Matrices

A matrix is a method of writing a set of numbers using rows and columns.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 \\ 1 & -5 \\ 7 & 2 \\ 5 & 4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & -5 & 10 \\ 12 & 8 & 4 & 9 \\ 25 & -30 & 1 & -1 \end{bmatrix}$$

Reading Information from a Matrix

- Cells in a matrix can be referenced in the form ________________________________.

$$\begin{bmatrix} 3 & -4 & 6 & 5 \\ 7 & 2 & 0 & 1 \\ 5 & 9 & 13 & -2 \\ -10 & -1 & 4 & 3 \end{bmatrix}$$

State the value for each of the following

(2, 3) = (1, 4) =

State the location of each of the following

4 –1 = 1 =

Matrix Addition & Subtraction

- To add matrices, they must be ____.

$$\begin{bmatrix} 2 & 1 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 12 \\ 5 & -2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} 10 & 3 & 2 \\ -5 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 & 14 \end{bmatrix}$$

Scalar Multiplication & Division

$$5 \times \begin{bmatrix} 2 & -7 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 6 & -8 & 24 \\ 18 & 22 & -4 \end{bmatrix} \div 2$$

Matrix Multiplication

- The number of ___________________ in the 1st matrix must equal the number of __________ in the 2nd matrix.

$$\begin{bmatrix} 3 & 5 & 1 \\ 2 & 8 & 4 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 & 5 & 1 \\ 2 & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 8 \\ 3 & 4 \\ 1 & 0 \\ 6 & 5 \end{bmatrix} \times \begin{bmatrix} 10 & 3 \\ 2 & 5 \end{bmatrix}$$

Zero Matrix

- All entries in the matrix are ___________________.

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ -1 \\ 3 \end{bmatrix}$$

Identity Matrix

- When multiplying by the identity matrix, the original matrix ___________________________.

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \qquad \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2 & 4\\3 & 5\end{bmatrix} \times \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Inverse Matrix

- When a matrix is multiplied by the inverse, the result is an identity matrix

Determine which of the following is the inverse of the matrix $\begin{bmatrix}3 & 8\\1 & 3\end{bmatrix}$:

$$\begin{bmatrix}8 & 3\\3 & 1\end{bmatrix} \qquad \begin{bmatrix}3 & -8\\-1 & 3\end{bmatrix} \qquad \begin{bmatrix}2 & 8\\1 & 2\end{bmatrix}$$

Gauss Jordan Elimination

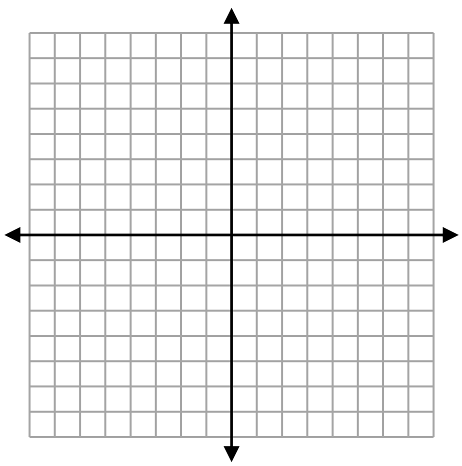
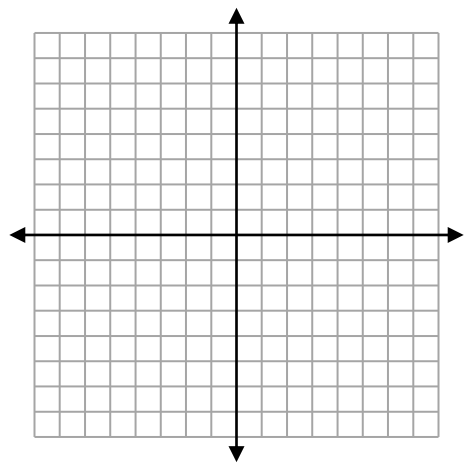
- You can multiply/divide the rows of an equation matrix by a constant
- You can add/subtract rows to eliminate entries

Find the inverse of the following matrices:

$$\begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 3 \\ 4 & 1 \end{bmatrix}$$

Equations of Lines in 2-Dimensions

Type	Scalar/Cartesian	Vector
Format		
Example	Find the equation of the line that has a slope of -3 and a y-intercept of 2. 	Find the equation of the line that passes through the point (-1, 5) and has the same direction as the vector $\vec{a}$ = (2, -6). 

Type	Parametric	Symmetric
Format		
Example	Find the equation of the line that passes through the point (-1, 5) and has the same direction as the vector $\vec{a}$ = (2, -6).	Find the equation of the line that passes through the point (-1, 5) and has the same direction as the vector $\vec{a}$ = (2, -6).

State the **direction vector** and **one point** on each of the following lines.

Equation	$\vec{r} = (2,5) + t(-3,1)$	$x = 5 + 3t$ $y = 10 - 2t$	$\frac{x-3}{7} = \frac{y+5}{3}$	2x + 3y = 12
Direction Vector				
Point				

Find the **direction vector** for each of the following:

a) A line passing through (4, 5) and (-2, 1)

b) A vertical line

c) A horizontal line

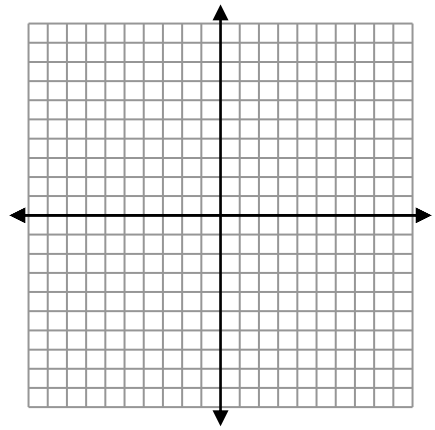
d) A line parallel to y = 4x – 2

e) A line perpendicular to y = 4x – 2

f) A line with a normal vector $\vec{N} = (2,7)$

Does the point (1, -7) lie on the line $\vec{r}$ = (3, 6) + t(2, -5)?

Find the scalar, vector, parametric, and symmetric equations of the line that passes through the points A (3, 0) and B (9, -4).



Determine whether the following vectors are parallel, perpendicular, or neither:

$x_1 = 6 + 2t$ $\qquad$ $x_2 = 10 + 9s$

$y_1 = 10 - 3t$ $\qquad$ $y_2 = -2 + 6s$

Equations of Lines in 3 Dimensions

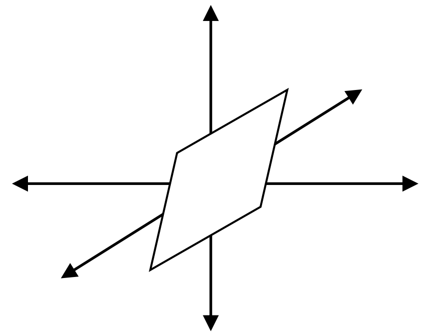
Scalar	Vector
Parametric	**Symmetric**

1. Find the vector, parametric, and symmetric equations of the line that passes through (2, 1, -3) and (5, -7, 4).

2. Find the parametric equation of the line that passes through (8, -10, 5) and is parallel to the line $\vec{r} = (5, -7, 4) + t(2, 1, -1)$.

3. Find the symmetric equation of the line that passes through (2, -1, 5) and has a direction vector of (12, 4, -1).

Vector Equation of a Plane



A plane is a flat surface that extends infinitely in all directions.

Vector Equation of a Plane

To find the equation of a plane we need:

- ______________________________
- ______________________________

Write the vector, parametric and symmetric equations of the plane that contains the lines $\vec{r} = (3,10,-4) + t(4,0,1)$ and $\vec{r} = (-6,6,7) + t(11,-3,-2)$.

Write the vector and parametric equations of the plane that passes through the point A(2, 5, 3) and contains the line $\vec{r} = (-2, 4, 7) + t(1, 4, -5)$.

Find the vector equation of a plane that passes through the points A(1, 2, 1), B(4, 5, 1) and C(4, 0, 1).

Scalar Equations of Planes

$$\mathbf{0 = Ax + By + Cz + D}$$

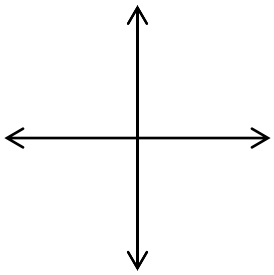
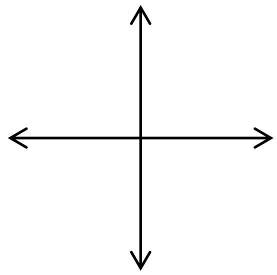
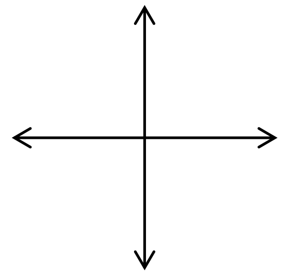
Normal = (A, B, C)

Find the scalar equation of the plane $\vec{r} = (2, -1, 7) + t(3, 1, 4) + s(1, 0, -4)$.

Find the scalar equation of the plane that passes through the points A(5, 14, -3), B(2, 1, -2), C(0, 4, -1).

Intersection of 2 Lines (2D)

Types of Intersection/Solutions

Type			
Graph			
Description			
Number of Solutions			
Scalar Equations			
Vector Equations			
Solving by Sub or Elim			
Solving by Matrices			
Solving by Vectors			

Scalar Equations

- To determine the intersection point of two lines, you can solve by **substitution, elimination** or **matrices.**

Solve: $3x + 2y + 5 = 0$
$x - y - 10 = 0$

Method 1: Substitution

Method 2: Elimination

Method 3: Matrices

Vector Equations

Solve: $\vec{r} = (2, 3) + t(-1, 4)$

$\vec{r} = (-10, 7) + s(9, 8)$

Intersection of a Line and a Plane

Determine the intersection of the following lines with the plane $4x + 3y - 2z - 5 = 0$.

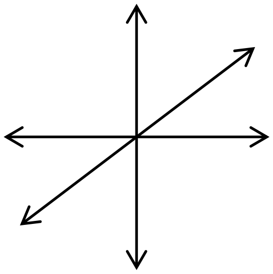
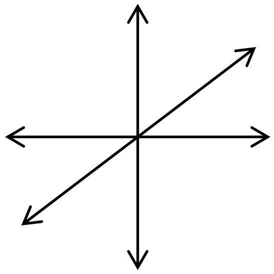
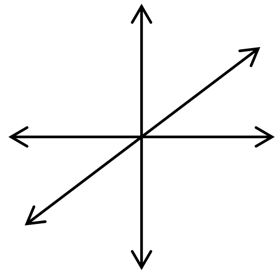
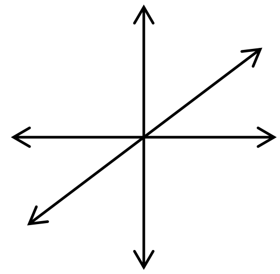
a) $\vec{r} = (5, 1, -3) + t(2, -4, 1)$

b) $\vec{r} = (-2, 3, 1) + t(-3, 4, 0)$

c) $\vec{r} = (1, 3, 4) + t(-1, -2, -5)$

Intersection of 2 Lines (3D)

<u>Types of Intersection/Solutions</u>

Type				
Graph				
Description				
Number of Solutions				
Vector Equations				
Solving				

Solve: $\vec{r} = (2, 4, -1) + t(2, 1, -1)$

$\vec{r} = (4, 5, 7) + s(-2, -1, 1)$

Solve: $\vec{r} = (10, -3, 1) + t(1, 1, -1)$

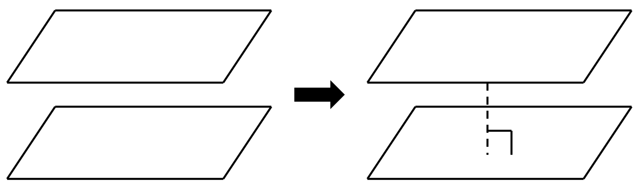
$\vec{r} = (7, -6, 4) + s(2, 2, -2)$

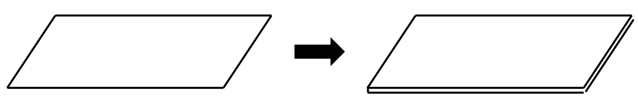
Solve: $\vec{r} = (4, 2, -1) + t(3, 0, 1)$

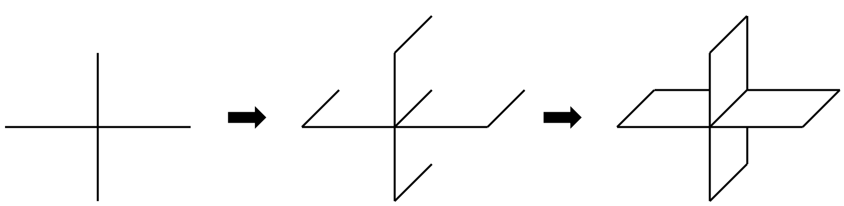
$\vec{r} = (-3, 3, -7) + s(1, -1, 4)$

Solve: $\frac{x-1}{4} = \frac{y-3}{3} = \frac{z-3}{1}$ $\qquad$ $\frac{x-2}{3} = \frac{y-2}{3} = \frac{z-10}{2}$

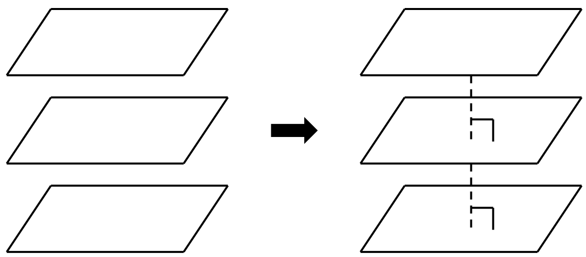
Intersection of Planes – 2 Planes

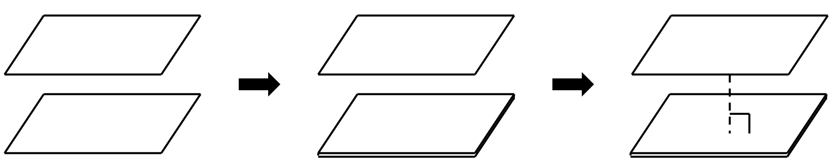
Type	Solution	Description
How To Draw		You Try
		

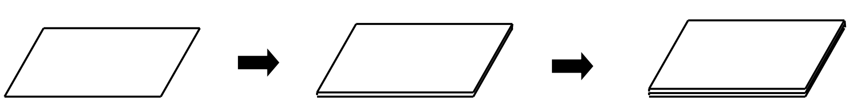
Type	Solution	Description
How To Draw		You Try
		

Type	Solution	Description
How To Draw		You Try
		

Intersection of Planes – 3 Planes

Type	Solution	Description
How To Draw		You Try
		

Type	Solution	Description
How To Draw		You Try
		

Type	Solution	Description
How To Draw		You Try
		

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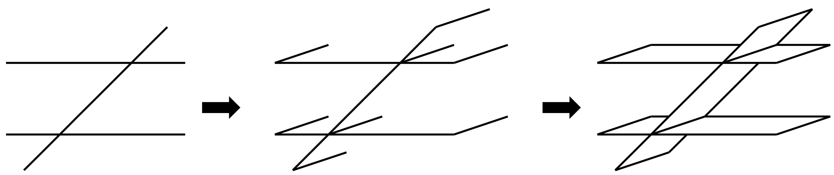</td><td>You Try</td></tr>
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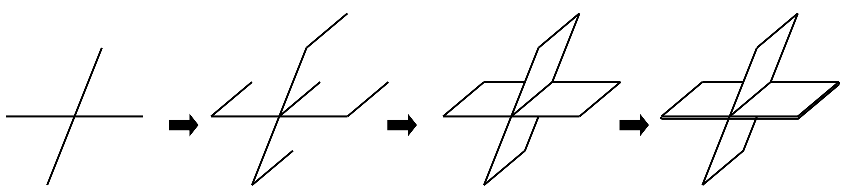</td><td>You Try</td></tr>
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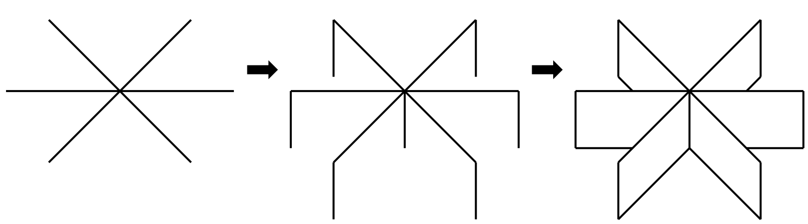</td><td>You Try</td></tr>
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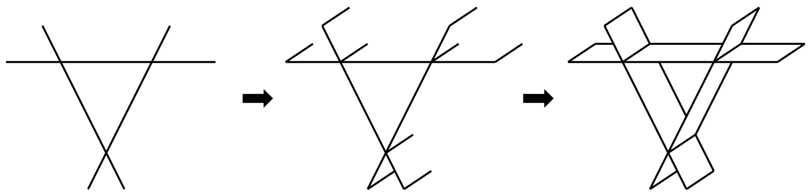</td><td>You Try</td></tr>
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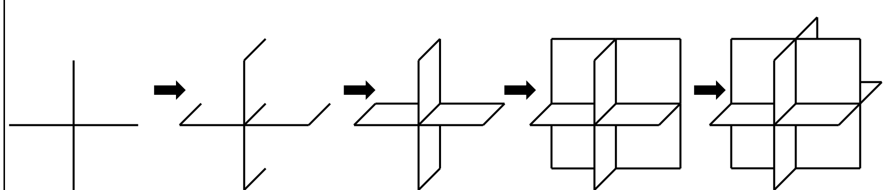</td><td>You Try</td></tr>
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Configurations of Planes

To determine whether or not planes intersect, they need to be written in ___________________ form.

Helpful Information		
<u>Parallel Planes</u>	<u>Coincident Planes</u>	<u>Intersecting Planes</u>
The ___________________ are the same or scalar multiples	The ___________________ are the same or scalar multiples	The ___________________ are different.
$2x + 5y - 3z + 10 = 0$	$2x + 5y - 3z + 10 = 0$	$2x + 5y - 3z + 10 = 0$

Determine the configuration of each of the following sets of planes and state the solution. Draw a geometric representation of each.

a) $4x - 2y + z - 3 = 0$
$8x - 4y + 2z - 3 = 0$

b) $10x - 6y + 4z - 8 = 0$
$15x - 9y + 6z - 12 = 0$

c) $3x + 5y - 2z + 1 = 0$
$2x + 5y + z - 6 = 0$

d) $2x + 3y + 5z - 4 = 0$
$4x + 6y + 10z + 3 = 0$
$10x + 15y + 25z - 7 = 0$

e) $5x + 3y - 4z + 2 = 0$
$5x + 3y - 4z + 10 = 0$
$35x + 21y - 28z + 14 = 0$

f) $x - y + 7z - 3 = 0$
$2x - 2y + 14z - 6 = 0$
$10x - 10y + 70z - 30 = 0$

g) $9x - 3y + 12z + 6 = 0$
$x - y + 3z + 2 = 0$
$15x - 5y + 20z - 10 = 0$

h) $x - 7y - 10z + 9 = 0$
$3x + 4y - z + 4 = 0$
$18x + 24y - 6z + 24 = 0$

Solving For The Intersection of 2 & 3 Planes

Solving means finding the ________________, ________________, or ________________ where two or more planes intersect.

1. Determine the **point** of intersection of the following planes. Include a geometric representation.

   $3x + y + 4z + 3 = 0$
   $2x - 5y + 3z + 13 = 0$
   $5x + 3y - 2z + 11 = 0$

2. Determine the equation of the **plane** of intersection of the following planes. Include a geometric representation.

   $2x - 7y + 4z - 3 = 0$
   $4x - 14y + 8z - 6 = 0$
   $10x - 35y + 20z - 15 = 0$

3. Determine the equation of the **line** of intersection of the following planes. Include a geometric representation.

   $2x + 10y - 3z + 1 = 0$
   $2x - 8y + 3z - 5 = 0$

4. Determine the equation of the **line** of intersection of the following planes. Include a geometric representation.

$$x + 4y + 3z - 6 = 0$$
$$3x + 2y - z + 2 = 0$$
$$3x + 14y + 11z - 22 = 0$$

5. Solve the following. Include a geometric representation.

$$10x - 3y - 7z + 18 = 0$$
$$12x - 11y - z + 66 = 0$$
$$22x - 14y - 8z + 158 = 0$$

Plane Practice

Solve for the intersection of the following planes. Draw a geometric representation of each.

a) $5x + 3y - 2z + 1 = 0$
$2x + 4y + 2z - 8 = 0$
$4x + 3y - z + 5 = 0$

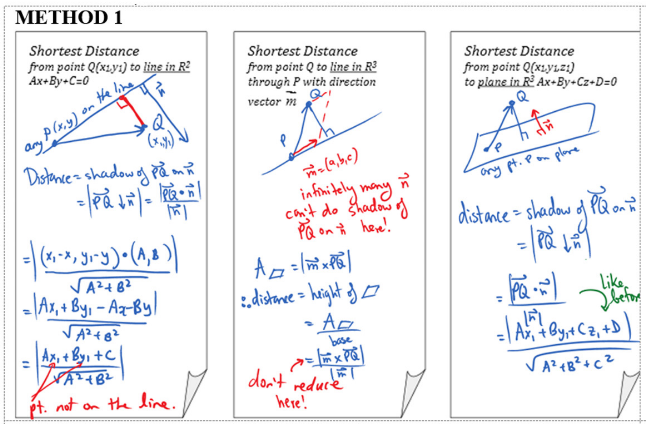
b) π_1 : $x + 7y - 6z + 17 = 0$

π_2 : $\vec{p} = (7, -2, 3) + s(1, 1, 2) + t(9, -5, -3)$

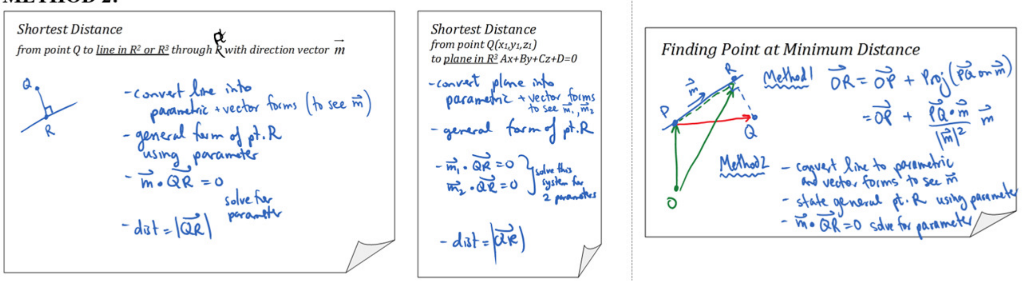
π_3 : $\begin{cases} x = 2 + 4s + t \\ y = 4 + s + 9t \\ z = 4 - 2s + 3t \end{cases}$

Distances

METHOD 1



METHOD 2:



b. Find the distance from point Q(5,8) to the line $7x + y - 23 = 0$ using METHOD 1 & 2

c. Calculate the distance between the two parallel planes $2x - y + 2z + 4 = 0$ and $2x - y + 2z + 16 = 0$

d. Determine the distance between point Q(-2,1,0) and line $\vec{r} = (0,1,0) + t(1,1,2), t \in R$

e. For question d. determine point R on the line at which minimum distance occurs (use METHOD 1 &2 then check if your answer in d. is correct).