Review

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MCV

1.) Find derivative using 1st principles

a) $y = \dfrac{3}{x-1}$

b) $f(x) = \sqrt{x-2}$

2.) Find an equation of the line that is tangent to $f(x) = x^3$ and parallel to the line $3x - y + 1 = 0$

3.) For each of the following, the limit represents $f'(c)$ for a function $f(x)$ and a number $x = c$. Find both f and c.

(a) $\lim_{h \to 0} \dfrac{[5 - 3(1+h)] - 2}{h}$

(b) $\lim_{h \to 0} \dfrac{(-2+h)^3 + 8}{h}$

(c) $\lim_{x \to 6} \dfrac{-x^2 + 36}{x - 6}$

(d) $\lim_{x \to 9} \dfrac{2\sqrt{x} - 6}{x - 9}$

4.) Use alternate form to discuss the differentiability of each of the following at the point.

a) $f(x) = x^3 + 2x$ at $x = 1$

b) $f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$ at $x = 2$

5.) True or False. If false, explain why or give a counterexample.

(a) The slope of the tangent line to the differentiable function f at the point $(2, f(2))$ is $\dfrac{f(2+h) - f(2)}{h}$.

(b) If a function is continuous at a point, then it is differentiable at that point.

(c) If a function has derivatives from both the right and the left at a point, then it is differentiable at that point.

(d) If a function is differentiable at a point, then it is continuous at that point.

6. find the value of k such that the given line is tangent to the graph of the given function.

$f(x) = k\sqrt{x}$, line $y = x + 4$

7. True of False. If False, explain why or provide a counterexample.

a) If $f'(x) = g'(x)$, then $f(x) = g(x)$.

b) If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.

c) If $y = \pi^3$, then $\frac{dy}{dx} = 3\pi^2$.

d) If $f(x) = \frac{1}{x^n}$, then $f'(x) = \frac{1}{nx^{n-1}}$

8. Find the values of a and b ($a \neq 0$) such that f is differentiable everywhere

$$f(x) = \begin{cases} ax^2, & x \le 1 \\ b\sqrt{x}, & x > 1 \end{cases}$$

9. Find the derivative of each. Show all steps, including rewriting and **simplifying**

(a) $f(x) = (6x+5)(x^3-3)$

(b) $h(t) = 2t\sin t + t^2 \cos t$

(c) $f(x) = 2x^2 \cot x$

(d) $y = \csc^3\left(\frac{3x}{2}\right)$

(e) $y = 3\sec^2(\pi t - 1)$

(f) $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x}$

(g) $y = e^{2x^2+2x}$

(h) $y = 6^{2x}$

(i) $y = \sin^2 x + 2^{\sin x}$

(j) $y = xe^2 - e^{x^2}$

10. Evaluate $f'\left(\frac{\pi}{4}\right)$ if $f(x) = \sin x(\sin x + \cos x)$, then find the equation of the tangent line at $x = \frac{\pi}{4}$.

11. Determine the point(s) (coordinates) at which the graph of the function has a horizontal tangent line.

$$f(x) = \frac{x^2}{x^2+1}$$

12. Find the equation of the tangent lines to the graph of $y = \frac{x+1}{x-1}$ that are parallel to the line $2y + x = 6$.

13. If $g(2) = 3$, $g'(2) = -2$, $h(2) = -1$, and $h'(2) = 4$, find $f'(2)$ for

(a) $f(x) = 2g(x) + h(x)$

(b) $f(x) = 4 - h(x)$

(c) $f(x) = \frac{g(x)}{h(x)}$

(d) $f(x) = 2g(x)h(x)$

14. Determine whether there exist any values of x in the interval $[0, 2\pi)$ such that the rate of change of $f(x) = \sec x$ and the rate of change of $g(x) = \csc x$ are equal.

15. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{\sqrt{t}}{2}$, where t is time in seconds and the dimensions are in inches. (Note: $V = \pi r^2 h$)

(a) Find an equation for the volume, $V(t)$, of the right circular cylinder as a function of time.

(b) Find the rate of change of volume with respect to time.

16. Determine the point(s) in the interval $(0, 2\pi)$ at which the graph of $f(x) = 2\cos x + \sin 2x$ has a horizontal tangent.

17. If $h(x) = \tan 2x$, evaluate $h''(x)$ at $\left(\frac{\pi}{6}, \sqrt{3}\right)$

18. Use the fact that $|g(x)| = \sqrt{g^2(x)}$ to prove that $\frac{d}{dx}\left[|g(x)|\right] = \frac{g(x)}{|g(x)|} \cdot g'(x)$, $g(x) \neq 0$. Now use this fact to find $\frac{d}{dx}\left[\left|x^2 - 4\right|\right]$.

19. Find $\frac{d^2y}{dx^2}$ for $y = 2\sin\left(4^{x^2}\right)$

AP

1. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(a) $x^2 + y^2 = 36$ (b) $1 - xy = x - y$

2. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

3. If $y^2 + \cos xy - 4x = 5$, find $\frac{dx}{dy}$, yes, that's $\frac{dx}{dy}$.

4. Find the derivative with respect to the appropriate variable. Simplify your expression

SHOW steps (don't use formulas → won't be provided)

(a) $y = \sec^{-1}\left(x^2\right)$

(b) $y = \cot^{-1}\sqrt{t-1}$

(c) $\frac{d}{dx}\left[e^{2\ln x}\right]$

(d) $\frac{d}{dx}\left[\log_a a^{\sin x}\right]$

e) $\frac{d}{dx}\left[\log_2 8^{x-5}\right]$:

f) $y = \ln\left|\frac{\cos x}{\cos x - 1}\right|$

g) $y = \log_3(1 + x\ln x)$

5. If a particle's position is given by $x(t) = \tan^{-1}(t^2)$, find the particle's velocity at $t = 1$.

6. If $h(x) = \cos x + 3x$, find a) $h(0)$ and b) $\left(h^{-1}\right)'(1)$.

7. Use LOG DIFF:

(a) $\frac{d}{dx}\left[\sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x-5)^3}}\right]$

(b) If $y = x^{1/\ln x}$, find $\frac{dy}{dx}$.

8. Let $f(x) = x^3 + x$. If h is the inverse function of f, the $h'(2) =$

9. Let $f(x) = \sin x$. Find $\frac{d}{dx}\left[f^{-1}\left(\frac{1}{2}\right)\right]$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

10. Let $f(x) = \frac{x^3}{x^2+1}$. Find x if $f^{-1}(x) = 2$

Multiple Choice

1. _____ The points P and Q on the graph of $y^2 - xy + 8 = 0$ have the same x-coordinate, $x = 6$. The point of intersection of the tangents to the graph at P and Q is

(A) $\left(\frac{8}{3}, \frac{16}{3}\right)$ (B) $\left(\frac{16}{3}, \frac{8}{3}\right)$ (C) $\left(\frac{16}{3}, \frac{16}{3}\right)$ (D) $\left(\frac{8}{3}, \frac{8}{3}\right)$ (E) $\left(\frac{8}{3}, \frac{2}{3}\right)$

2. _____ Find the value of $f(1)$ when $f(x) = 5\sin^{-1}x + 6\tan^{-1}x$.

(A) 3π (B) 2π (C) 4π (D) $\frac{7\pi}{2}$ (E) $\frac{5\pi}{2}$

3. _____ Simplify the expression $f(x) = \sin\left(\tan^{-1}x\right)$ by writing it in algebraic form.

(A) $f(x) = \frac{1}{\sqrt{1+x^2}}$ (B) $f(x) = \sqrt{1+x^2}$ (C) $f(x) = \frac{x}{\sqrt{1-x^2}}$ (D) $f(x) = \frac{x}{\sqrt{1+x^2}}$

(E) $f(x) = \frac{1}{\sqrt{1-x^2}}$ (F) $f(x) = \sqrt{1-x^2}$

4. ____ Suppose g is the inverse function of a differentiable function f and $G(x)=\dfrac{1}{g(x)}$. If $f(3)=7$ and $f'(3)=\dfrac{1}{9}$, find $G'(7)$.

(A) -5 (B) 4 (C) 6 (D) -1 (E) -4

5. ____ Find $\dfrac{dy}{dx}$ when $\tan(2x-y)=2x$

(A) $\dfrac{dy}{dx}=\dfrac{8x^2}{1+4x^2}$ (B) $\dfrac{dy}{dx}=-\dfrac{8x^2}{1+4x^2}$ (C) $\dfrac{dy}{dx}=-\dfrac{4y^2}{2+x^2}$ (D) $\dfrac{dy}{dx}=\dfrac{4y^2}{2+x^2}$

(E) $\dfrac{dy}{dx}=-\dfrac{8x^2}{1+4y^2}$ (F) $\dfrac{dy}{dx}=\dfrac{8y^2}{2+x^2}$

6. Use the properties of logs to simplify, as much as possible, the expression:

$$\log_a 32+\frac{4}{5}\log_a 4-\frac{4}{5}\log_a 2+\log_a \frac{1}{2^{\frac{14}{5}}}$$

(A) $\log_a 128$ (B) $\log_a 8$ (C) $\log_a 32$ (D) $\log_a 2^{-7}$ (E) 8

7. Simplify the expression as much as possible: $2^{5(\log_2 e)\ln x}$

(A) 5^x (B) e^{11} (C) x^5 (D) x^{10} (E) x^2

8. Find the derivative of f when $f(x)=x[7\sin(\ln x)+2\cos(\ln x)]$.

(A) $f'(x)=x[5\sin(\ln x)+9\cos(\ln x)]$ (B) $f'(x)=5\sin(\ln x)-9\cos(\ln x)$

(C) $f'(x)=5\sin(\ln x)+9\cos(\ln x)$ (D) $f'(x)=9\sin(\ln x)+5\cos(\ln x)$

(E) $f'(x)=x[9\sin(\ln x)+5\cos(\ln x)]$