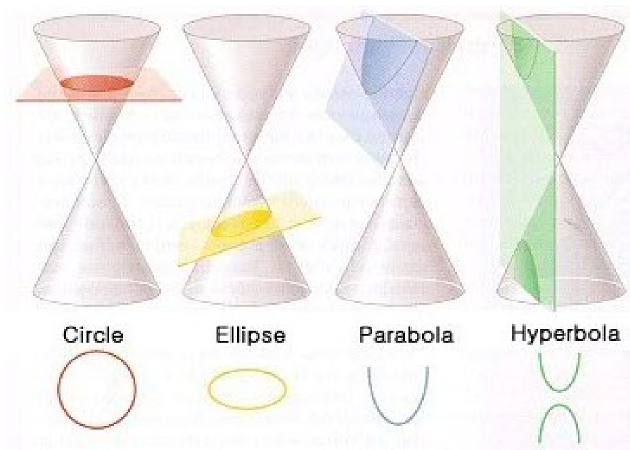
Conic Sections Assignment



Big idea

This unit presents several new types of graphs, called conic sections. These include circles, parabolas, ellipses, and hyperbolas. Not all of these graphs are functions and hence the equations will not necessarily have explicitly isolated output, y. These shapes are found in a variety of applications. For example, a reflecting telescope has a mirror whose cross section is in the shape of a parabola, and planetary orbits are modeled by ellipses. The Moon's orbit is almost a perfect circle. You will not need to memorize any of the formulas, they will be provided on your test.

Feedback & Assessment of Your Success

Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: ___________________ Questions to ask the teacher:
2days	2-6	Quadratic Word Problems (MCR) Journal #1				
	7-11	Circles & Ellipses (AP) Journal #2				
	12-14	Hyperbolas (AP) Journal #3				
	15-17	Parabolas (AP) Journal #4				
	18-19	Non Linear Systems (MCR) Journal #5				
	20-24	Applications (AP)				

ASSIGNMENT **Quadratic Word Problems (gr10 & MCR)**

Look over your journal PreCalculus UNIT C - Quadratics and answer the following questions.

1. The hypotenuse of a right triangle is 6 cm more than the shorter leg. The longer leg is three more than the shorter leg. Find the lengths of all three sides.

2. Find three consecutive integers such that four times the sum of all three is 2 times the product of the larger two

3. **Revenue problem**
The circus sells 1000 tickets for $6 each. The circus owners want to increase their revenues, so they increase prices. They have noticed that ticket sales decrease by 45 tickets every time the price increases by $0.50.
a) What is the revenue equation?
b) What price will maximize the revenue?
c) What is the quantity at maximum?
d) What is the maximum revenue?

4. **Falling object problem**
A water balloon is catapulted into the air so that its height, in meters after t seconds is given by a quadratic function. The balloon's initial speed is 27 meters/second and it was released from the height of 3 meters. When does the balloon reach the height of 35m?

5. **Profit problem**
If the total costs are $600 flat and $90 per item, and the revenue is given by $R(x) = 150x - x^2$, where x is the number of items sold.
a) What is the profit equation?
b) Determine the items sold that would produce the maximum profit.
c) Find the number of items to sell to have the profit of $100.
d) What is the initial profit?

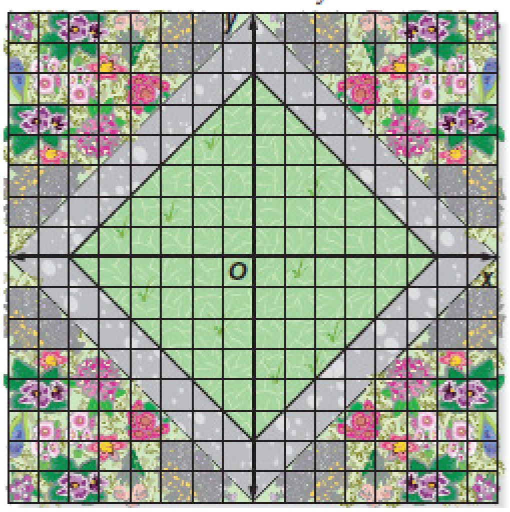
6. **Fence (or rope) off an area problem**
For a park swimming area, 4500 m of line is used to mark off the permissible area. One side not roped off is next to the beach. Find the dimensions of the swimming area that will make it a maximum

7. This is a diagram of a practice field. The track and field coach wants two laps around the field to be 1000m. But the Phys. Ed. department wants the rectangular field to be as large as possible. What dimensions would satisfy both conditions?

8. Dan and Sue set off at the same time on a 42 km go-cart race. Dan, drives 0.4 km/min faster than Sue, but has to stop en route and fix his go-cart for one-half hour. This stop costs Dan to arrive 15 min after Sue. How fast was each person driving?

ASSIGNMENT Circles & Ellipses (gr10 & AP)

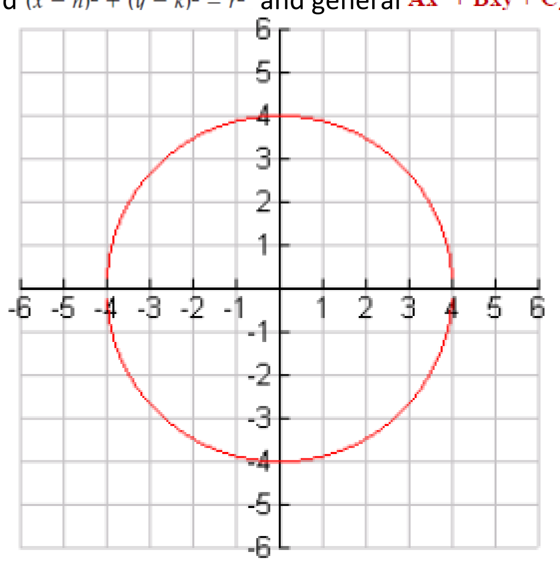
1. **LANDSCAPING** The design of a garden is shown at the right. A pond is to be built in the center region. What is the equation of the largest circular pond centered at the origin that would fit within the walkways?



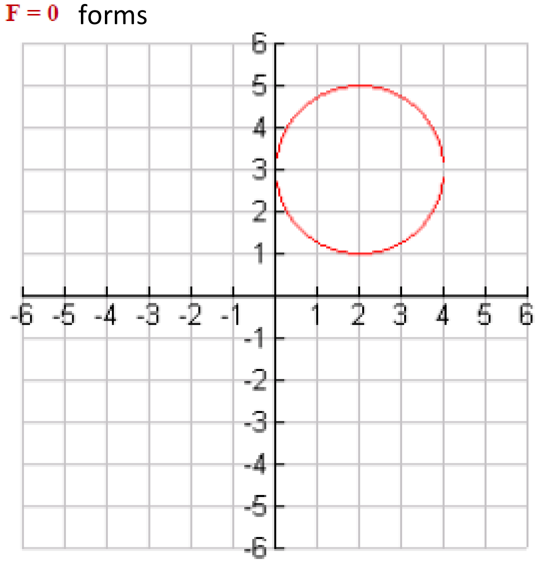
2. **EARTHQUAKES** The University of Southern California is located about 2.5 miles west and about 2.8 miles south of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 40 miles from the university. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.

Write equations for the following circle graphs in both standard $(x-h)^2+(y-k)^2=r^2$ and general $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ forms

3.

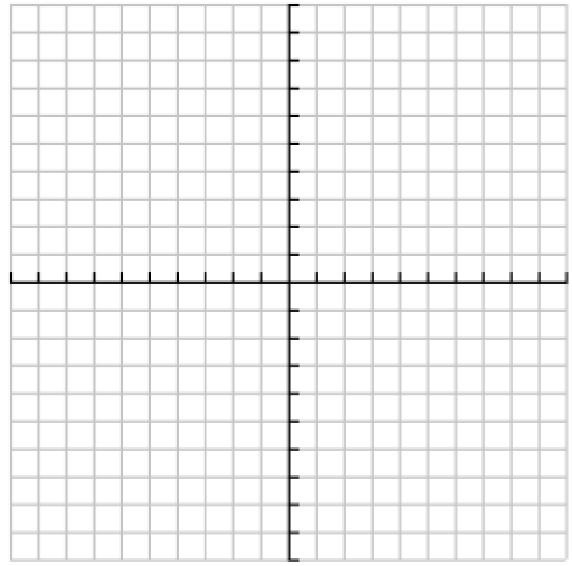


4.

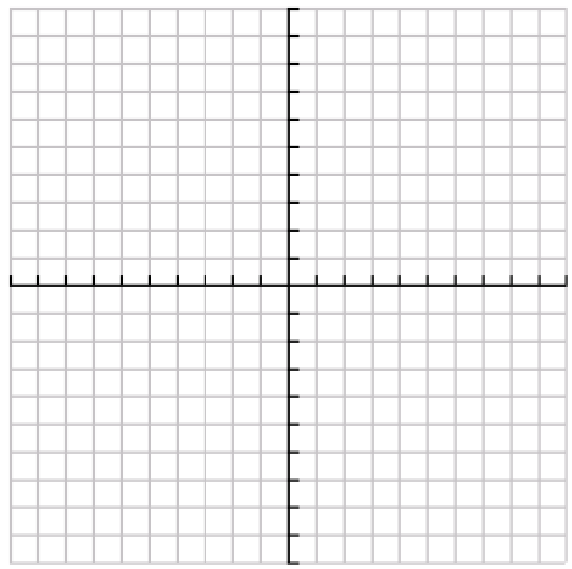


Change the following equations to standard form. Graph the circles, identify the centres and radii

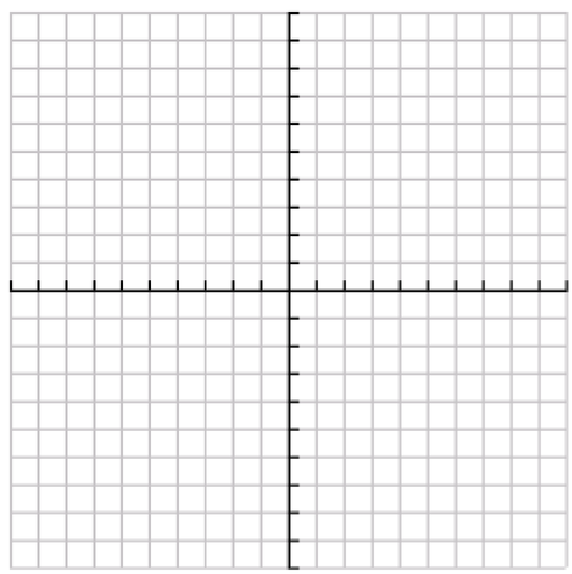
5. $x^2 + y^2 + 2x + 4y - 20 = 0$



6. $x^2 + y^2 - 4y = 0$

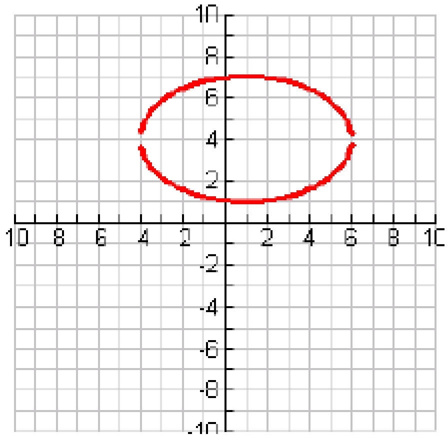


7. $x^2 + y^2 - 6x - 10y = 2$



Write an equations in both standard and general forms for each ellipse.

8.



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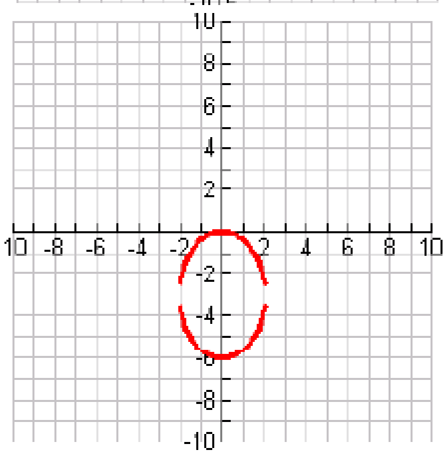

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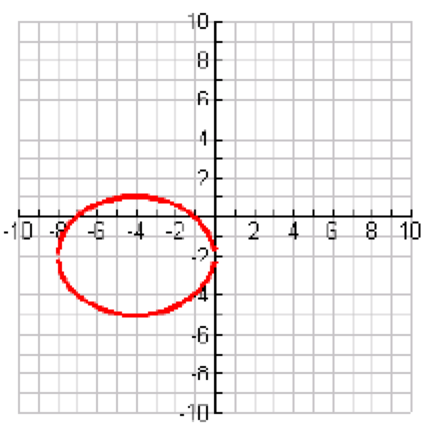
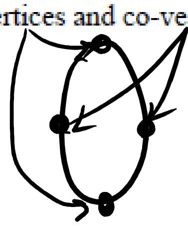
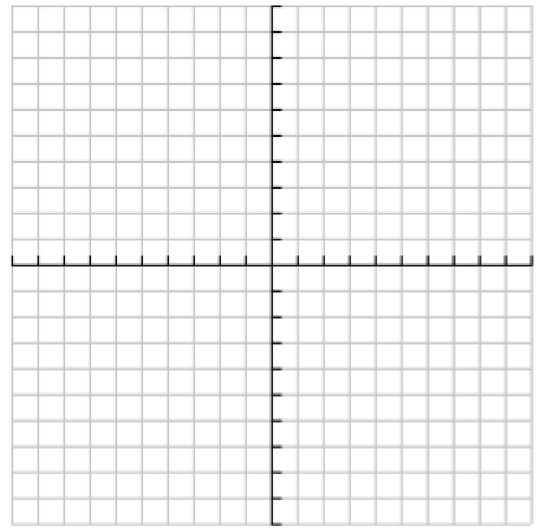


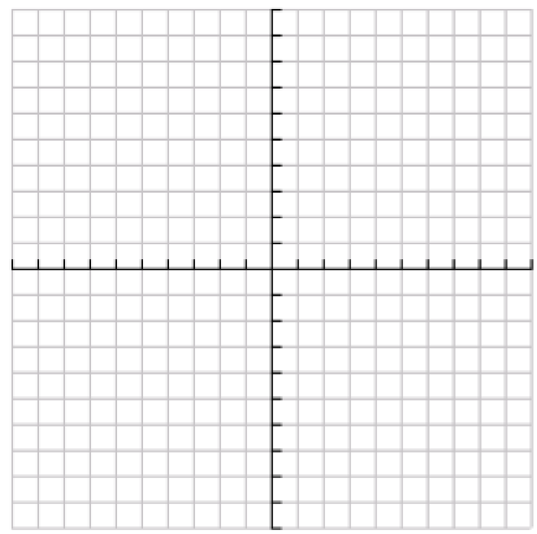
Graph each ellipse. Identify the vertices and co-vertices. Find the lengths of the major and minor axes.



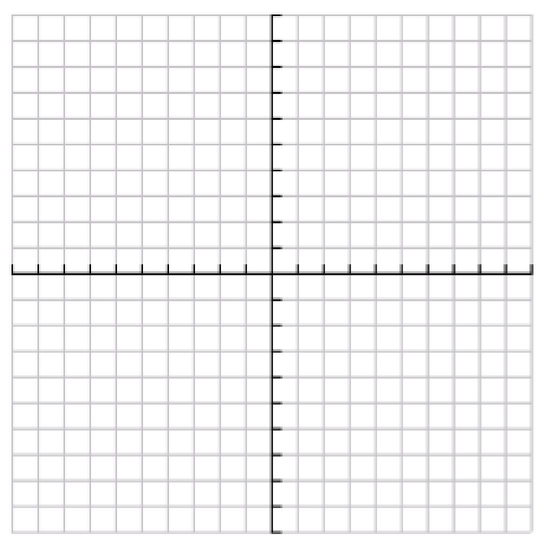
11. $\frac{x^2}{9} + \frac{y^2}{16} = 1$



12. $4x^2 + 9y^2 = 36$



13. $4(x - 3)^2 + 9(y + 2)^2 = 36$



Use completing the square to change each general form ellipse to standard form.

14. $9x^2 + 4y^2 + 8y - 32 = 0$

15. $x^2 + 4y^2 + 6x - 8y - 3 = 0$

16. Find the equation for the following using both forms
major axis 20 units long and parallel to *y*-axis, minor axis 6 units long, center at (4, 2)

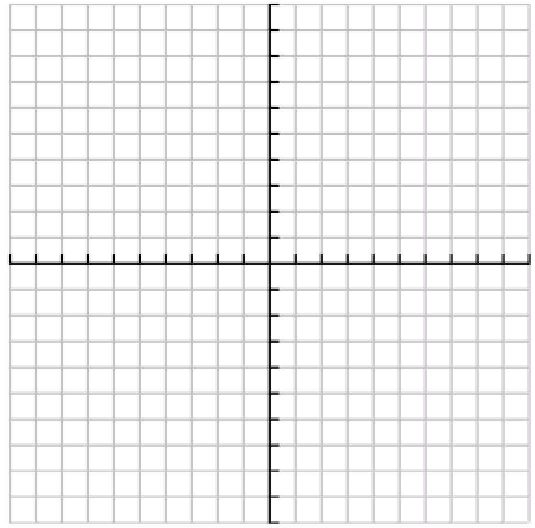
17. Consider the following ellipse equations. Compare the graphs. How are they similar? How do they differ?

$$\frac{x^2}{2.25} + \frac{y^2}{6.25} = 1 \qquad \frac{x^2}{9} + \frac{y^2}{25} = 1 \qquad \frac{x^2}{900} + \frac{y^2}{2500} = 1$$

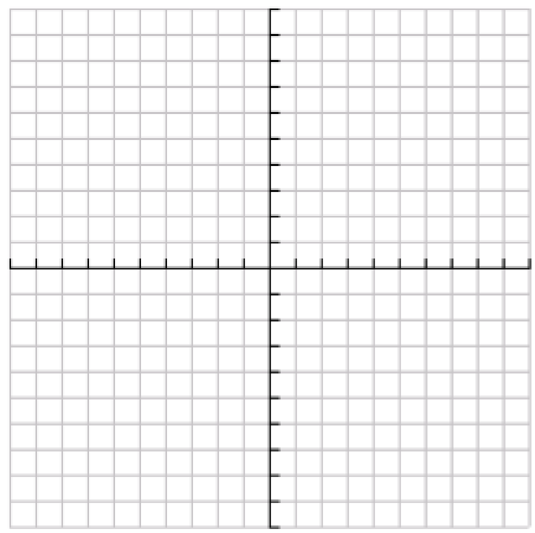
ASSIGNMENT Hyperbolas (AP)

For each of the following hyperbolas, find the equation in standard form, draw the graph, and locate the foci.

1. vertices at (-3, 0) and (3, 0) and asymptotes of $y = \pm \frac{4}{3}x$

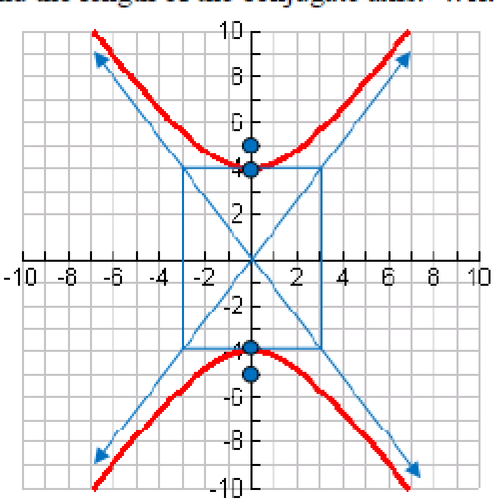


2. vertices at (0, 1) and (0, -1) and asymptotes of y $\pm \frac{1}{3}x$



For the hyperbola graph, identify the vertices, the asymptotes, the length of the transverse axis and the length of the conjugate axis. Write an equation in standard form for the graph.

3.



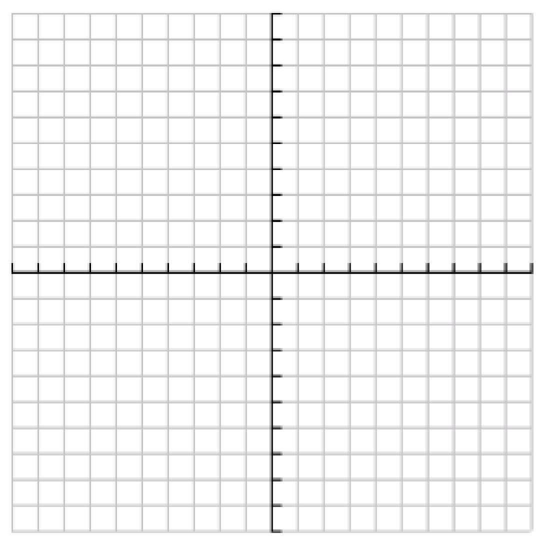
Write the standard equation for each hyperbola, give the coordinates of the center, vertices, and foci. What direction does the transverse axis lie?

4. $4x^2 - 9y^2 - 8x + 54y - 113 = 0$

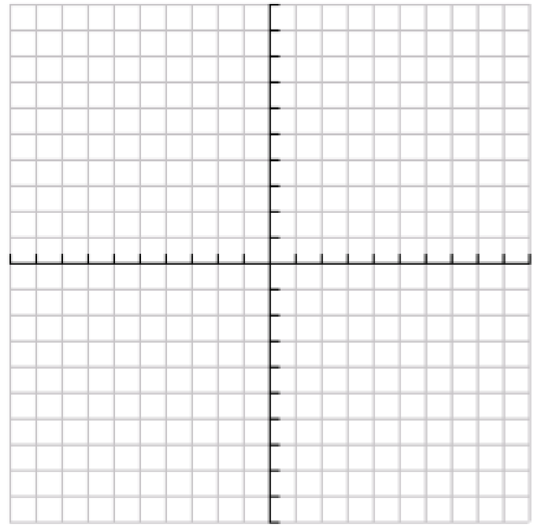
5. $y^2 - 9x^2 - 6y - 36x - 36 = 0$

For each of the following hyperbolas, give the coordinates of the center, and the vertices. Write the equations of the asymptotes. Sketch the graph and include all of the values you found as well as identifying the transverse and conjugate axes.

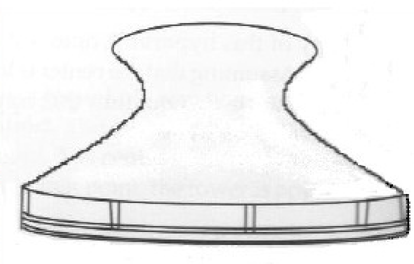
6. $4x^2 - 25y^2 = 100$



7. $\frac{(y-1)^2}{4}-\frac{(x+2)^2}{9}=1$



Hyperbolic shapes are used for horns, street lamps, space heaters, and cooling towers for nuclear reactors. Rays emanating from one focus point, A , reflect off point P on the hyperbola as if they had emanated from the other focus point B. This has the effect of spreading out an waves coming from a focus point.

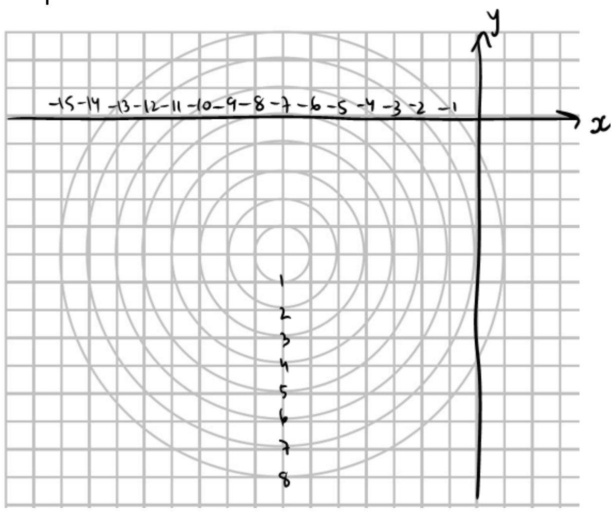


The architecture of the James S. McDonnell Planetarium of the St. Louis Science Center

and the natural draft wet cooling hyperbolic towers at Didcot Power Station, UK show hyperbolic designs.

ASSIGNMENT Parabolas (gr10 & AP)

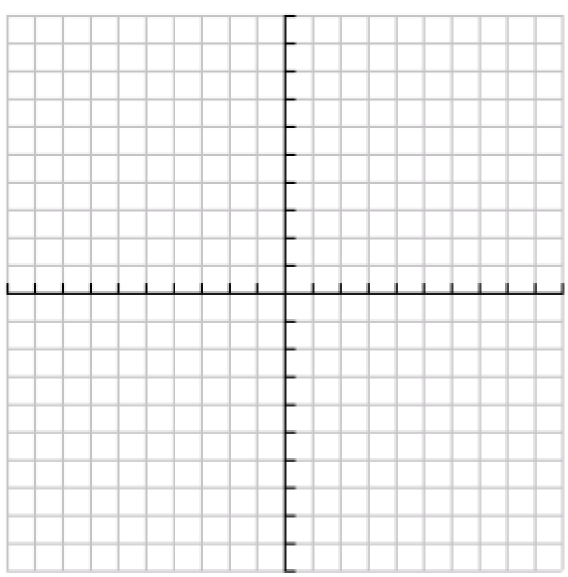
1. Mark off the intersections between circle 4 and line -11, then circle 5 and line -10, etc. (Each time the difference between circle number and line number should be constant at 15). Draw the parabola.



2. Mark off the Focus with point F (-7 ,-5) the Directrix with D (-15 , y) and the general point on the parabola as P(x, y). Using the relationship PF=PD and the distance formula find the equation of this parabola. Write it in the form where vertex is visible.

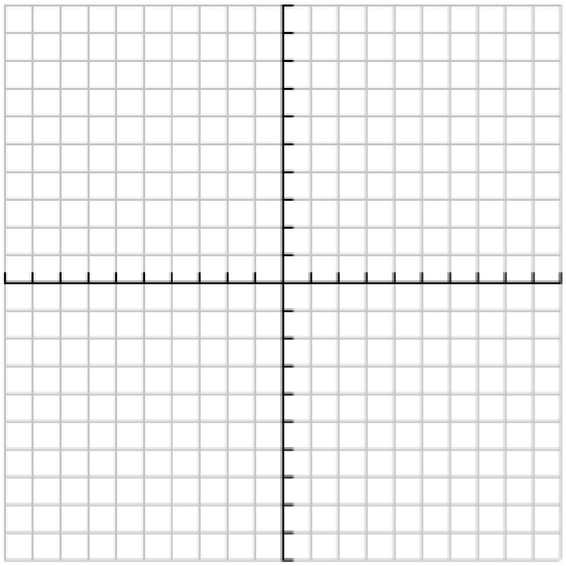
In each of the following, list the coordinates of the vertex, the coordinates of the focus, the equation of the directrix, and graph the parabola (ensure the length of the latus rectum is shown)

3. $(y-3)^2 = -12(x+2)$

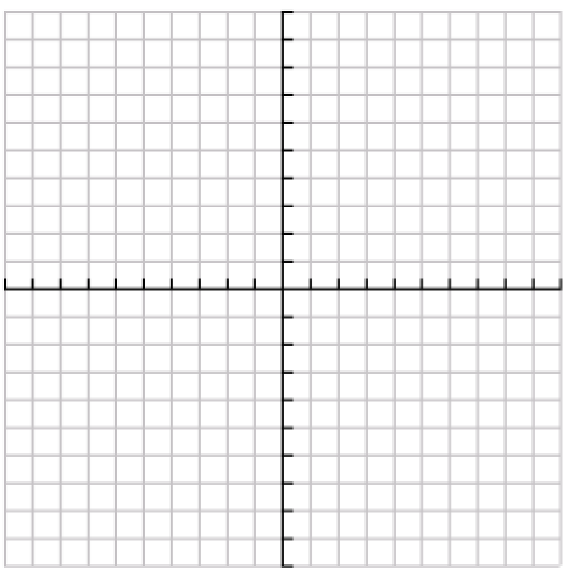


In each of the following, list the coordinates of the vertex, the coordinates of the focus, the equation of the directrix, and graph the parabola (ensure the length of the latus rectum is shown)

4. $(x + 1)^2 = 2(y + 3)$

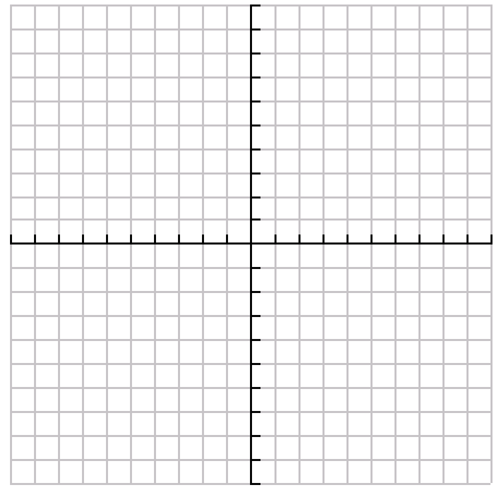


5. $x = (y - 4)^2$

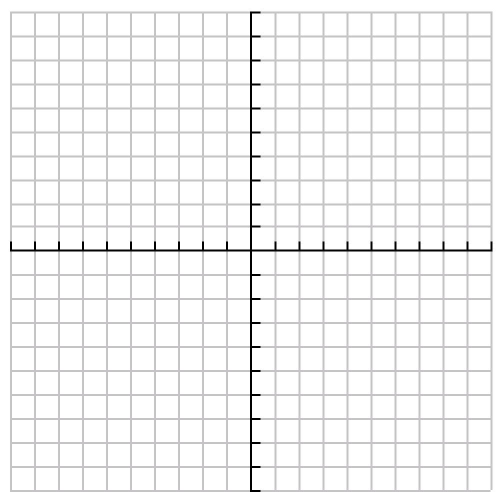


Write each of the following equations in standard form. List the vertex, coordinates of the focus and equation of the directrix. Graph the parabola.

6. $y^2 - 8y + 8x + 8 = 0$



7. $x^2 - 6x + 12y + 21 = 0$



ASSIGNMENT Non Linear Systems (MCR)

1. Find the point(s) of intersection of the parabola and the line. Explain the solution using pictures.

$y^2 + x^2 = 64$

$y - 2x = -3$

2. Find k so that the system has only one solution.

$y = -3x^2 + x - 1$

$y = -x + k$

3. Predict the number of solutions the following system will have. Explain your reasoning.

$y = -2x^2 + x - 3$

$y = -x - 2$

Find the point(s) of intersection

4. $x^2 + 7y = 16$
$3x^2 - 2y = 25$

5. $x^2 + y^2 = 8$
$16x + 4y^2 = 80$

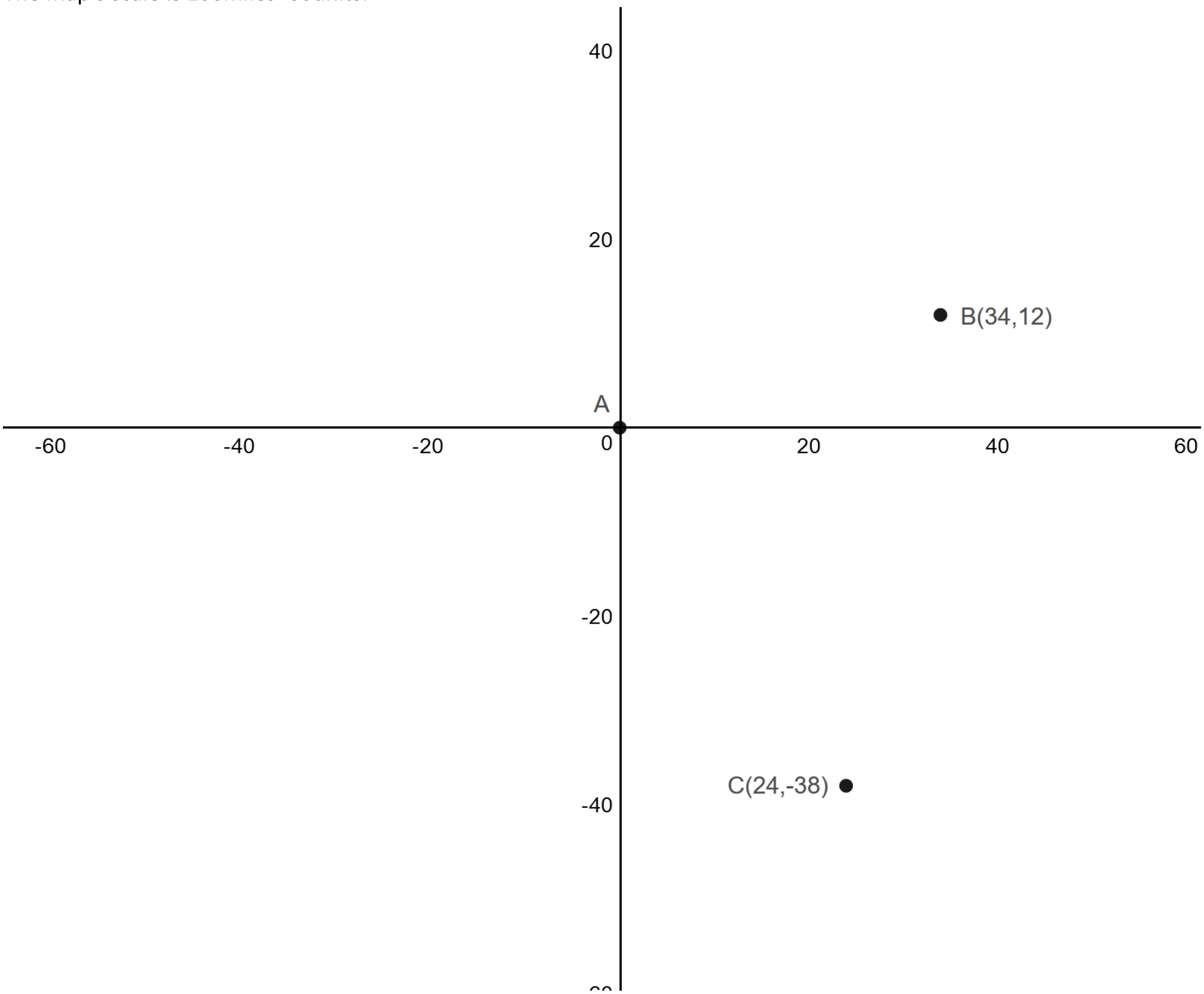
6. A toy rocket's flight path is given by $h(t) = -5t^2 + 3t + 2$ and a bird's flight path is $h(t) = -5t + 4$ where t is time in seconds. What time did the bird and the toy rocket collide?

ASSIGNMENT Applications (AP)

RADIO STATIONS – application of circles

Radio signals emitted from a transmitter form a pattern of concentric circles (circles with the same centre).
Randy listens to a radio station, A, from Atlanta. His home is located 24 miles east and 32 miles south of the radio transmitter. His house is located on the edge of the radio station A's maximum broadcast. On the map shown, let (0,0) be the radio station A, in Atlanta, let (34,12) be the station B, in Athens, and let (24,-38) be the station C, in Macon.

The map's scale is 100miles=60units.



1. On the map, draw two concentric circles from the radio station A, these represent the traveling signal. State their equations below

2. What are the coordinates of Randy's house? Hint: Convert miles to units.

3. Draw that point and draw another concentric circle showing the maximum range of the station A. When a radio signal reaches Randy's house, how far has it traveled?

4. Find an equation which represents the station's maximum listening area. Do a version in both miles and units of the map.

Randy likes to listen to country music. His friends have suggested that in addition to radio station A, in Atlanta, he can try station B, in Athens, and station C, in Macon. Stations B and C have a broadcast range of 40 miles.

5. Can Randy expect to pick up signals from station B and C? Explain.

6. What are the coordinates of the intersections of the broadcast areas of station A and C? Does it matter whether you find the intersections using miles or units of the map?

LOng RAnge Navigation (LORAN) – application of hyperbolas

For a more detailed account of how hyperbolas are used to locate ships at sea using the LORAN system, go to *http://mathcentral.uregina.ca/beyond/articles/LoranGPS/Loran.html*

Two radio stations located at A and B transmit simultaneously to a ship located at P. The on-board-computer converts the time difference $|PA - PB|$ between the time the ship receives a signal from each station and this locates the ship on one branch of a hyperbola. Suppose the ship receives the signal from station A 1200 microseconds before it receives the signal from station B. The radio signal travels at a speed of 980feet/microsec.

1. Station A is located 400 miles due east of station B. Draw these points assuming the origin is at A. Then draw a horizontal hyperbola with A and B as foci. Don't worry about accuracy, just a sketch is ok.

2. Will point P be closer to A or to B? Put in point P somewhere on the corresponding sketch of a branch of the hyperbola.

3. Using the difference in time and the speed of the signal, find the difference in distances in miles. (1mi=5280ft)

4. Use $|PA - PB|$ and your answer form 3. to find the equation of the hyperbola where the ship lies.

Application of parabolas

1. The paper clip chain with no weight attached is a **catenary curve**. When weight is attached, the curve becomes a **parabola**. Catenary curves and parabolas are often mistaken for each other. The Golden Gate bridge is a suspension bridge in San Francisco, California. The towers are 1280 meters apart and rise 160 meters above the road. The cable just touches the sides of the road midway between the towers.

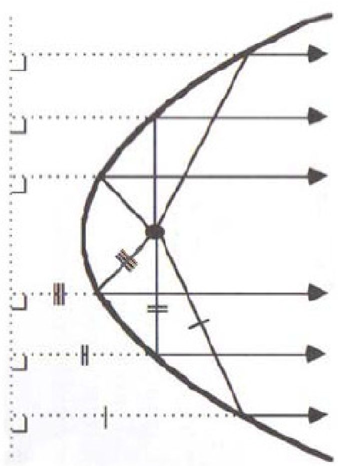
a. Place origin on the left edge of the road where the bridge begins. Find the equation that represents the parabola.

b. What is the height of the cable 200 meters from a tower?

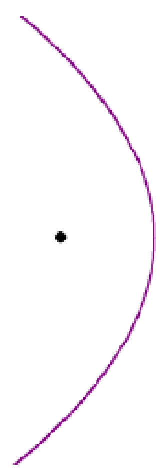
2. Parabolas have special reflective properties which make them useful shapes for many items including flashlights, car headlights, suspension bridges such as the Golden Gate Bridge, solar cookers, and satellite dishes. Two properties are especially important when considering applications of parabolas. First, all rays in the interior of a parabola parallel to the axis of symmetry are reflected toward the focus. And, all rays emitted from the focus are reflected so that each reflected ray runs parallel to the axis of symmetry and perpendicular to the directrix.

a. Using the approximate the location of the focus point draw the directrix on the following parabola
b. Show how rays emitted from the directrix would travel:

Rays emitted **from focus**:



Rays emitted **from the directrix**:



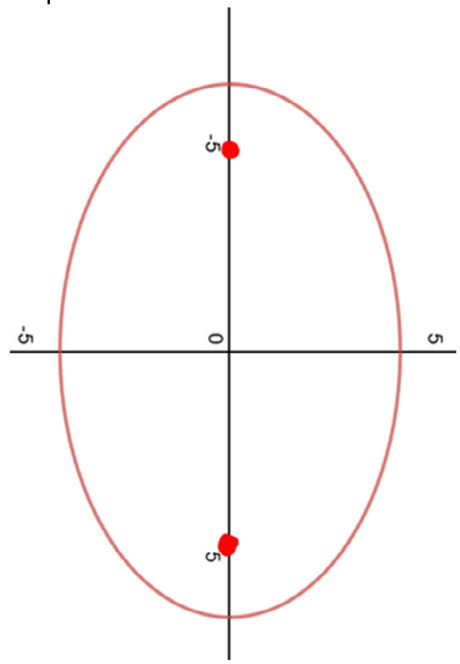
Application of ellipses

Ellipses have special reflective properties. Energy waves emanating from one focus point will bounce off the ellipse and travel to the other focus.

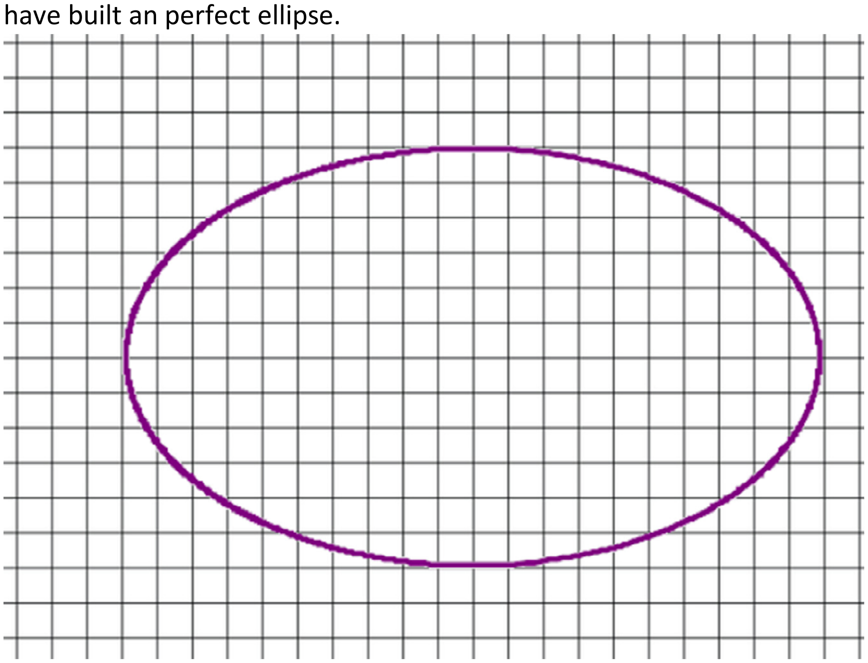
"Whispering galleries" are elliptical domes where any sound waves produces at one focus can be heard at the other focus and nowhere in between. Statuary Hall in the US Capital building is elliptical. John Quincy Adams, while a member of the House of Representatives, discovered this acoustical phenomenon. He situated his desk at a focal point of the elliptical ceiling and could easily hear conversations being held at the opposing party leader's desk located near the other focal point.

A nonsurgical treatment for kidney stones, Extracorporeal Shock Wave Lithotripsy, uses the same reflection property of the ellipse to break up the stones without damaging any tissue around them. A patient is positioned in an elliptical chamber so that the kidney stone is at one focus and high energy sound waves are created at the other focus. The sound waves reflect off the wall of the chamber and converge to break up the kidney stone.

1. Draw several sound waves from one focus point and show how the path bounces off to arrive at the other focus point



Elliptical pool tables also depend on the reflection qualities of ellipses. A ball moving across one focus point and hitting the wall of the table will travel to a pocket located at the other focus point. The game would be very easy to play as long as you locate the focus point correctly and have built an perfect ellipse.



2. Verify if the picture on the left is a true ellipse or not.
   a. Locate the centre and draw the x and y axis through it.
   b. What would be the equation of an ellipse that has the vertices and co-vertices of the one shown in the diagram?
   c. Use your equation to find two other points which should be on the diagram if it is a true ellipse. Are these points on the figure?
   d. If you believe the diagram is a true ellipse, where would you place the pocket, and from where would you shoot the ball to ensure it always goes in?