Algebraic Vectors

Geometric Vectors

- Geometric vectors are vectors with no fixed location
- Geometric vectors are written as a ______________________ and a ________________________

i.e. ___________________________

Algebraic Vectors

- Algebraic vectors are vectors that are drawn on a coordinate plane with the tail at ____________.

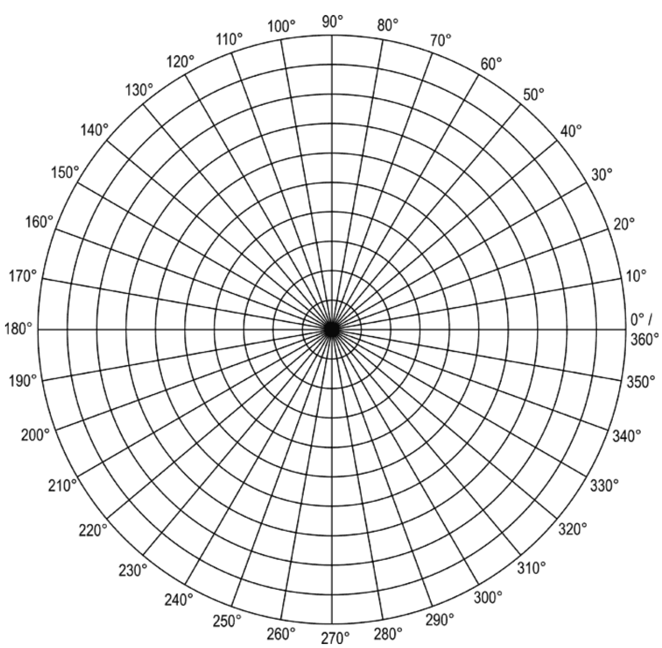
Polar Coordinates

- Algebraic vectors can be written using polar coordinates in the form ________________________,

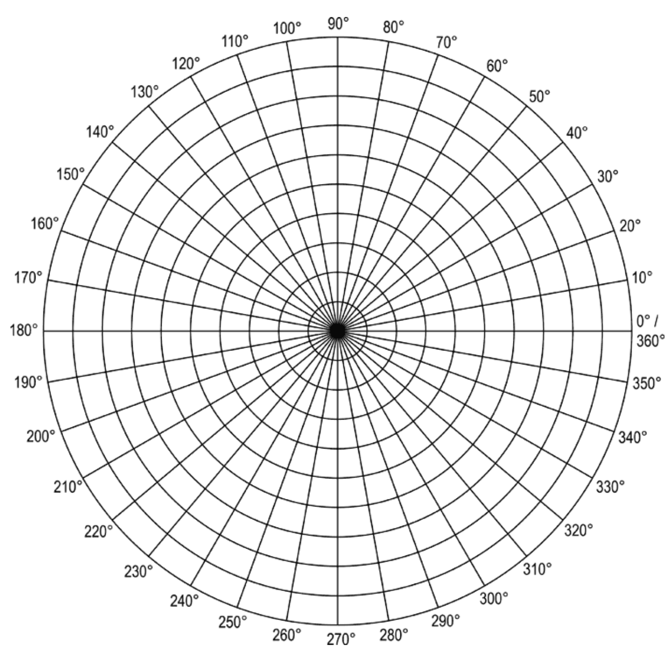
  where the angle is measured from the terminal arm (positive x-axis).

  i.e. _____________________ or ______________________

1. Plot the vector $\vec{a} = (4, 120^o)$



2. Plot the vector $\vec{a}$ = 9 units [S45^{o}W]



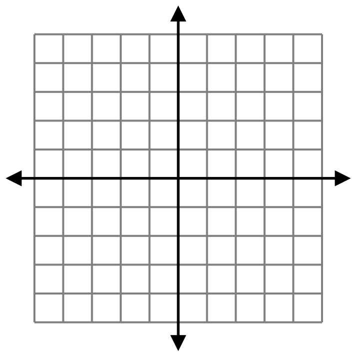
Cartesian Coordinates

- Algebraic vectors can be written using Cartesian coordinates in ______________________________ or ___________________________ form.

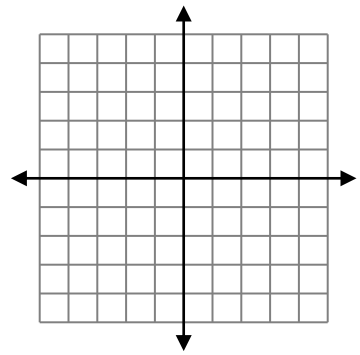
  i.e. _______________________ or ________________________

1. Draw the following algebraic vectors:

a) $\vec{a} = (3, 5)$



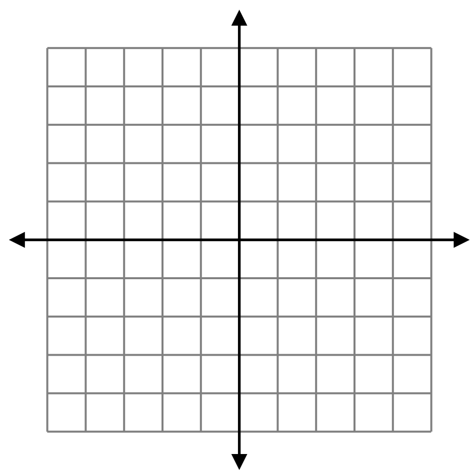
b) $\vec{a} = (-4, 1)$



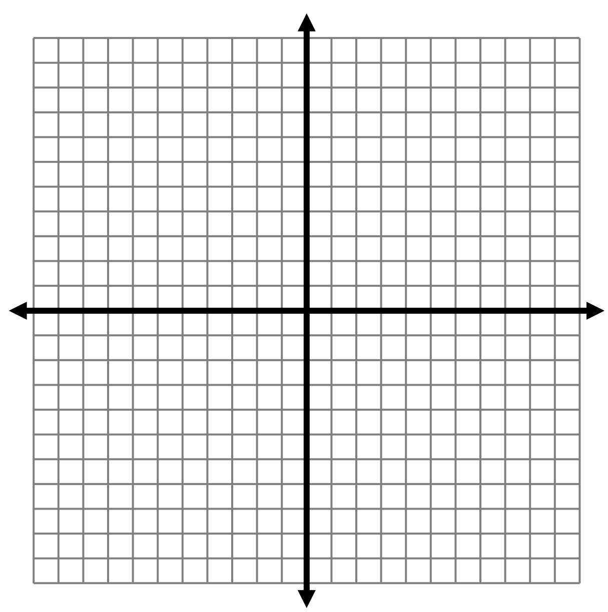
2. Write the vector $\overrightarrow{AB}$ in component form if:

a) A = (4, 3) and B = (-1, 1)

b) A = (25, 70) and B = (15, 100)



3. Determine $\vec{a} + \vec{b}$ if $\vec{a} = (1, 5)$ and $\vec{b} = (3, -2)$



4. Determine $2\vec{a}$ if $\vec{a} = (-3, -1)$

5. Simplify $10\vec{a} - 3\vec{b}$ if $\vec{a}$ = (-2, 7) and $\vec{b}$ = (3, 1)

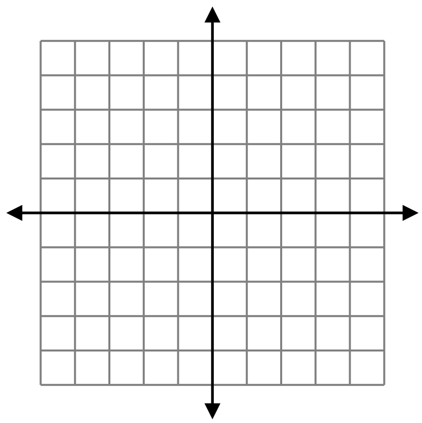
<u>Unit Vector Form</u>

- Algebraic vectors can be written using ______________________.

$\vec{i}$ = ____

$\vec{j}$ = ____

1. Write each of the following vectors in unit vector form:

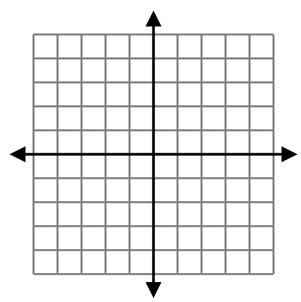


a) $\vec{a}$ = (2, 5)

b) $\vec{b}$ = (-3, 10)

<u>Magnitude of an Algebraic Vector</u>

Calculate the magnitude of the following algebraic vectors:



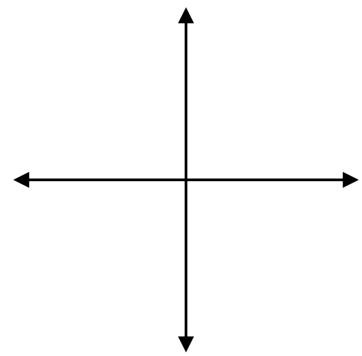
a) $\vec{a}$ = (5, 2)

b) $\vec{a}$ = (-7, 3)

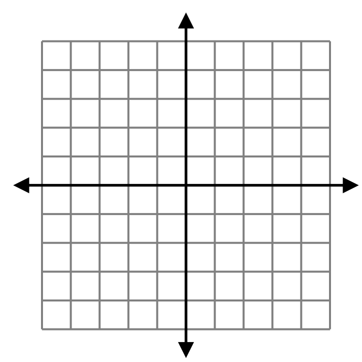
Converting Between Forms

1. Write $\vec{a} = -4\vec{i} + 3\vec{j}$ in component form:

2. Write the vector $\vec{a}$ = 6 m [N30^{o}W] as an algebraic vector in component form.



3. Write the vector $\vec{a}$ = (5, -2) as a geometric vector.



Drawing Vectors in Three Dimensions

Draw each of the following vectors:

a) $\vec{u} = (2, 6, 5)$

b) $\vec{u} = (6, 3, 1)$

c) $\vec{u} = (3, 1, -4)$

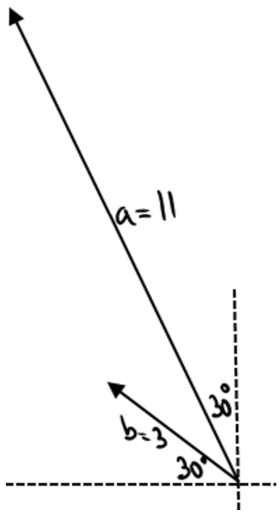
d) $\vec{u} = (-5, 4, 3)$

e) $\vec{u} = (3, -1, 4)$

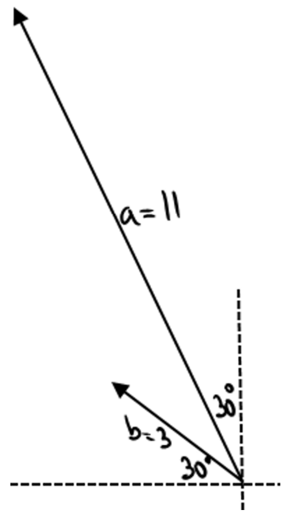
f) $\vec{u} = -5\hat{i} - 2\hat{j} - 7\hat{k}$

Velocity Again

Find $\vec{b}-\vec{a}$ using Geometric vectors with Sine and Cosine laws



Find $\vec{b}-\vec{a}$ using Algebraic Vectors then check if the answers are the same.



A search and rescue aircraft, travelling at a speed of 240km/h, starts out at a heading of $N20^o W$. After travelling for one hour and fifteen minutes, it turns to a heading of $N80^o E$ and continues for another 2 hours before returning to base.

a) Determine the displacement vector for each leg of the trip. – use a method of your choice
b) Find the total distance the aircraft travelled and how long it took.

The **relative velocity** of the object B traveling at $\vec{v}_B$ relative to the object A traveling at $\vec{v}_A$ is given by:

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

For **boat in water** (similarly for **plane with wind**) questions: $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$ where $\vec{v}_{BG}$ is velocity of boat relative to ground, $\vec{v}_{BW}$ is velocity of boat in still water, and $\vec{v}_{WG}$ is velocity of water relative to ground.

A car is traveling at $\vec{v}_c = 100 km/h[E]$, a motorcycle is traveling at $\vec{v}_m = 80 km/h[W]$, a truck is traveling at $\vec{v}_t = 120 km/h[N]$ and an SUV is traveling at $\vec{v}_s = 100 km/h[SW]$. Find the relative velocity of the car relative to:

a) motorcycle
b) truck
c) SUV

Give answers in both algebraic and geometric forms

Vectors in Three Dimensions

Algebraic Form

Component Form	Unit Vector Form
$\vec{u} = (x, y, z)$	$\vec{u} = x\vec{i} + y\vec{j} + z\vec{k}$

Write $\vec{u} = (2, -3, 1)$ in unit vector form

Write $\vec{u} = -7\vec{i} + 5\vec{j} + 9\vec{k}$ in component form

Geometric Form

Magnitude

$$|\vec{u}| = \sqrt{x^2 + y^2 + z^2}$$

Direction (Direction Cosines)

$$\cos\alpha = \frac{x}{|\vec{u}|} \qquad \cos\beta = \frac{y}{|\vec{u}|} \qquad \cos\gamma = \frac{z}{|\vec{u}|}$$

Write the vector $\vec{u} = (3, 1, -4)$ as a geometric vector.

Scalar Multiplication

a) $3(4\vec{i} + 2\vec{j} - \vec{k})$

b) $10(3, 7, 1)$

Vector Addition

a) $(5, 7, -3) + (6, 2, 4)$

b) $(2\hat{i} + 15\hat{j} + 3\hat{k}) - (6\hat{i} - 4\hat{j} + 2\hat{k})$

Parallel Vectors

- Parallel vectors have the same or opposite direction.
- Two vectors are parallel if ____

a) For the points A(3, 2, 7), B(4, 5, 1), C(-4, 7, 1), D(-6, 1, 13), determine whether $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.

Collinear Points

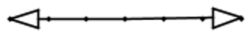
- Three points (A, B, C) are collinear if _____ and _____ are ____________________ .

a) Determine whether the points A (4, -2, 3), B(3, 2, 7), C(1, 10, 15) are collinear.

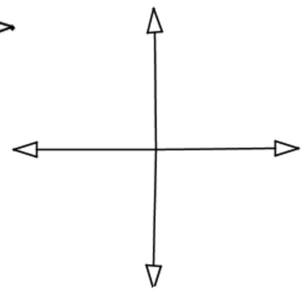
Linear Combinations

Describe what $x = 3$ would mean in each dimension.

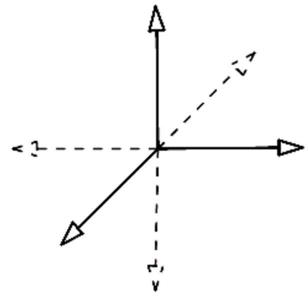
R – line

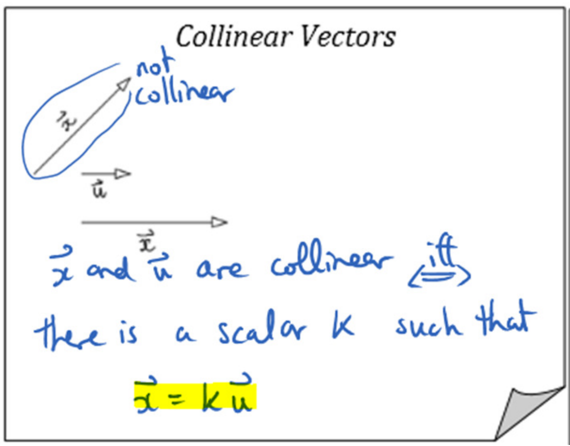


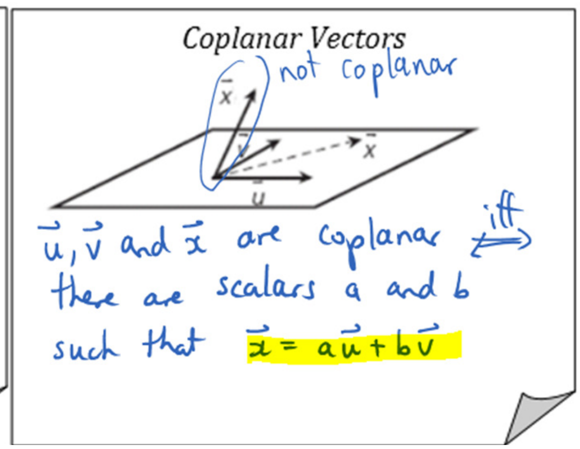
R^2 – plane



R^3 – space







Parallel/Collinear Vectors can be written as ______________________________ of each other

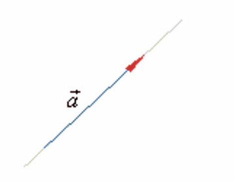
Ex.

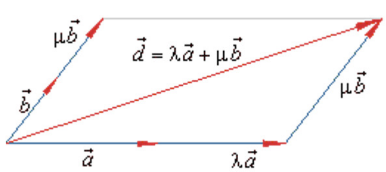
Coplanar Vectors can be written as ______________________________ of each other.

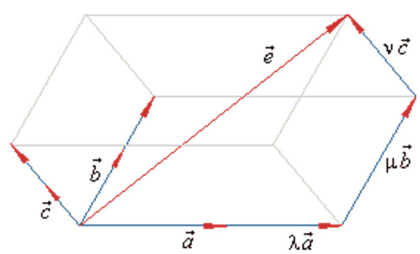
Vectors can also be said to be **Linearly Dependent**

Ex.

Spanning Set for 1D – LINE in R^2 or R^3	Spanning Set for 2D – PLANE in R^3	Spanning Set for 3D – SPACE=R^3







What is the set of all linear combinations of (1, 0, 0) and (0, 1, 1)?

Prove that the vectors $\vec{a} = (-1, 2, -3)$, $\vec{b} = (2, 0, -1)$, and $\vec{c} = (-7, 6, -7)$ are linear dependant.

Dot Product

Dot product is the _________________________________ of one vector with the scalar projection of the other vector.

Determine $\vec{a} \bullet \vec{b}$, if $|\vec{a}| = 20$, $|\vec{b}| = 45$ and $\theta = 55^o$.

Perpendicular Vectors

If two vectors are perpendicular, their dot product ____________________________

Determine $\vec{a} \bullet \vec{b}$, if $|\vec{a}| = 5$, $|\vec{b}| = 10$ and $\theta = 90^o$.

Algebraic Vectors

1. If $\vec{u} = (2, -3, 1)$ and $\vec{v} = (-5, 2, 4)$, calculate $\vec{u} \cdot \vec{v}$.

2. If $\vec{u} = 3\vec{i} + 2\vec{j} + 7\vec{k}$ and $\vec{v} = 5\vec{i} - 9\vec{k}$, find $\vec{u} \cdot \vec{v}$.

3. Find the angle between the vectors $\vec{u} = (-2, 3, 4)$ and $\vec{v} = (1, 5, 2)$.

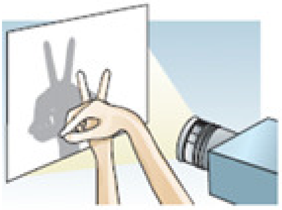
Properties of Dot Product

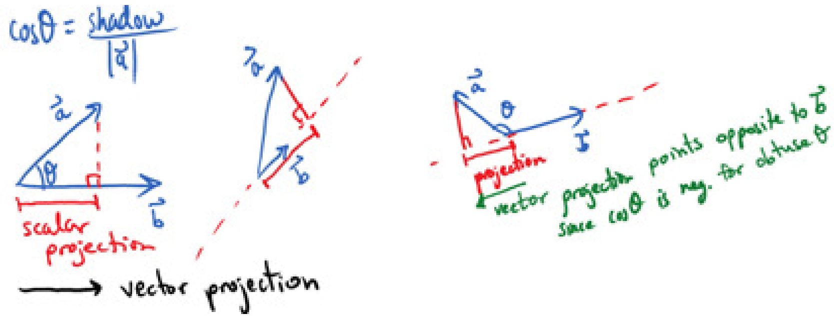
$k(\vec{u} \cdot \vec{v}) =$

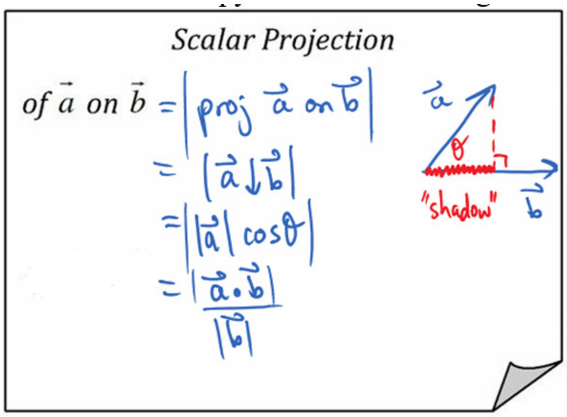
$\vec{u} \cdot (\vec{v} + \vec{w}) =$

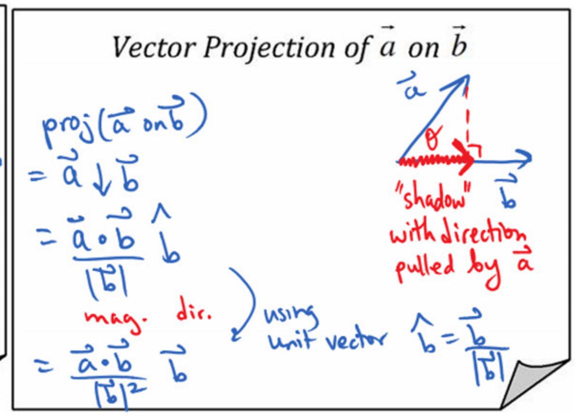
$\vec{u} \cdot \vec{u} =$

Projections

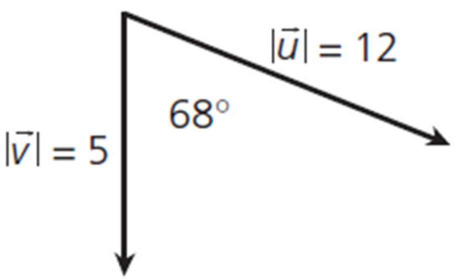




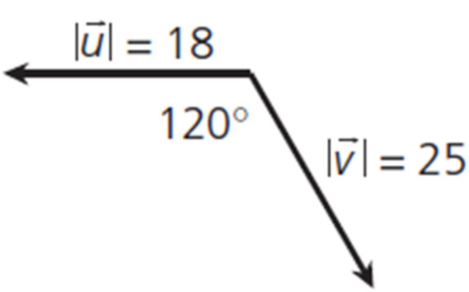




Explain and show how to draw the scalar projection $\left|\vec{v} \downarrow \vec{u}\right|$ on the diagram. Find the magnitude



Explain and show how to draw the vector projection $\vec{v} \downarrow \vec{u}$. Find the vector.



$\vec{a} = [5, 4, -1]$ and $\vec{b} = [1, -2, 3]$.

a) Find $|\vec{a} \downarrow \vec{b}|$

b) Find $\vec{b} \downarrow \vec{a}$

Under what circumstances is

a. $\text{Proj}(\vec{u} \text{ onto } \vec{v}) = \text{Proj}(\vec{v} \text{ onto } \vec{u})$?

b. $\left|\text{Proj}(\vec{u} \text{ onto } \vec{v})\right| = \left|\text{Proj}(\vec{v} \text{ onto } \vec{u})\right|$?

Applications of Dot Product

Work

- Work occurs when a force is applied on an object, resulting in the displacement of the object.
- Measured in Joules

1. A 30 kg child slides down a 3 m long slide that makes an angle of 35º with the ground. Determine the work done by the force of gravity.

2. A mover pushes a washing machine 7.5 m across a basement floor. Calculate the work done by the mover, if he is pushing against a frictional force of 300 N.

Cross Product

The cross product is the ___________________ multiplication of two vectors, which results in a vector that is _______________________________ to the original two vectors.

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\text{Sin}\theta$$

$$\vec{a} \times \vec{b} = (y_a z_b - z_a y_b, z_a x_b - x_a z_b, x_a y_b - y_a x_b)$$

1. If $|\vec{u}| = 10$, $|\vec{v}| = 13$, and the angle between them is 25^{o}, find $|\vec{u} \times \vec{v}|$.

2. If $\vec{a} = (2, 3, 5)$ and $\vec{b} = (4, 7, 6)$, find $\vec{a} \times \vec{b}$.

3. Find a vector perpendicular to $\vec{u} = 10\vec{i} + 3\vec{j} + 7\vec{k}$ and $\vec{v} = -2\vec{i} - 1\vec{j}$.

4. Calculate the magnitude of $\vec{a} \times \vec{b}$ when $\vec{a} = (3, 2, 1)$ and $\vec{b} = (4, -5, 7)$

Properties of Cross Product

$\vec{a} \times \vec{a} =$

$\vec{a} \times \vec{b} =$

$\vec{a} \times (\vec{b} + \vec{c}) =$

$k(\vec{a} \times \vec{b}) =$

1. Simplify $(\vec{a} + \vec{b}) \times (\vec{a} + \vec{b})$.

Applications of Cross Product

Area of a Parallelogram

Find the equation for the area of a parallelogram with sides $\vec{a}$ and $\vec{b}$.

1. Calculate the area of the parallelogram with sides $\vec{u}$ = (10, 4, -1) and $\vec{v}$ = (-2, 5, 3).

Area of a Triangle

Find the equation for the area of a triangle

1. Calculate the area of a triangle with vertices A(1, 5, 0), B(2, 2, 2), and C(5, -1, 3).

Torque

- Torque occurs when a force is applied on an object, resulting in rotational motion.
- Measured in N m

1. Find the torque produced by a mechanic exerting a force of 95 N on the end of a 20 cm long wrench at an angle of 30^o.