRATION by PARTIAL FRACTIONS (BC)
   a. Record the partial fractions set up for the following, no need to solve

      i. $\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}$

      ii. $\frac{6x+7}{(x+2)^2}$

      iii. $\frac{-2x+4}{(x^2+1)(x-1)^2}$

   b. Solve fully, explaining steps and any shortcuts you use.

$$\int \frac{3x^2+4x+4}{x^3+x} dx$$

6. TRIG INTEGRATION (BC)

a. Copy/Paste the strategies:

Sine & Cosine	**Tangent & Secant**
• One power is odd – split up the odd power like $\cos x\cos^2 x$ keep one cosine and change the rest into sine using $\sin^2 x+\cos^2 x=1$ (if keep one sine change the rest into cosines) • Both powers odd – split up the smallest power • Both powers even – use $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$ $\cos^2(x)=\frac{1}{2}(1+\cos(2x))$ $\sin 2x=2\sin x\cos x$ $\cos^2 x-\sin^2 x=\cos 2x$	• Secant has even power – split off one $\sec^2 x$ change the rest to tangent using $\sec^2 x=1+\tan^2 x$ • Tangent has odd power – split off $\tan x\sec x$ change the rest to secant **Cosecant & Cotangent** Similar to tangent & secant but use $\csc^2 x=1+\cot^2 x$ **Other types** Convert to sine and cosine then apply the rules given

b. Explain and show how to integrate $\int \tan^2 x\sin x\,dx$

7. INTEGRATION BY TRIG SUBSTITUTION (BC)

a. Summarize the type of substitutions will go with what expressions, include key identities and triangles that help.

b. Explain and show how to integrate $\int_{1/\sqrt{3}}^{1} \frac{dx}{x^2\sqrt{1+x^2}}$

8. APPLICATION of DE (AB)

EXPONETIAL relationships come from Proportional Statements

Set up proportion statement as a DE, then solve the equation for each of the following real life applications:

MEDICINE

a. The rate of change of the amount of morphine in the human bloodstream is proportional to the amount of morphine present. Set up a DE equation. If initially there is 0.4 mg of morphine in the bloodstream, and only 0.2 mg remains after 3 hours, find an equation for the amount in the bloodstream at any time.

POPULATION

b. The rate of change of the population of bacteria is proportional to the amount of bacteria present. Set up a DE equation. The count in the culture of bacteria is 200 after 2 hours and 350 after 3 hours, develop an equation of the amount of bacteria at any time.

INTEREST

c. A 18 year graduate of Notre Dame is given \$50 000 which is invested at 5% per year compounded continuously. Assuming the interest rate remains constant, find the equation for the amount of money in terms of years. (Show the steps of developing the formula from the statement: "change is always 5% of output" instead of just using the $y=Pe^{rt}$ formula.)

NEWTON's LAW of COOLING

d. State the Newton's Law of cooling in words, and record the result as a DE. Use the DE to develop a function of temperature of coffee as a function of time if a 90ºC cup of coffee is placed into a 20ºC room. Assume that the coffee is in a Styrofoam cup for which the proportional constant is $k\approx -0.05$, if time is measured in minutes, what is the temperature of the coffee after 10 minutes?

MOTION

e. An arrow is shot upward from the origin with an initial velocity of 90 m/sec. Assume that there is no air resistance and use the DE model: $m\frac{dv}{dt}=-mg$, where m is mass and g is acceleration due to gravity. Find the velocity and position as a function of time. Find ascent time. Find descent time. Find maximum height. Find impact velocity.

MIXING SOLUTIONS *you can read up on this topic and all of the above at* www.mrsk.ca/AP/LessonALLtypesAPPofDE.pdf

f. Consider a tank with volume 600 liters containing a well-mixed salt solution with 40kg of salt initially. Suppose pure water flows into the tank at a rate of 50 liters/min. Solution flows out of the tank at a rate of 25 liters/min. Find volume equation as a function of time, then find amount of salt as a function of time.