

Algebraic Vectors

Geometric Vectors

- Geometric vectors are vectors with no fixed location
- Geometric vectors are written as a _____ and a _____

i.e. _____

Algebraic Vectors

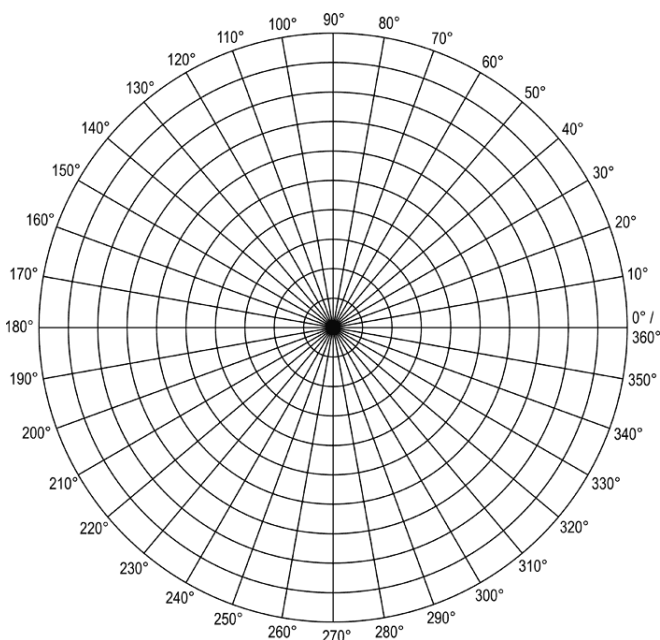
- Algebraic vectors are vectors that are drawn on a coordinate plane with the tail at _____.

Polar Coordinates

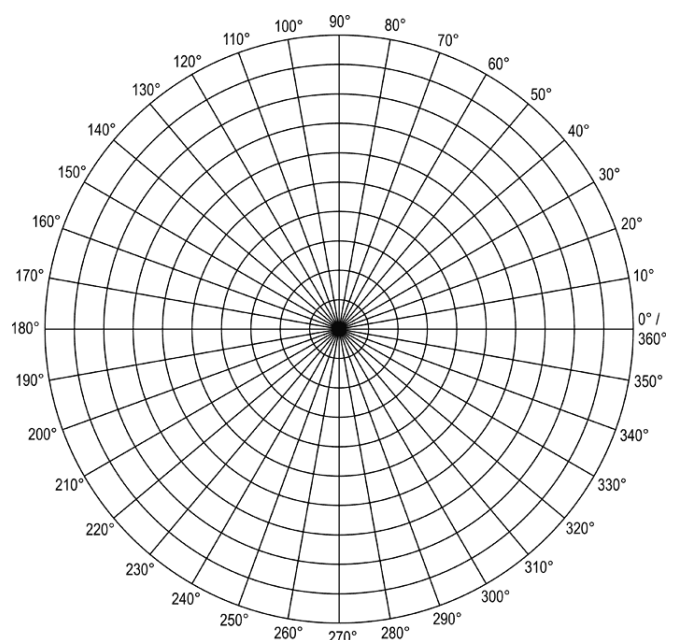
- Algebraic vectors can be written using polar coordinates in the form _____, where the angle is measured from the terminal arm (positive x-axis).

i.e. _____ or _____

1. Plot the vector $\vec{a} = (4, 120^\circ)$



2. Plot the vector $\vec{a} = 9 \text{ units [S45°W]}$



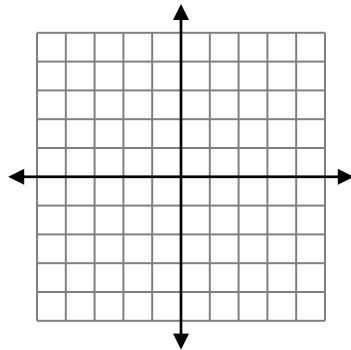
Cartesian Coordinates

- Algebraic vectors can be written using Cartesian coordinates in _____
or _____ form.

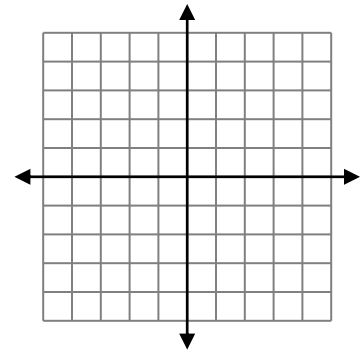
i.e. _____ or _____

1. Draw the following algebraic vectors:

a) $\vec{a} = (3, 5)$



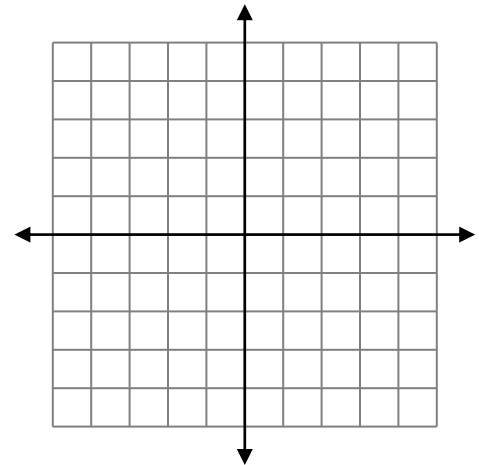
b) $\vec{a} = (-4, 1)$



2. Write the vector \overrightarrow{AB} in component form if:

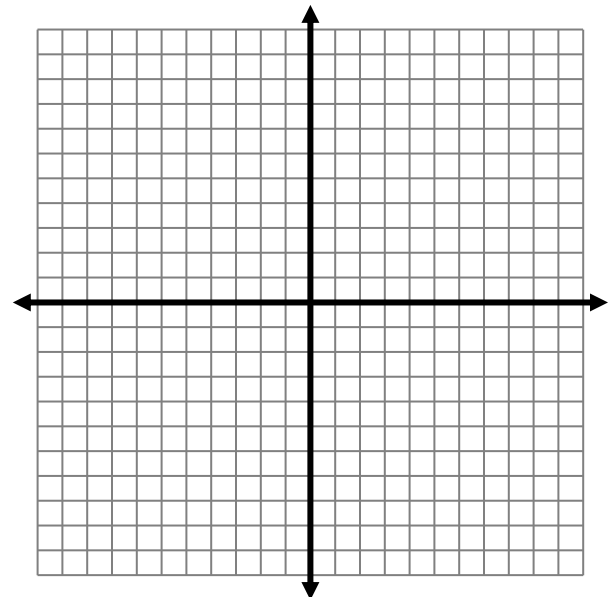
a) $A = (4, 3)$ and $B = (-1, 1)$

b) $A = (25, 70)$ and $B = (15, 100)$



3. Determine $\vec{a} + \vec{b}$ if $\vec{a} = (1, 5)$ and $\vec{b} = (3, -2)$

4. Determine $2\vec{a}$ if $\vec{a} = (-3, -1)$



5. Simplify $10\vec{a} - 3\vec{b}$ if $\vec{a} = (-2, 7)$ and $\vec{b} = (3, 1)$

Unit Vector Form

- Algebraic vectors can be written using _____.

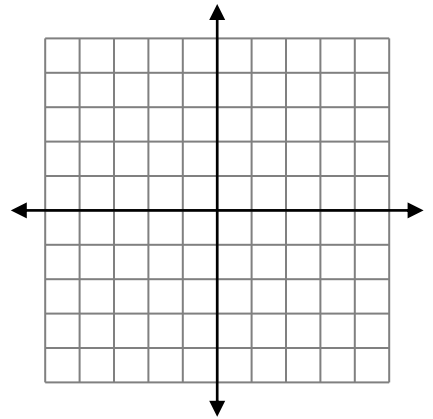
$$\vec{i} = \underline{\hspace{15em}}$$

$$\vec{j} = \underline{\hspace{15em}}$$

1. Write each of the following vectors in unit vector form:

a) $\vec{a} = (2, 5)$

b) $\vec{b} = (-3, 10)$

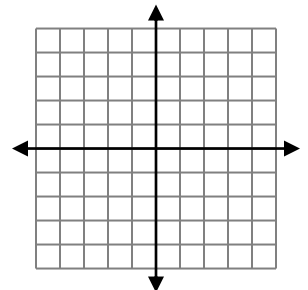


Magnitude of an Algebraic Vector

Calculate the magnitude of the following algebraic vectors:

a) $\vec{a} = (5, 2)$

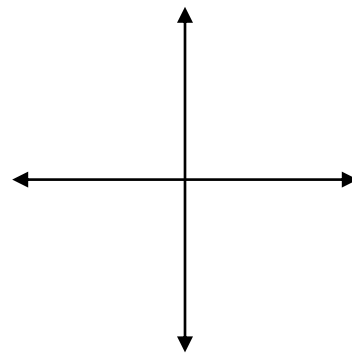
b) $\vec{a} = (-7, 3)$



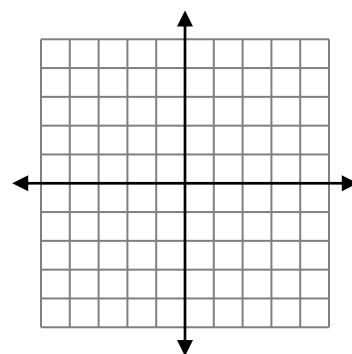
Converting Between Forms

1. Write $\vec{a} = -4\vec{i} + 3\vec{j}$ in component form:

2. Write the vector $\vec{a} = 6 \text{ m [N30°W]}$ as an algebraic vector in component form.



3. Write the vector $\vec{a} = (5, -2)$ as a geometric vector.



Drawing Vectors in Three Dimensions

Draw each of the following vectors:

a) $\bar{u} = (2, 6, 5)$

b) $\bar{u} = (6, 3, 1)$

c) $\bar{u} = (3, 1, -4)$

d) $\bar{u} = (-5, 4, 3)$

e) $\bar{u} = (3, -1, 4)$

f) $\bar{u} = -5\hat{i} - 2\hat{j} - 7\hat{k}$

Vectors in Three Dimensions

Algebraic Form

Component Form

$$\vec{u} = (x, y, z)$$

Unit Vector Form

$$\vec{u} = x \vec{i} + y \vec{j} + z \vec{k}$$

Write $\vec{u} = (2, -3, 1)$ in unit vector form

Write $\vec{u} = -7 \vec{i} + 5 \vec{j} + 9 \vec{k}$ in component form

Geometric Form

Magnitude

$$|\vec{u}| = \sqrt{x^2 + y^2 + z^2}$$

Direction (Direction Cosines)

$$\cos\alpha = \frac{x}{|\vec{u}|} \quad \cos\beta = \frac{y}{|\vec{u}|} \quad \cos\gamma = \frac{z}{|\vec{u}|}$$

Write the vector $\vec{u} = (3, 1, -4)$ as a geometric vector.

Scalar Multiplication

a) $3(4\vec{i} + 2\vec{j} - \vec{k})$

b) $10(3, 7, 1)$

Vector Addition

a) $(5, 7, -3) + (6, 2, 4)$

b) $(2\hat{i} + 15\hat{j} + 3\hat{k}) - (6\hat{i} - 4\hat{j} + 2\hat{k})$

Parallel Vectors

- Parallel vectors have the same or opposite direction.
 - Two vectors are parallel if _____
- a) For the points A(3, 2, 7), B(4, 5, 1), C(-4, 7, 1), D(-6, 1, 13), determine whether \overline{AB} is parallel to \overline{CD} .

Collinear Points

- Three points (A, B, C) are collinear if ____ and ____ are _____.
- a) Determine whether the points A (4, -2, 3), B(3, 2, 7), C(1, 10, 15) are collinear.

What is the set of all linear combinations of $(1, 0, 0)$ and $(0, 1, 1)$?

Prove that the vectors $\vec{a} = (-1, 2, -3)$, $\vec{b} = (2, 0, -1)$, and $\vec{c} = (-7, 6, -7)$ are linear dependant.

Dot Product

Dot product is the _____ of one vector with the scalar projection of the other vector.

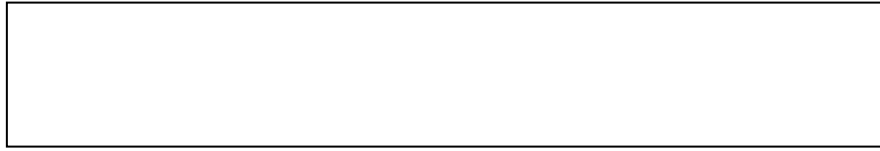
Determine $\vec{a} \cdot \vec{b}$, if $|\vec{a}| = 20$, $|\vec{b}| = 45$ and $\theta = 55^\circ$.

Perpendicular Vectors

If two vectors are perpendicular, their dot product _____

Determine $\vec{a} \cdot \vec{b}$, if $|\vec{a}| = 5$, $|\vec{b}| = 10$ and $\theta = 90^\circ$.

Algebraic Vectors



1. If $\vec{u} = (2, -3, 1)$ and $\vec{v} = (-5, 2, 4)$, calculate $\vec{u} \cdot \vec{v}$.

2. If $\vec{u} = 3\vec{i} + 2\vec{j} + 7\vec{k}$ and $\vec{v} = 5\vec{i} - 9\vec{k}$, find $\vec{u} \cdot \vec{v}$.

3. Find the angle between the vectors $\vec{u} = (-2, 3, 4)$ and $\vec{v} = (1, 5, 2)$.

Properties of Dot Product

$$k(\vec{u} \cdot \vec{v}) =$$

$$\vec{u} \cdot (\vec{v} + \vec{w}) =$$

$$\vec{u} \cdot \vec{u} =$$