PreCalculus UNIT 0

CONICS– journal questions – MCR + AP

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. HOW EQUATIONS AFFECT THE PICTURE?
   a. Copy/Paste the following

Conic sections can be identified directly from their equations of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, assuming $B = 0$.

Conic Section	Relationship of A and C
Parabola	$A = 0$ or $C = 0$, but not both.
Circle	$A = C$
Ellipse	A and C have the same sign and $A \neq C$.
Hyperbola	A and C have opposite signs.

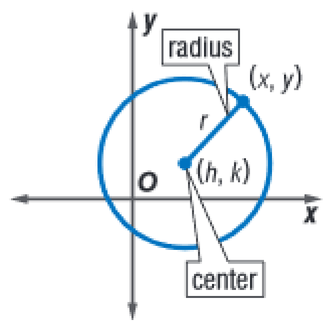
   b. Classify each of the following as either circle, ellipse, parabola or hyperbola. Then use desmos.com to check your work.

i $-2x^2 + y^2 + 4x + 6y + 3 = 0$	vii $2x^2 + 2y^2 - 4x + 16y + 2 = 0$
ii $x^2 + 16y^2 - 64y = 0$	viii $3x^2 + 3y^2 = 36$
iii $x^2 + y^2 = -4x + 6y + 3$	ix $16x^2 - 25y^2 - 32x + 100y - 484 = 0$
iv $16x^2 + 4y^2 + 32x - 8y = 44$	x $x - 2 = y^2 - 10y$
v $y = x^2 + 2x - 4$	xi . $9x^2 - 3 = 18x + 4y$
vi . $x^2 + 4y^2 + 6x - 8y = 3$	xii . $7x^2 - 5y^2 = 48 - 20y - 14x$

2. CIRCLES & ELLIPSES
   a. Definition of a CIRCLE is ______________________________
   b. Copy/Paste the following into your journal

Key Concept *Equation of a Circle*

The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

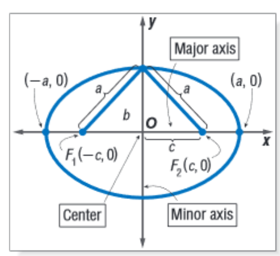


   c. Complete the square then sketch $x^2 + 4x - 6y + y^2 = 3$

   d. Definition of an ELLIPSE is ________________________________
   e. Copy/Paste the following into your journal.

Key Concept *Equations of Ellipses with Centers at the Origin*

Standard Form of Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units



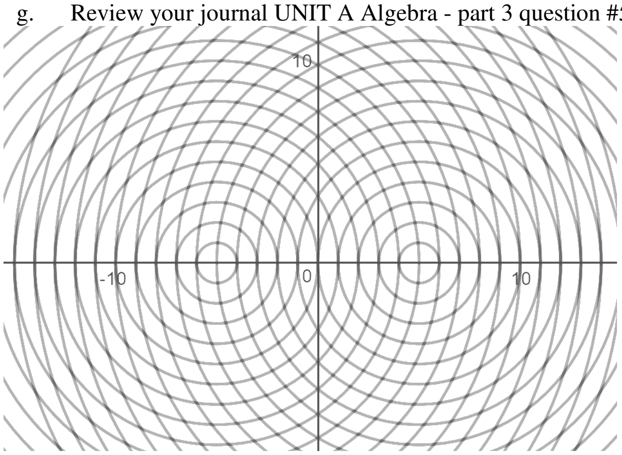
Key Concept *Equations of Ellipses with Centers at (h, k)*

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$

$a^2 \geq b^2$ and $c^2 = a^2 - b^2$.

   f. Show all the information and sketch $16x^2 + 4y^2 + 32x - 8y = 44$

g. Review your journal UNIT A Algebra - part 3 question #5 on how to solve root equations then follow the instructions below



Draw points on the intersections between circle 11 and circle 2, then circle 10 and circle 3, etc. to create the ellipse.
(Each time the sum between circle numbers should be constant at 13)

Now mark off the foci with A (5, 0) and B (-5, 0), and the general point on the ellipse as P (x, y).
Using the relationship $PA + PB = 13$ and the distance formulas develop the following equation of the ellipse: $\frac{4}{169}x^2 + \frac{4}{69}y^2 = 1$

Verify the location of the foci from the equation

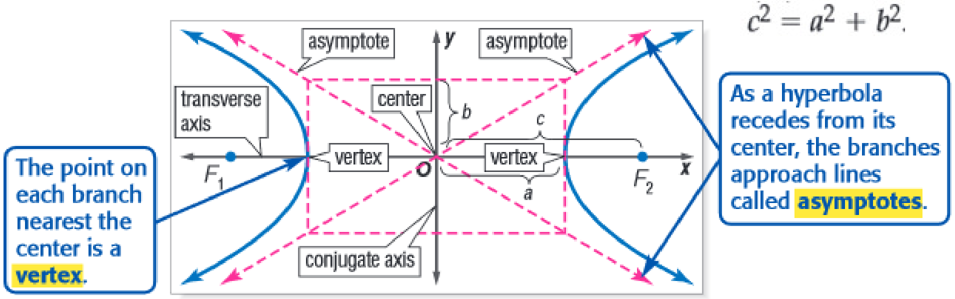
3. HYPERBOLAS
   a. Definition of HYPERBOLA is ____________________
   b. Copy/Paste the following into your journal, then show an example that uses this information.

Key Concept *Equations of Hyperbolas with Centers at the Origin*

Standard Form of Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Length of Transverse Axis	$2a$ units	$2a$ units
Length of Conjugate Axis	$2b$ units	$2b$ units
Equations of Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

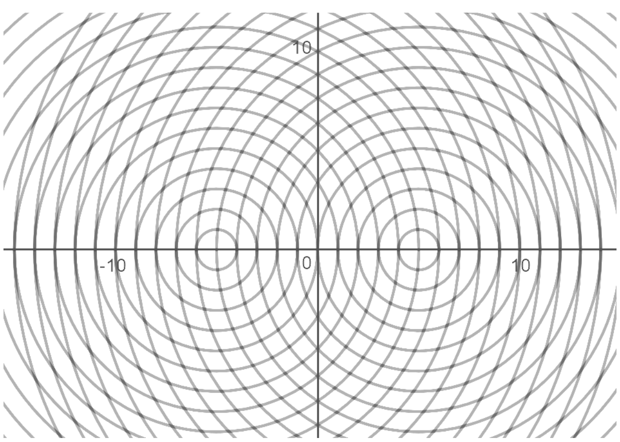
Key Concept *Equations of Hyperbolas with Centers at (h, k)*

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Equations of Asymptotes	$y - k = \pm\frac{b}{a}(x - h)$	$y - k = \pm\frac{a}{b}(x - h)$



$c^2 = a^2 + b^2$.

c. Show all the information and sketch $-2x^2 + y^2 + 4x + 6y + 3 = 0$



d. Draw the intersections points between circle 9 and circle 2, then circle 10 and circle 3, etc. to create the hyperbola.
(Each time the difference between circle numbers should be constant at 7)

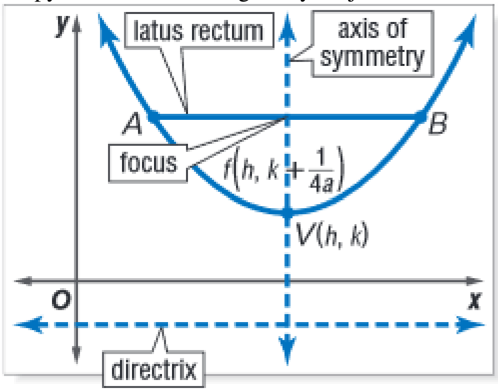
Now mark off the foci with A (5, 0) and B (-5, 0), and the general point on the ellipse as P (x, y).
Using the relationship $|PA - PB| = 7$ and the distance formulas
start with $PA - PB = 7$ or $-PA + PB = 7$
and develop the following equation of the ellipse: $\frac{4}{49}x^2 - \frac{4}{51}y^2 = 1$

Verify the location of the foci from the equation

4. PARABOLAS

a. Definition of a PARABOLA is the set of all points that are ________________________________

b. Copy/Paste the following into your journal

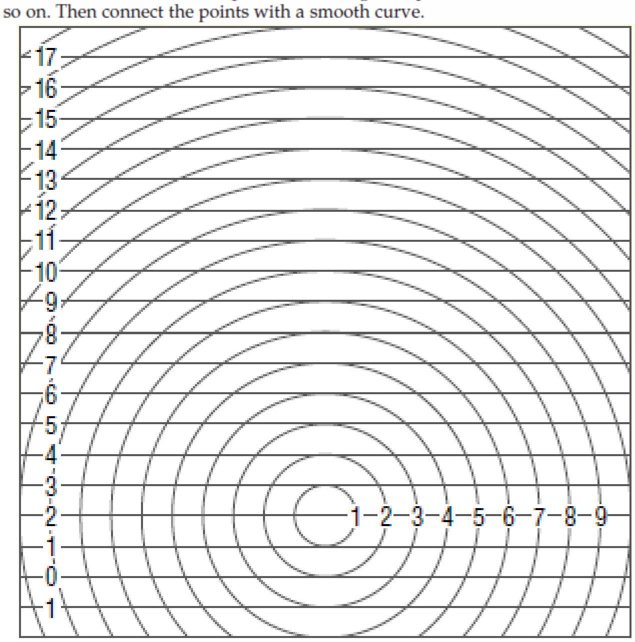


Concept Summary	Information About Parabolas	
Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Vertex	(h, k)	(h, k)
Axis of Symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Length of Latus Rectum	$\left\|\frac{1}{a}\right\|$ units	$\left\|\frac{1}{a}\right\|$ units

c. Show all the information and sketch $y = \frac{1}{4}(x-2)^2 - 5$ and $x = -\frac{1}{2}(y+4)^2 - 7$

d.

Mark the point at the intersection of circle 1 and line 1. Mark both points that are on line 2 and circle 2. Continue this process, marking both points on line 3 and circle 3, and so on. Then connect the points with a smooth curve.



On the diagram place y-axis at the centre of inner circle and x-axis at line 0. Mark off the centre of the inner circle as the focus with F (0, 2), any general point on the parabola as P (x, y) and right underneath the point P label the point on the directrix with D (x, 0). Using the relationship PF=PD and the distance formula show how

PF = $\sqrt{(x-0)^2 + (y-2)^2}$ and PD = $\sqrt{(x-x)^2 + (y-0)^2}$

becomes $y = \frac{1}{4}x^2 + 1$

5. NON LINEAR SYSTEMS (MCR)

a. Describe ways Line and Parabola can meet show pictures (secant, tangent, no intersection)

b. Describe what is a related equation and how it helps to find the POIs. Explain how to use the DISCRIMINANT to determine if there is an intersection

c. Show ex of elimination method with circle and ellipse $\boxed{\begin{array}{l} x^2 + y^2 = 7 \\ 4x^2 + 3y^2 = 24 \end{array}}$ Show a picture why 4 POI makes sense.

d. Show ex of substitution method with two hyperbolas $\boxed{\begin{array}{l} 4x^2 - y^2 = 5 \\ xy - 3 = 0 \end{array}}$ Show a picture why 2 POI makes sense.