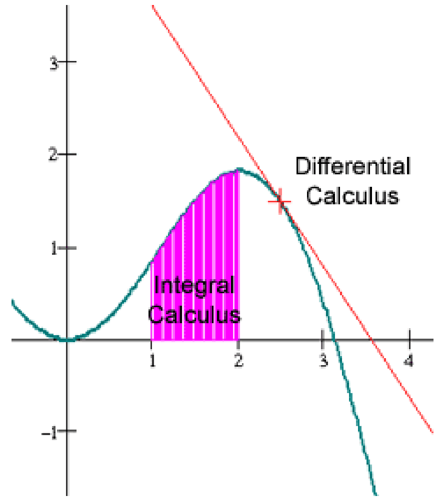
Big idea



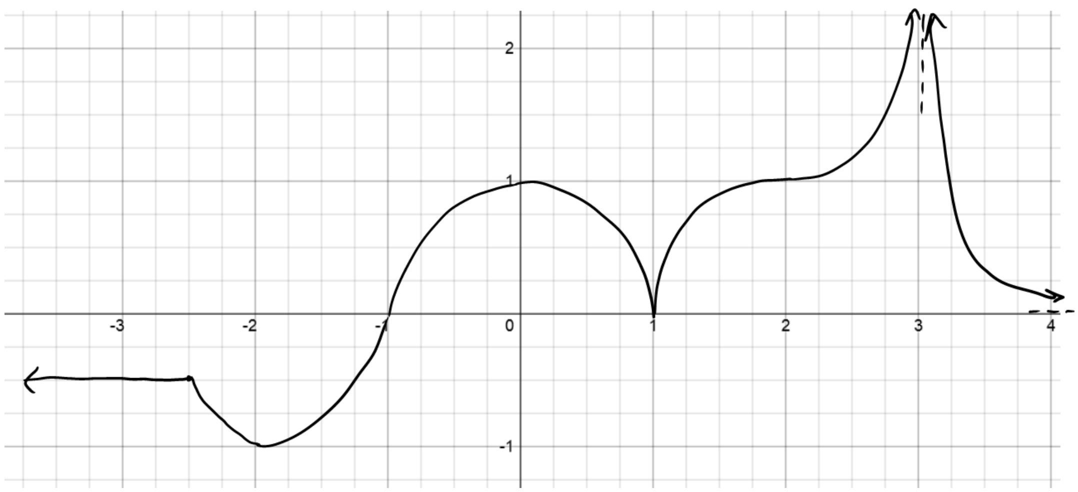
Calculus is an entire branch of mathematics. Calculus is built on two major complementary ideas. The first is differential calculus, which is concerned with the instantaneous rate of change. This can be illustrated by the slope of a tangent to a function's graph. The second is integral calculus, which studies the areas under a curve. These two processes act inversely to each other. The development of the mathematical methods of calculus has been credited to two great mathematicians; Sir Isaac Newton (1642 –1727) and Gottfried Wilhelm von Leibniz (1646 –1716). Although others as far back as 200B.C. had been working on solutions to these types of problems, Newton and Leibniz developed the process of differentiation and integration. Calculus allows you to find optimal solutions to mathematical expressions and is used in medicine, engineering, economics, computer science, business, physical sciences, statistics, and many more areas.

Feedback & Assessment of Your Success

Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: ________________ Questions to ask the teacher:
2days	2-6	Derivative Sketches & Definition of a Derivative (MCV) Journal #1				
	7-8	Basic Derivatives (PowerRule)(MCV) Journal #2				
	9-11	Differentiability (MCV) Journal #3				
1.5days	12-14	Chain Rule (MCV) Journal #4				
1.5days	15-17	Product & Quotient Rules (MCV) Journal #5				
	18-19	Derivatives of Trig (MCV) Journal #6				
1.5days	20-22	Implicit Differentiation (AP) Journal #7				
2.5days	23-26	Derivatives of Exponentials & Logarithms (MCV) Journal #8				
2days	27-29	Derivatives of Inverse Functions and Inverse Trig (AP) Journal #9				
		Extra Review Day?				

ASSIGNMENT Derivative Sketches & Definition of a Derivative (MCV)

1. For the graph below sketch the derivative in the space above and a possible antiderivative in the space below

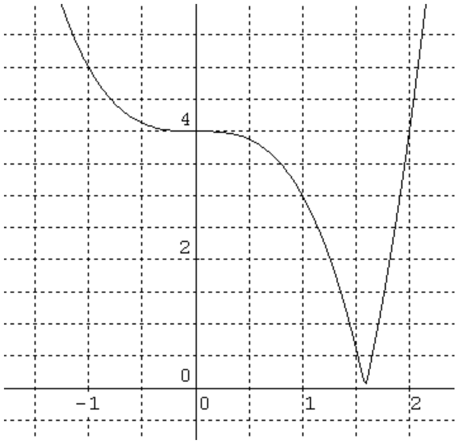


$f(x)$	$f'(x)$
• MAX Turning point	• +, Zero, –
• MIN Turning point	• – , Zero, +
• Inflection point (but not VT or Saddle pt)	• Turning point NOT AT ZERO
• Saddle point (is a Pt. of Inf. too)	• Turning point or cusp AT ZERO – , Zero, – *or* + , Zero, +
• Vertical tangent (is a Pt. of Inf. too)	• VA with even symmetry
• Vertical Cusp (is a MAX/MIN pt too)	• VA with odd symmetry
• Sharp point but not with ∞ slopes	• Hole with Jump discontinuity
• Inc (slope is pos)	• Pos (above x-axis)
• Dec (slope is neg)	• Neg (below x-axis)
• CU	• Inc (slope is positive)
• CD	• Dec (slope is negative)
• VA	• Stays VA
• HA at any constant	• Becomes HA at y=0
• Flat graph at any constant	• Becomes flat graph at y=0
• If graph resembles x^n polyn	• Will resemble x^{n-1} polyn

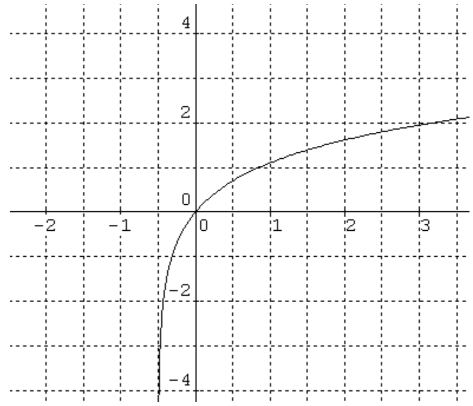
3 Calculus AP U2 – Derivatives (MCV) Name: _______________

Sketch the derivative functions

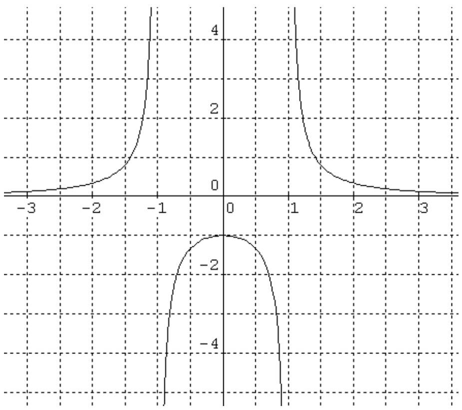
2.



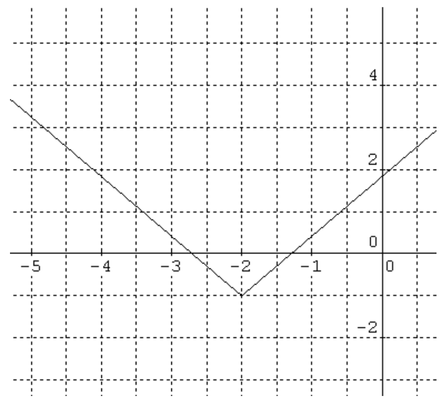
3.



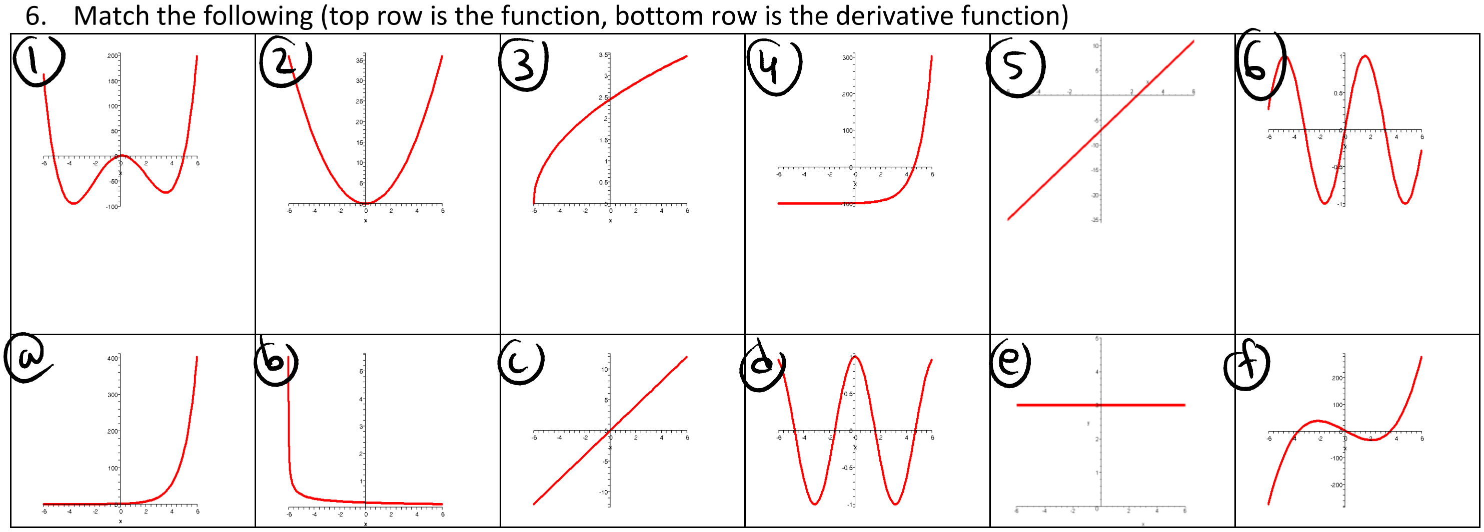
4.



5.

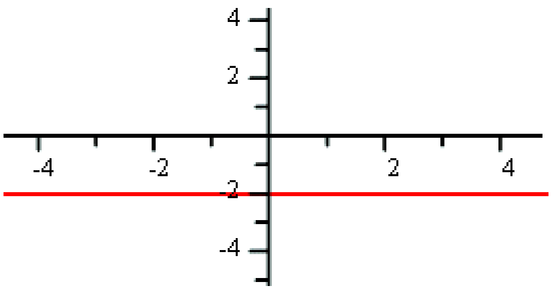


6. Match the following (top row is the function, bottom row is the derivative function)

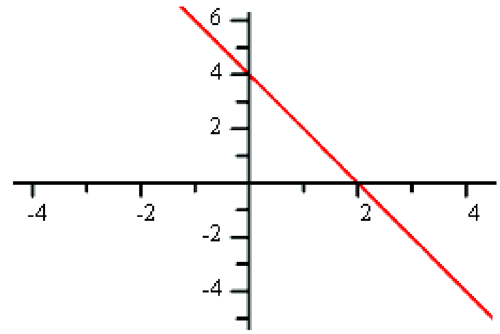


Use the graphs of the first derivative $f'(x)$ to sketch a possible function $f(x)$

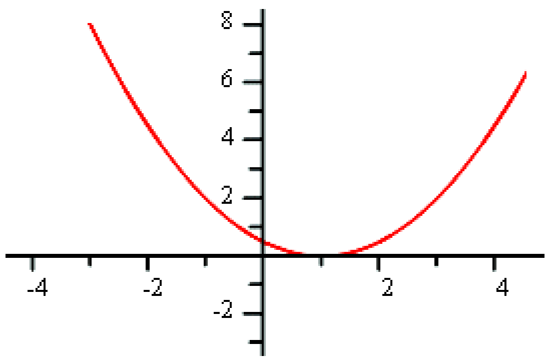
7.



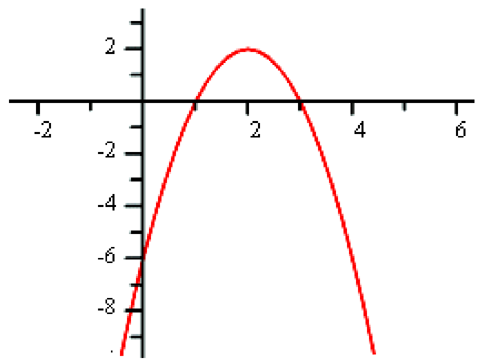
8.



9.



10.



Determine the derivatives of the following, from first principles

11. $f(x) = \sqrt[3]{x}$ **using alternate definition**

12. $f(x) = \dfrac{1}{x^2}$ **using direct definition**

13. Find the slope of $y = \frac{3}{x}$ at point (3, 1) **using alternate definition**

14. Find the slope of the tangent to $y = \sqrt{x-2}$ at x=6 **using direct definition**

15. If $f(x) = \frac{5}{x-2}$, find $f'(x)$ **using alternate definition** and its domain. What is $f'(3)$

16. If the position of a particle is given by $s(t) = t^2$ (measured in metres), find the instantaneous velocity at $t = 1$, where *t* is time measured in seconds **using direct definition**

17. Find the equation of the normal line to $y=\sqrt{x^2-9}$ at x=5 **using alternate definition**

18. Determine the equation of the tangent to $y=\frac{-4}{x^2+1}$ at point (1, -2) **using direct definition**

19. If $f(a)=0$ and $f'(a)=4$, find $\lim\limits_{h\to 0}\frac{f(a+h)}{3h}$

ASSIGNMENT Basic Derivatives (Power Rule) (MCV)

Use the derivative rules to find the derivative

1. $f(x)=5x^8-9x+\dfrac{8}{x^6}+\sqrt[4]{x^7}+3x^{\pi}-\sqrt{2}$

2. $f(x)=x^{57}-6x^{14}-\dfrac{2}{x^3}+\sqrt[5]{x^2}+3$

3. Rewrite the Leibniz notation with prime notation and find the following to see the importance of knowing what variable you're taking the derivative 'with respect to'. $f=\dfrac{a^3x^2}{z}$ $\quad g=x^2$ $\quad p=t^3+2t$

a) $\dfrac{df}{dx}$ b) $\dfrac{df}{dy}$ c) $\dfrac{df}{da}$ d) $\dfrac{df}{dz}$

e) $\dfrac{dg}{dx}$ f) $\dfrac{dg}{dt}$ g) $\dfrac{dp}{dx}$ h) $\dfrac{dp}{dt}$

4. For $y=\sqrt{x}+4x$ find $y(1), y'(1), y''(1)$ and interpret what they mean in terms of graph. If y was displacement what do the values mean?

5. Find the derivatives

a) $y=5x^6-4x^3+6$

b) $f(x)=-3x^5+8\sqrt{x}-\dfrac{9}{x}$

6. $g(x)=(2x-3)(x+1)$ Find $g'(x)$

7. $h(x)=\dfrac{-8x^6+8x^2}{4x^5}$ Find $\dfrac{d}{dx}h$

8. Determine the equation of the tangent to the curve $f(x) = 4x^3 + 3x^2 - 5$ at $x = -1$

9. Determine the point(s) on the graph of $y = x^2(x+3)$ where the slope of the tangent is 24.

10. A ball is dropped from a height of 800 feet and its height after follows the model: $s(t) = -16t^2 + 800$ where s(t) is height and t is time

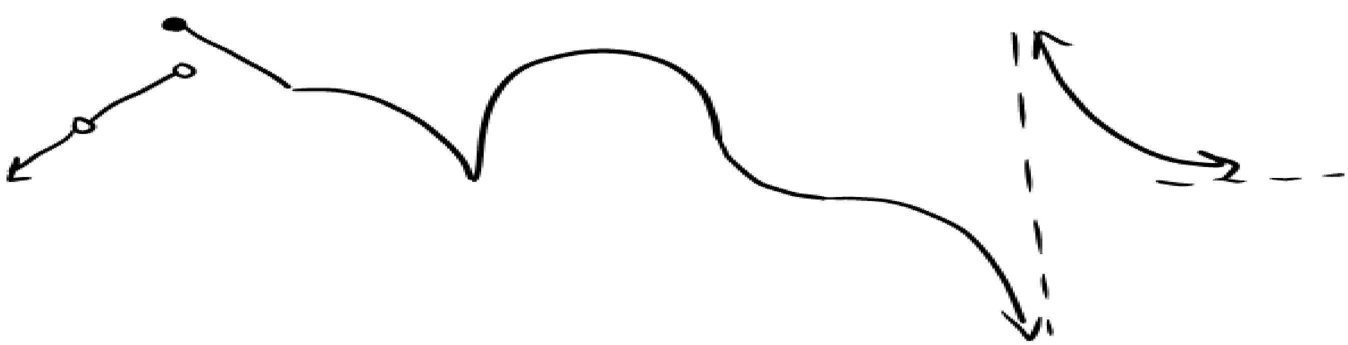
a) Find how fast the ball is falling at t=3 seconds.
b) At what time does it hit the ground?
c) What speed is it traveling at when it hits the ground?

ASSIGNMENT Differentiability (MCV)

1. Find the derivatives, then using limits show that these are not differentiable at x=0

a) $y=\sqrt[3]{x}$	b) $y=\sqrt[3]{x^2}$	c) $y=\lvert x\rvert$	d) $y=\frac{1}{x}$

2. Sketch the derivative of the following.



3. Find the values of a and b that make $g(x)$ differentiable everywhere

$$g(x)=\begin{cases} 5x+a; \; x<2 \\ x^2-bx+3; \; x\geq 2 \end{cases}$$

4. Find the constants so that the function is differentiable.

$$f(x)=\begin{cases} 3ax+b & x<1 \\ x^2+2x+5 & x\geq 1 \end{cases}$$

5. Connect smoothly the point (0,1) and ray $y = -2x + 10$ that starts at point (3, 4)

6. Create a third piece so that the function is continuous and differentiable everywhere.

$$f(x) = \begin{cases} x^3 & x < 2 \\ -x^2 + 18 & x > 3 \end{cases}$$

ASSIGNMENT Chain Rule (MCV)

1. For $u(x)=4x^2-3x$ and $p(u)=\sqrt{u}$.
   a) Find $p(u(x))$ composition
   b) Use the chain rule to find $\frac{dp}{dx}$
   c) Look at part a) and your answer to b) and record the key words to say to yourself in order to find $\frac{dp}{dx}$ directly

2. Differentiate
   a) $y=\frac{1}{x^3+5x}$
   b) $y=\sqrt{x^2+3}$
   c) $y=(x^3-4)^5$
   d) $y=(4x^3-2x^2+x-10)^5$
   e) $y=\frac{1}{\sqrt[3]{6-5x^3}}$

3. If $y=\sqrt[4]{u}$ and $u=1+2x+x^3$ determine $\left.\frac{dy}{dx}\right|_{x=1}$

4. If $a=b(2-b^2)$ and $b=\frac{1}{c}$ determine $\left.\frac{da}{dc}\right|_{c=2}$

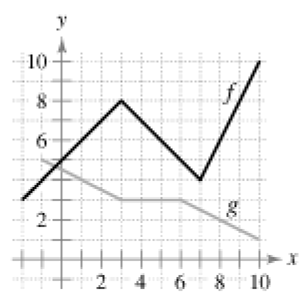
5. If $y=-\sqrt{u}$ and $u=4x^3-3x^2+1$ determine $\left.\dfrac{dy}{dx}\right|_{x=0}$

6. If $F(x)=f(g(x))$, where $f(-2)=8$, $f'(-2)=4$, $f'(5)=3$, $g(5)=-2$, and $g'(5)=6$, find $F'(5)$.

7. $f(x)=-x^5+15x$, Find $h'(-2)$ if $h(x)=f(g(x))$, $g(-2)=3$, and $g'(-2)=-6$

8. The graph of f and g is shown. Let $h(x)=f(g(x))$ and $s(x)=g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

(a) Find $h'(1)$. (b) Find $s'(5)$.



9. If $x=\sqrt{t}$ and $y=t^3+5$ find $\left.\frac{dy}{dx}\right|_{t=4}$

10. $y=\frac{3}{2x^3-8x}$. Sketch the rational function using the algorithm you learned before, label all intercepts, asymptotes, find and label turning points

11. $f(x)=\left[\left(\left[3x^2+x\right]^{-1}-5\right)^3+10\right]^6$ differentiate

ASSIGNMENT Product & Quotient Rules (MCV)

Differentiate and simplify if you can common factor

1. $y=(3x+1)(8x^4+5x)$

2. $y=(x^3+4x^2-6x+1)(-3x^2+9x-2)$

3. $y=(x^3+5x)(x^2-3)^4$

4. $y=(x^2-3x)^5(3-2x)^4$

5. $y=3x\sqrt{x^2-1}$

6. $y=\dfrac{\sqrt{x}}{x^2+1}$ then find the points where there's a horizontal tangent

Differentiate and simplify if you can common factor

7. $y=\dfrac{6x-5}{x^3+4}$

8. $y=\dfrac{x^2}{\sqrt{x^2+1}}$

9. $y=\left(\dfrac{1+x^3}{2x-x^2}\right)^7$

10. $y=\dfrac{5x-3}{x^2+4x}$ using product rule

Differentiate

11. $y=\dfrac{3x^2}{7}$ and $y=\dfrac{x^2+3}{x}$ look for shortcuts!

12. $y=(x^2+1)(1-x)(2x-x^2)$

13. Determine the x value when $\dfrac{dy}{dx}=0$ for

$$y=\frac{(5x+4)^2}{(3x^2-6)^3}$$

14. Suppose the function $V(t)=\dfrac{50000+6t}{1+0.4t}$ represents the value, in dollars, of a new car *t* years after it is purchased.

a. What is the rate of change of the value of the car at 2years? 5years? 7years?
b. What is the initial value of the car?
c. Explain how the values in a. can be used to support an argument in favour of purchasing a used car, rather than a new one.

ASSIGNMENT Derivatives of Trig (MCV)

1. Sketch the derivatives of the sine, cosine and tangent. (To verify the trig derivatives given in journal.)

Differentiate

2. $f(x) = x^5 \sin x - 3\cos x + 10x$

3. $f(x) = \dfrac{1+\sin x}{x+\cos x}$ and simplify

4. $y = (\sin x - 5x^3)^6 + \cos x$

5. $y = \tan^2(x^2)$

6. $y = \sin(\cos(x^2))$

7. $y = \sin^2(\cos(x))$

Differentiate

8. $$y=\left(\frac{5x-2}{\sin^2 x+10x}\right)$$

9. $$f(x)=x^2\tan x-\frac{1}{\sec x+1}$$

10. $$y=\left[\cos\left(x+\frac{1}{\sqrt{x+3}}\right)+5x\right]^2-6x$$

11. Find y' in each case, where A, B, m, and n are constants.

A). $y=\cos(Ax+B)$

B). $y=A\cos^n(Bx)$

C). $y=\sin^m(x^n)$

12. Is there a value of b that will make

$$g(x)=\begin{cases} x+b, & x<0 \\ \cos(x), & x\geq 0 \end{cases}$$

continuous at $x=0$? Differentiable at $x=0$?

13. On a sunny day, a 20 m tall flagpole casts a shadow that changes with the angle of elevation of the Sun. The length of the shadow *s* is related to the angle of elevation of the Sun, θ, by the formula $s=20\cot\theta$. Find the rate at which the length of the shadow is changing with respect to θ when θ is 45°.

ASSIGNMENT Implicit Differentiation (AP)

1. Up to now, we have worked explicitly, solving an equation for one variable in terms of another. For example, if you were asked to find $\frac{dy}{dx}$ for the **implicit** equation (where y is not isolated) $2x^2 + y^2 = 4$, you would
   a) solve for y and get an **explicit** equation:

   b) and then take the derivative:

2. Sometimes it is difficult or impossible to solve for y. In which case, we use implicit differentiation. You assume y is defined in terms of x, ie. every time you see y, think x is hidden inside of it. Thus, you must apply the ________________ rule.

   Notice what you must do when variables agree versus when they do not.

   $\frac{d}{dx}\left[x^3\right]=$

   $\frac{d}{dx}\left[y^3\right]=$

use Leibniz notation on left

3. Find the derivative of the relation in 1. implicitly. $2x^2 + y^2 = 4$

Use prime notation on right

4. $y^3+y^2-6y+x^2=-8$ find $\frac{dy}{dx}$

5. $\sin x+\sin(2y)=1$ Find $\frac{dy}{dx}$

6. Find $\frac{dy}{dx}$ for $\cos(xy)=x$

7. $xy=3x^2+1$ find $\frac{dy}{dx}$

8. $x^2y+2xy^2=4$ find $\frac{d^2y}{dx^2}$

9. Find $\frac{dy}{dx}$ from $\sqrt{x+y} = 1 + x^2 y^2$

10. Find $\frac{dy}{dx}$ from $x^4(x + y) = y^2(3x - y)$

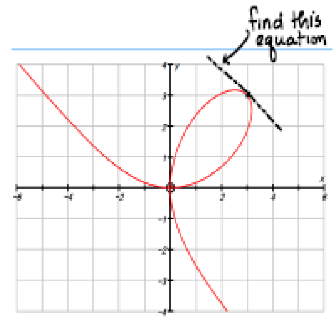
11. $-4a^3 + 6b + a^2 b = 7$ where $a(b)$ Differentiate with respect to b:

12. $4t^5 + y - 2y^{-3} - 2ty + t^4 y^2 - 9 = 0$ where $y(t)$ Differentiate with respect to t

13. Find $\frac{dy}{dx}$ from $x^2 + y^2 = 3x^3 + y$ at the point (1, 2)

14. Find $\frac{dy}{dx}$ from $x^{2/3} + y^{2/3} = 4$ at the point $\left(-3\sqrt{3}, 1\right)$

15. ‘Folium of Descartes’ $x^3 + y^3 = 6xy$ graph is shown.
Find the tangent line at the point (3, 3).



16. At a certain factory, approximately $q(t) = t^3 - \dfrac{2}{\sqrt{t}}$ units are manufactured during the first *t* hours of a production run, and it is estimated that the total cost of producing *q* units is $C(q) = 300q + 0.2\sqrt{q} + \dfrac{20}{q}$ dollars. Find the rate at which the cost is changing with respect to time 4 hours after production commences.

ASSIGNMENT Derivatives of Exponentials & Logarithms (MCV)

Differentiate

1. a) $y = e^{5x}$

   b) $s = 3^t$

   c) $y = 10e^{3x^2}$

   d) $s = 8.10^{1/t}$

2. $y = e^{x^2+1}\sqrt{5x+2}$ and simplify

3. $y = (x^6 6^x + 4x)^2 + 4^x$

4. $y = \sqrt{x\tan 5x - 3x} + e^{x^2-x}$

5. $f(x) = 3e^x - 17x$ then find HT points

6. $y = 2^{x^5}$ then find HT points

7. Find $\dfrac{d}{dx}5x^2e^x$, then find HT points

8. Find $\dfrac{d}{dx}\dfrac{5}{4e^x+3x}$

9. Discuss the effect of absolute value on the log
Draw $y=\ln x$ and $y=\ln|x|$, then draw the derivative graphs and state the rule in general.

10. $y=\log_5|3x|$

11. $y=\ln(\cos x)$

12. $y=\log_{\frac{1}{2}}(x^2)$

13. $y=\ln(\ln x^3)$

14. $f(x)=x^2\log x$ then find the points where there's a horizontal tangent

15. $f(x)=\log(x+\sqrt{4+x^2})$

16. $f(x)=\ln\dfrac{x}{\sqrt{x^2+2}}$

17. $f(x)=\ln\left|\dfrac{\sin x}{\sin x+1}\right|$

18. If $x^2 \sin y = (y^3 - e^y)$, find y'.

19. T/F?

a) $\ln(x+25) = \ln x + \ln 25$

b) $\ln\left(\dfrac{x+4}{\sqrt{x^2+1}}\right) = \dfrac{\ln(x+4)}{0.5\ln(x^2+1)}$

c) If $y = \ln \pi$ then $y' = 1/\pi$

d) If $y = \ln(3x)$ then $y' = \dfrac{1}{3x}$

Using logarithmic differentiation

20. When to use?

steps:

21. $\dfrac{d}{dx}(x^2+1)^{\sin x}$

22. $\dfrac{d}{dx}(x^3+1)^{\ln x}$

23. $\dfrac{d}{dx}x(3x)^{x^2}$

Differentiate (faster if you use what you just learned)

24. $$y=\frac{(3x+4)^5(2x-1)^{\frac{1}{2}}}{(2x+1)^{14}}$$

25. $$y=\frac{\sqrt[3]{2x-1}(3x+2)}{(x-5)^4}$$

26. The motion of a spring subject to a damping force is described by $f(t)=e^{-2t}\sin 3t$. What is the velocity of the spring at time *t*?

27. Based on data published in 1990 by the International Panel on Climate Change, if the trends of burning fossil fuel and deforestation were to continue, the atmospheric carbon dioxide level in parts per million would be approximated by $C(x)=353(1.0061)^{x-1990}$ where *x* is the year. Find $C'(2007)$ and interpret its meaning.

ASSIGNMENT Derivatives of Inverse Functions and Inverse Trig (AP)

1. Review how to simplify

a) $\sin(\arccos(\frac{-5}{13}))$

b) $\arcsin(\cos\frac{7\pi}{6})$

c) $\sec(\arctan(4x))$

d) $\lim_{x\to\infty}\csc^{-1}x$

e) $\tan(\sin^{-1}\frac{3}{5})$

f) $\arccos(\cos\frac{5\pi}{4})$

g) $\cot(\arcsin(x+2))$

h) $\lim_{x\to -1^+}\cos^{-1}x$

2. Recap how to find the inverse for

$$y=\frac{x}{x-4}$$

3. Sometimes finding the inverse algebraically is too hard like for $f(x)=x^5+x^3+1$. Find the slope of the inverse at 3. Suppose you were told that point on f is (1, 3)

3a) Find a derivative of the inverse of $y=f(x)=2x^3+5x+2$ at $y=9$.

3b) Let $f(x)=9^x$ Find $g'\left(\frac{1}{3}\right)$ where g is inverse of f

Differentiate, show the derivation steps here – this skill is needed later.

4. $y = \sin^{-1}(3x^2 - 4)$

5. $y = \sec^{-1}(x^2)$

6. $y = x\sin^{-1} x$

7. $y = \ln\left(\tan^{-1} x\right)$

8. $y = \cos^{-1}\dfrac{1}{x}$

9. $y = \csc^{-1}\left(e^t\right)$

For these questions, go ahead and use the formula shortcuts

10. A toy car moves along a track so that its horizontal position in meteres for $0 \le t \le 10$ min is given by $x(t) = \tan^{-1}\sqrt{3t+1}$. What is the velocity of the car when t=5min?

11. Find the equation for the line tangent to the graph of $y = \cot^{-1}(x)$ at x = – 1.

12. A particle moves along the x-axis so that its position at any time is given by $x = \arctan(t)$.
    a) Prove that the particle is always moving to the right
    b) Prove that the particle is always decelerating
    c) What is the limiting position of the particle as time goes on?

13. Find all points where $y = \arcsin(\frac{x}{x^2+1})$ has a horizontal tangent.