A.2   Exponents and Radicals

What you should learn

- Use properties of exponents.
- Use scientific notation to represent real numbers.
- Use properties of radicals.
- Simplify and combine radicals.
- Rationalize denominators and numerators.
- Use properties of rational exponents.

Why you should learn it

Real numbers and algebraic expressions are often written with exponents and radicals. For instance, in Exercise 105 on page A22, you will use an expression involving rational exponents to find the time required for a funnel to empty for different water heights.

Integer Exponents

Repeated *multiplication* can be written in **exponential form.**

Repeated Multiplication	*Exponential Form*
$a \cdot a \cdot a \cdot a \cdot a$	a^5
$(-4)(-4)(-4)$	$(-4)^3$
$(2x)(2x)(2x)(2x)$	$(2x)^4$

Exponential Notation

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

where n is the **exponent** and a is the **base.** The expression a^n is read "a to the nth **power.**"

An exponent can also be negative. In Property 3 below, be sure you see how to use a negative exponent.

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

Property	*Example*
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{x^7}{x^4} = x^{7-4} = x^3$
3. $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$	$y^{-4} = \dfrac{1}{y^4} = \left(\dfrac{1}{y}\right)^4$
4. $a^0 = 1, \quad a \neq 0$	$(x^2 + 1)^0 = 1$
5. $(ab)^m = a^m b^m$	$(5x)^3 = 5^3 x^3 = 125x^3$
6. $(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \dfrac{1}{y^{12}}$
7. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{2}{x}\right)^3 = \dfrac{2^3}{x^3} = \dfrac{8}{x^3}$
8. $|a^2| = |a|^2 = a^2$	$|(-2)^2| = |-2|^2 = (2)^2 = 4$

Technology

You can use a calculator to evaluate exponential expressions. When doing so, it is important to know when to use parentheses because the calculator follows the order of operations. For instance, evaluate $(-2)^4$ as follows

Scientific:

(2 +/−) y^x 4 =

Graphing:

((−) 2) ^ 4 ENTER

The display will be 16. If you omit the parentheses, the display will be −16.

It is important to recognize the difference between expressions such as $(-2)^4$ and -2^4. In $(-2)^4$, the parentheses indicate that the exponent applies to the negative sign as well as to the 2, but in $-2^4 = -(2^4)$, the exponent applies only to the 2. So, $(-2)^4 = 16$ and $-2^4 = -16$.

The properties of exponents listed on the preceding page apply to *all* integers m and n, not just to positive integers as shown in the examples in this section.

Example 1 Using Properties of Exponents

Use the properties of exponents to simplify each expression.

a. $(-3ab^4)(4ab^{-3})$ **b.** $(2xy^2)^3$ **c.** $3a(-4a^2)^0$ **d.** $\left(\dfrac{5x^3}{y}\right)^2$

Solution

a. $(-3ab^4)(4ab^{-3}) = (-3)(4)(a)(a)(b^4)(b^{-3}) = -12a^2b$

b. $(2xy^2)^3 = 2^3(x)^3(y^2)^3 = 8x^3y^6$

c. $3a(-4a^2)^0 = 3a(1) = 3a, \quad a \ne 0$

d. $\left(\dfrac{5x^3}{x}\right)^2 = \dfrac{5^2(x^3)^2}{y^2} = \dfrac{25x^6}{y^2}$

✓CHECKPOINT   Now try Exercise 25.

Example 2 Rewriting with Positive Exponents

Rewrite each expression with positive exponents.

a. x^{-1} **b.** $\dfrac{1}{3x^{-2}}$ **c.** $\dfrac{12a^3b^{-4}}{4a^{-2}b}$ **d.** $\left(\dfrac{3x^2}{y}\right)^{-2}$

Solution

a. $x^{-1} = \dfrac{1}{x}$ Property 3

b. $\dfrac{1}{3x^{-2}} = \dfrac{1(x^2)}{3} = \dfrac{x^2}{3}$ The exponent -2 does not apply to 3.

c. $\dfrac{12a^3b^{-4}}{4a^{-2}b} = \dfrac{12a^3 \cdot a^2}{4b \cdot b^4} = \dfrac{3a^5}{b^5}$ Properties 3 and 1

d. $\left(\dfrac{3x^2}{y}\right)^{-2} = \dfrac{3^{-2}(x^2)^{-2}}{y^{-2}}$ Properties 5 and 7

              $= \dfrac{3^{-2}x^{-4}}{y^{-2}}$ Property 6

              $= \dfrac{y^2}{3^2x^4}$ Property 3

              $= \dfrac{y^2}{9x^4}$ Simplify.

✓CHECKPOINT   Now try Exercise 33.

STUDY TIP

Rarely in algebra is there only one way to solve a problem. Don't be concerned if the steps you use to solve a problem are not exactly the same as the steps presented in this text. The important thing is to use steps that you understand and, of course, steps that are justified by the rules of algebra. For instance, you might prefer the following steps for Example 2(d).

$$\left(\frac{3x^2}{y}\right)^{-2} = \left(\frac{y}{3x^2}\right)^2 = \frac{y^2}{9x^4}$$

Note how Property 3 is used in the first step of this solution. The fractional form of this property is

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m.$$

Scientific Notation

Exponents provide an efficient way of writing and computing with very large (or very small) numbers. For instance, there are about 359 billion billion gallons of water on Earth—that is, 359 followed by 18 zeros.

$$359,000,000,000,000,000,000$$

It is convenient to write such numbers in **scientific notation.** This notation has the form $\pm c \times 10^n$, where $1 \leq c < 10$ and n is an integer. So, the number of gallons of water on Earth can be written in scientific notation as

$$3.59 \times 100,000,000,000,000,000,000 = 3.59 \times 10^{20}.$$

The *positive* exponent 20 indicates that the number is *large* (10 or more) and that the decimal point has been moved 20 places. A *negative* exponent indicates that the number is *small* (less than 1). For instance, the mass (in grams) of one electron is approximately

$$9.0 \times 10^{-28} = 0.0000000000000000000000000009.$$

<div align="center">28 decimal places</div>

Example 3   **Scientific Notation**

Write each number in scientific notation.

a. 0.0000782   **b.** 836,100,000

Solution

a. $0.0000782 = 7.82 \times 10^{-5}$ **b.** $836,100,000 = 8.361 \times 10^8$

✓CHECKPOINT   Now try Exercise 37.

Example 4   **Decimal Notation**

Write each number in decimal notation.

a. 9.36×10^{-6} **b.** 1.345×10^2

Solution

a. $9.36 \times 10^{-6} = 0.00000936$ **b.** $1.345 \times 10^2 = 134.5$

✓CHECKPOINT   Now try Exercise 41.

Technology

Most calculators automatically switch to scientific notation when they are showing large (or small) numbers that exceed the display range.

To *enter* numbers in scientific notation, your calculator should have an exponential entry key labeled

EE   or   EXP.

Consult the user's guide for your calculator for instructions on keystrokes and how numbers in scientific notation are displayed.

Radicals and Their Properties

A **square root** of a number is one of its two equal factors. For example, 5 is a square root of 25 because 5 is one of the two equal factors of 25. In a similar way, a **cube root** of a number is one of its three equal factors, as in $125 = 5^3$.

Definition of *n*th Root of a Number

Let a and b be real numbers and let $n \geq 2$ be a positive integer. If

$$a = b^n$$

then b is an **nth root of a.** If $n = 2$, the root is a **square root.** If $n = 3$, the root is a **cube root.**

Some numbers have more than one nth root. For example, both 5 and -5 are square roots of 25. The *principal square root* of 25, written as $\sqrt{25}$, is the positive root, 5. The **principal nth root** of a number is defined as follows.

Principal *n*th Root of a Number

Let a be a real number that has at least one nth root. The **principal nth root of a** is the nth root that has the same sign as a. It is denoted by a **radical symbol**

$$\sqrt[n]{a}. \qquad \text{Principal } n\text{th root}$$

The positive integer n is the **index** of the radical, and the number a is the **radicand.** If $n = 2$, omit the index and write $\sqrt{a}$ rather than $\sqrt[2]{a}$. (The plural of index is *indices*.)

A common misunderstanding is that the square root sign implies both negative and positive roots. This is not correct. The square root sign implies only a positive root. When a negative root is needed, you must use the negative sign with the square root sign.

Incorrect: $\sqrt{4} = \pm 2$ *Correct:* $-\sqrt{4} = -2$ *and* $\sqrt{4} = 2$

Example 5   Evaluating Expressions Involving Radicals

a. $\sqrt{36} = 6$ because $6^2 = 36$.

b. $-\sqrt{36} = -6$ because $-\left(\sqrt{36}\right) = -\left(\sqrt{6^2}\right) = -(6) = -6$.

c. $\sqrt[3]{\dfrac{125}{64}} = \dfrac{5}{4}$ because $\left(\dfrac{5}{4}\right)^3 = \dfrac{5^3}{4^3} = \dfrac{125}{64}$.

d. $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$.

e. $\sqrt[4]{-81}$ is not a real number because there is no real number that can be raised to the fourth power to produce -81.

✓**CHECKPOINT**   Now try Exercise 51.

Here are some generalizations about the nth roots of real numbers.

Generalizations About nth Roots of Real Numbers

Real Number a	Integer n	Root(s) of a	Example
$a > 0$	$n > 0$, is even.	$\sqrt[n]{a}, \ -\sqrt[n]{a}$	$\sqrt[4]{81} = 3, \ -\sqrt[4]{81} = -3$
$a > 0$ or $a < 0$	n is odd.	$\sqrt[n]{a}$	$\sqrt[3]{-8} = -2$
$a < 0$	n is even.	No real roots	$\sqrt{-4}$ is not a real number.
$a = 0$	n is even or odd.	$\sqrt[n]{0} = 0$	$\sqrt[5]{0} = 0$

Integers such as 1, 4, 9, 16, 25, and 36 are called **perfect squares** because they have integer square roots. Similarly, integers such as 1, 8, 27, 64, and 125 are called **perfect cubes** because they have integer cube roots.

Properties of Radicals

Let a and b be real numbers, variables, or algebraic expressions such that the indicated roots are real numbers, and let m and n be positive integers.

Property	*Example*
1. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$\sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{5} \cdot \sqrt{7} = \sqrt{5 \cdot 7} = \sqrt{35}$
3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}, \quad b \neq 0$	$\dfrac{\sqrt[4]{27}}{\sqrt[4]{9}} = \sqrt[4]{\dfrac{27}{9}} = \sqrt[4]{3}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{10}} = \sqrt[6]{10}$
5. $\left(\sqrt[n]{a}\right)^n = a$	$\left(\sqrt{3}\right)^2 = 3$
6. For n even, $\sqrt[n]{a^n} = |a|$.	$\sqrt{(-12)^2} = |-12| = 12$
For n odd, $\sqrt[n]{a^n} = a$.	$\sqrt[3]{(-12)^3} = -12$

A common special case of Property 6 is $\sqrt{a^2} = |a|$.

Example 6   **Using Properties of Radicals**

Use the properties of radicals to simplify each expression.

a. $\sqrt{8} \cdot \sqrt{2}$ **b.** $\left(\sqrt[3]{5}\right)^3$ **c.** $\sqrt[3]{x^3}$ **d.** $\sqrt[6]{y^6}$

Solution

a. $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$
b. $\left(\sqrt[3]{5}\right)^3 = 5$
c. $\sqrt[3]{x^3} = x$
d. $\sqrt[6]{y^6} = |y|$

✓**CHECKPOINT**   Now try Exercise 61.

Simplifying Radicals

An expression involving radicals is in **simplest form** when the following conditions are satisfied.

1. All possible factors have been removed from the radical.

2. All fractions have radical-free denominators (accomplished by a process called *rationalizing the denominator*).

3. The index of the radical is reduced.

To simplify a radical, factor the radicand into factors whose exponents are multiples of the index. The roots of these factors are written outside the radical, and the "leftover" factors make up the new radicand.

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STUDY TIP

When you simplify a radical, it is important that both expressions are defined for the same values of the variable. For instance, in Example 7(b), $\sqrt{75x^3}$ and $5x\sqrt{3x}$ are both defined only for nonnegative values of x. Similarly, in Example 7(c), $\sqrt[4]{(5x)^4}$ and $5|x|$ are both defined for all real values of x.

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Example 7   **Simplifying Even Roots**

Perfect 4th power   Leftover factor

a. $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = \sqrt[4]{2^4 \cdot 3} = 2\sqrt[4]{3}$

Perfect square   Leftover factor

b. $\sqrt{75x^3} = \sqrt{25x^2 \cdot 3x}$ Find largest square factor.

$\qquad = \sqrt{(5x)^2 \cdot 3x}$

$\qquad = 5x\sqrt{3x}$ Find root of perfect square.

c. $\sqrt[4]{(5x)^4} = |5x| = 5|x|$

✓CHECKPOINT   Now try Exercise 63(a).

Example 8   **Simplifying Odd Roots**

Perfect cube   Leftover factor

a. $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{2^3 \cdot 3} = 2\sqrt[3]{3}$

Perfect cube   Leftover factor

b. $\sqrt[3]{24a^4} = \sqrt[3]{8a^3 \cdot 3a}$ Find largest cube factor.

$\qquad = \sqrt[3]{(2a)^3 \cdot 3a}$

$\qquad = 2a\sqrt[3]{3a}$ Find root of perfect cube.

c. $\sqrt[3]{-40x^6} = \sqrt[3]{(-8x^6) \cdot 5}$ Find largest cube factor.

$\qquad = \sqrt[3]{(-2x^2)^3 \cdot 5}$

$\qquad = -2x^2\sqrt[3]{5}$ Find root of perfect cube.

✓CHECKPOINT   Now try Exercise 63(b).

Radical expressions can be combined (added or subtracted) if they are **like radicals**—that is, if they have the same index and radicand. For instance, $\sqrt{2}$, $3\sqrt{2}$, and $\frac{1}{2}\sqrt{2}$ are like radicals, but $\sqrt{3}$ and $\sqrt{2}$ are unlike radicals. To determine whether two radicals can be combined, you should first simplify each radical.

Example 9   Combining Radicals

a. $2\sqrt{48} - 3\sqrt{27} = 2\sqrt{16 \cdot 3} - 3\sqrt{9 \cdot 3}$ Find square factors.

$\qquad\qquad\qquad = 8\sqrt{3} - 9\sqrt{3}$ Find square roots and multiply by coefficients.

$\qquad\qquad\qquad = (8 - 9)\sqrt{3}$ Combine like terms.

$\qquad\qquad\qquad = -\sqrt{3}$ Simplify.

b. $\sqrt[3]{16x} - \sqrt[3]{54x^4} = \sqrt[3]{8 \cdot 2x} - \sqrt[3]{27 \cdot x^3 \cdot 2x}$ Find cube factors.

$\qquad\qquad\qquad = 2\sqrt[3]{2x} - 3x\sqrt[3]{2x}$ Find cube roots.

$\qquad\qquad\qquad = (2 - 3x)\sqrt[3]{2x}$ Combine like terms.

✔*CHECKPOINT*   Now try Exercise 71.

Rationalizing Denominators and Numerators

To rationalize a denominator or numerator of the form $a - b\sqrt{m}$ or $a + b\sqrt{m}$, multiply both numerator and denominator by a **conjugate:** $a + b\sqrt{m}$ and $a - b\sqrt{m}$ are conjugates of each other. If $a = 0$, then the rationalizing factor for $\sqrt{m}$ is itself, $\sqrt{m}$. For cube roots, choose a rationalizing factor that generates a perfect cube.

Example 10   Rationalizing Single-Term Denominators

Rationalize the denominator of each expression.

a. $\dfrac{5}{2\sqrt{3}}$ **b.** $\dfrac{2}{\sqrt[3]{5}}$

Solution

a. $\dfrac{5}{2\sqrt{3}} = \dfrac{5}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$ $\sqrt{3}$ is rationalizing factor.

$\qquad = \dfrac{5\sqrt{3}}{2(3)}$ Multiply.

$\qquad = \dfrac{5\sqrt{3}}{6}$ Simplify.

b. $\dfrac{2}{\sqrt[3]{5}} = \dfrac{2}{\sqrt[3]{5}} \cdot \dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$ $\sqrt[3]{5^2}$ is rationalizing factor.

$\qquad = \dfrac{2\sqrt[3]{5^2}}{\sqrt[3]{5^3}}$ Multiply.

$\qquad = \dfrac{2\sqrt[3]{25}}{5}$ Simplify.

✔*CHECKPOINT*   Now try Exercise 79.

Additional Examples

a. $\dfrac{3}{\sqrt{7}} = \dfrac{3}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{3\sqrt{7}}{7}$

b. $\dfrac{2}{\sqrt[3]{4}} = \dfrac{2}{\sqrt[3]{4}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$= \dfrac{2\sqrt[3]{2}}{\sqrt[3]{8}}$

$= \dfrac{2\sqrt[3]{2}}{2}$

$= \sqrt[3]{2}$

c. $\dfrac{6}{\sqrt{2} + \sqrt{3}}$

$= \dfrac{6}{\sqrt{2} + \sqrt{3}} \cdot \dfrac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}}$

$= \dfrac{6(\sqrt{2} - \sqrt{3})}{-1}$

$= -6\sqrt{2} + 6\sqrt{3}$

Example 11   **Rationalizing a Denominator with Two Terms**

$\dfrac{2}{3 + \sqrt{7}} = \dfrac{2}{3 + \sqrt{7}} \cdot \dfrac{3 - \sqrt{7}}{3 - \sqrt{7}}$ Multiply numerator and denominator by conjugate of denominator.

$= \dfrac{2(3 - \sqrt{7})}{3(3) + 3(-\sqrt{7}) + \sqrt{7}(3) - (\sqrt{7})(\sqrt{7})}$ Use Distributive Property.

$= \dfrac{2(3 - \sqrt{7})}{(3)^2 - (\sqrt{7})^2}$ Simplify.

$= \dfrac{2(3 - \sqrt{7})}{9 - 7}$ Square terms of denominator.

$= \dfrac{2(3 - \sqrt{7})}{2} = 3 - \sqrt{7}$ Simplify.

✓**CHECKPOINT**   Now try Exercise 81.

Sometimes it is necessary to rationalize the numerator of an expression. For instance, in Appendix A.4 you will use the technique shown in the next example to rationalize the numerator of an expression from calculus.

STUDY TIP

Do not confuse the expression $\sqrt{5} + \sqrt{7}$ with the expression $\sqrt{5 + 7}$. In general, $\sqrt{x + y}$ does not equal $\sqrt{x} + \sqrt{y}$. Similarly, $\sqrt{x^2 + y^2}$ does not equal $x + y$.

Example 12   **Rationalizing a Numerator**   ∫

$\dfrac{\sqrt{5} - \sqrt{7}}{2} = \dfrac{\sqrt{5} - \sqrt{7}}{2} \cdot \dfrac{\sqrt{5} + \sqrt{7}}{\sqrt{5} + \sqrt{7}}$ Multiply numerator and denominator by conjugate of numerator.

$= \dfrac{(\sqrt{5})^2 - (\sqrt{7})^2}{2(\sqrt{5} + \sqrt{7})}$ Simplify.

$= \dfrac{5 - 7}{2(\sqrt{5} + \sqrt{7})}$ Square terms of numerator.

$= \dfrac{-2}{2(\sqrt{5} + \sqrt{7})} = \dfrac{-1}{\sqrt{5} + \sqrt{7}}$ Simplify.

✓**CHECKPOINT**   Now try Exercise 85.

Rational Exponents

> **Definition of Rational Exponents**
>
> If a is a real number and n is a positive integer such that the principal nth root of a exists, then $a^{1/n}$ is defined as
>
> $a^{1/n} = \sqrt[n]{a}$, where $1/n$ is the **rational exponent** of a.
>
> Moreover, if m is a positive integer that has no common factor with n, then
>
> $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$ and $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$.

The symbol ∫ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

The numerator of a rational exponent denotes the *power* to which the base is raised, and the denominator denotes the *index* or the *root* to be taken.

$$\underset{\text{Index}}{\underset{\downarrow}{}}\ \ \overset{\text{Power}}{\overset{\downarrow}{}}$$

$$b^{m/n} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

When you are working with rational exponents, the properties of integer exponents still apply. For instance,

$$2^{1/2}2^{1/3} = 2^{(1/2)+(1/3)} = 2^{5/6}.$$

Example 13 Changing from Radical to Exponential Form

a. $\sqrt{3} = 3^{1/2}$

b. $\sqrt{(3xy)^5} = \sqrt[2]{(3xy)^5} = (3xy)^{(5/2)}$

c. $2x\sqrt[4]{x^3} = (2x)(x^{3/4}) = 2x^{1+(3/4)} = 2x^{7/4}$

✓**CHECKPOINT**   Now try Exercise 87.

Example 14 Changing from Exponential to Radical Form

a. $(x^2 + y^2)^{3/2} = \left(\sqrt{x^2 + y^2}\right)^3 = \sqrt{(x^2 + y^2)^3}$

b. $2y^{3/4}z^{1/4} = 2(y^3z)^{1/4} = 2\sqrt[4]{y^3z}$

c. $a^{-3/2} = \dfrac{1}{a^{3/2}} = \dfrac{1}{\sqrt{a^3}}$ **d.** $x^{0.2} = x^{1/5} = \sqrt[5]{x}$

✓**CHECKPOINT**   Now try Exercise 89.

Rational exponents are useful for evaluating roots of numbers on a calculator, for reducing the index of a radical, and for simplifying expressions in calculus.

Example 15 Simplifying with Rational Exponents

a. $(-32)^{-4/5} = \left(\sqrt[5]{-32}\right)^{-4} = (-2)^{-4} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}$

b. $(-5x^{5/3})(3x^{-3/4}) = -15x^{(5/3)-(3/4)} = -15x^{11/12}, \quad x \neq 0$

c. $\sqrt[9]{a^3} = a^{3/9} = a^{1/3} = \sqrt[3]{a}$ Reduce index.

d. $\sqrt[3]{\sqrt{125}} = \sqrt[6]{125} = \sqrt[6]{(5)^3} = 5^{3/6} = 5^{1/2} = \sqrt{5}$

e. $(2x - 1)^{4/3}(2x - 1)^{-1/3} = (2x - 1)^{(4/3)-(1/3)}$

$$= 2x - 1, \quad x \neq \dfrac{1}{2}$$

f. $\dfrac{x - 1}{(x - 1)^{-1/2}} = \dfrac{x - 1}{(x - 1)^{-1/2}} \cdot \dfrac{(x - 1)^{1/2}}{(x - 1)^{1/2}}$

$$= \dfrac{(x - 1)^{3/2}}{(x - 1)^0}$$

$$= (x - 1)^{3/2}, \quad x \neq 1$$

✓**CHECKPOINT**   Now try Exercise 99.

A.2 | Exercises

VOCABULARY CHECK: Fill in the blanks.

1. In the exponential form a^n, n is the _____ and a is the _____.
2. A convenient way of writing very large or very small numbers is called _____ _____.
3. One of the two equal factors of a number is called a _____ _____ of the number.
4. The _____ _____ _____ of a number is the nth root that has the same sign as a, and is denoted by $\sqrt[n]{a}$.
5. In the radical form, $\sqrt[n]{a}$ the positive integer n is called the _____ of the radical and the number a is called the _____.
6. When an expression involving radicals has all possible factors removed, radical-free denominators, and a reduced index, it is in _____ _____.
7. The expressions $a + b\sqrt{m}$ and $a - b\sqrt{m}$ are _____ of each other.
8. The process used to create a radical-free denominator is know as _____ the denominator.
9. In the expression $b^{m/n}$, m denotes the _____ to which the base is raised and n denotes the _____ or root to be taken.

In Exercises 1 and 2, write the expression as a repeated multiplication problem.

1. 8^5
2. $(-2)^7$

In Exercises 3 and 4, write the expression using exponential notation.

3. $(4.9)(4.9)(4.9)(4.9)(4.9)(4.9)$
4. $(-10)(-10)(-10)(-10)(-10)$

In Exercises 5–12, evaluate each expression.

5. (a) $3^2 \cdot 3$ (b) $3 \cdot 3^3$
6. (a) $\dfrac{5^5}{5^2}$ (b) $\dfrac{3^2}{3^4}$
7. (a) $(3^3)^0$ (b) -3^2
8. (a) $(2^3 \cdot 3^2)^2$ (b) $\left(-\tfrac{3}{5}\right)^3\left(\tfrac{5}{3}\right)^2$
9. (a) $\dfrac{3 \cdot 4^{-4}}{3^{-4} \cdot 4^{-1}}$ (b) $32(-2)^{-5}$
10. (a) $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$ (b) $(-2)^0$
11. (a) $2^{-1} + 3^{-1}$ (b) $(2^{-1})^{-2}$
12. (a) $3^{-1} + 2^{-2}$ (b) $(3^{-2})^2$

In Exercises 13–16, use a calculator to evaluate the expression. (If necessary, round your answer to three decimal places.)

13. $(-4)^3(5^2)$
14. $(8^{-4})(10^3)$
15. $\dfrac{3^6}{7^3}$
16. $\dfrac{4^3}{3^{-4}}$

In Exercises 17–24, evaluate the expression for the given value of x.

	Expression	*Value*
17.	$-3x^3$	$x = 2$
18.	$7x^{-2}$	$x = 4$
19.	$6x^0$	$x = 10$
20.	$5(-x)^3$	$x = 3$
21.	$2x^3$	$x = -3$
22.	$-3x^4$	$x = -2$
23.	$4x^2$	$x = -\tfrac{1}{2}$
24.	$5(-x)^3$	$x = -\tfrac{1}{3}$

In Exercises 25–30, simplify each expression.

25. (a) $(-5z)^3$ (b) $5x^4(x^2)$
26. (a) $(3x)^2$ (b) $(4x^3)^0$
27. (a) $6y^2(2y^0)^2$ (b) $\dfrac{3x^5}{x^3}$
28. (a) $(-z)^3(3z^4)$ (b) $\dfrac{25y^8}{10y^4}$
29. (a) $\dfrac{7x^2}{x^3}$ (b) $\dfrac{12(x + y)^3}{9(x + y)}$
30. (a) $\dfrac{r^4}{r^6}$ (b) $\left(\dfrac{4}{y}\right)^3\left(\dfrac{3}{y}\right)^4$

In Exercises 31–36, rewrite each expression with positive exponents and simplify.

31. (a) $(x + 5)^0, \quad x \neq -5$ (b) $(2x^2)^{-2}$
32. (a) $(2x^5)^0, \quad x \neq 0$ (b) $(z + 2)^{-3}(z + 2)^{-1}$

33. (a) $(-2x^2)^3(4x^3)^{-1}$ (b) $\left(\dfrac{x}{10}\right)^{-1}$

34. (a) $(4y^{-2})(8y^4)$ (b) $\left(\dfrac{x^{-3}y^4}{5}\right)^{-3}$

35. (a) $3^n \cdot 3^{2n}$ (b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3$

36. (a) $\dfrac{x^2 \cdot x^n}{x^3 \cdot x^n}$ (b) $\left(\dfrac{a^{-3}}{b^{-3}}\right)\left(\dfrac{a}{b}\right)^3$

In Exercises 37–40, write the number in scientific notation.

37. Land area of Earth: 57,300,000 square miles

38. Light year: 9,460,000,000,000 kilometers

39. Relative density of hydrogen: 0.0000899 gram per cubic centimeter

40. One micron (millionth of a meter): 0.00003937 inch

In Exercises 41–44, write the number in decimal notation.

41. Worldwide daily consumption of Coca-Cola: 4.568×10^9 ounces (Source: The Coca-Cola Company)

42. Interior temperature of the sun: 1.5×10^7 degrees Celsius

43. Charge of an electron: 1.6022×10^{-19} coulomb

44. Width of a human hair: 9.0×10^{-5} meter

In Exercises 45 and 46, evaluate each expression without using a calculator.

45. (a) $\sqrt{25 \times 10^8}$ (b) $\sqrt[3]{8 \times 10^{15}}$

46. (a) $(1.2 \times 10^7)(5 \times 10^{-3})$ (b) $\dfrac{(6.0 \times 10^8)}{(3.0 \times 10^{-3})}$

In Exercises 47–50, use a calculator to evaluate each expression. (Round your answer to three decimal places.)

47. (a) $750\left(1 + \dfrac{0.11}{365}\right)^{800}$

     (b) $\dfrac{67,000,000 + 93,000,000}{0.0052}$

48. (a) $(9.3 \times 10^6)^3(6.1 \times 10^{-4})$

     (b) $\dfrac{(2.414 \times 10^4)^6}{(1.68 \times 10^5)^5}$

49. (a) $\sqrt{4.5 \times 10^9}$ (b) $\sqrt[3]{6.3 \times 10^4}$

50. (a) $(2.65 \times 10^{-4})^{1/3}$ (b) $\sqrt{9 \times 10^{-4}}$

In Exercises 51–56, evaluate each expression without using a calculator.

51. (a) $\sqrt{9}$ (b) $\sqrt[3]{\frac{27}{8}}$

52. (a) $27^{1/3}$ (b) $36^{3/2}$

53. (a) $32^{-3/5}$ (b) $\left(\frac{16}{81}\right)^{-3/4}$

54. (a) $100^{-3/2}$ (b) $\left(\frac{9}{4}\right)^{-1/2}$

55. (a) $\left(-\dfrac{1}{64}\right)^{-1/3}$ (b) $\left(\dfrac{1}{\sqrt{32}}\right)^{-2/5}$

56. (a) $\left(-\dfrac{125}{27}\right)^{-1/3}$ (b) $-\left(\dfrac{1}{125}\right)^{-4/3}$

In Exercises 57–60, use a calculator to approximate the number. (Round your answer to three decimal places.)

57. (a) $\sqrt{57}$ (b) $\sqrt[5]{-27^3}$

58. (a) $\sqrt[3]{45^2}$ (b) $\sqrt[6]{125}$

59. (a) $(-12.4)^{-1.8}$ (b) $(5\sqrt{3})^{-2.5}$

60. (a) $\dfrac{7 - (4.1)^{-3.2}}{2}$ (b) $\left(\dfrac{13}{3}\right)^{-3/2} - \left(-\dfrac{3}{2}\right)^{13/3}$

In Exercises 61 and 62, use the properties of radicals to simplify each expression.

61. (a) $\left(\sqrt[3]{4}\right)^3$ (b) $\sqrt[5]{96x^5}$

62. (a) $\sqrt{12} \cdot \sqrt{3}$ (b) $\sqrt[4]{(3x^2)^4}$

In Exercises 63–74, simplify each radical expression.

63. (a) $\sqrt{8}$ (b) $\sqrt[3]{54}$

64. (a) $\sqrt[3]{\frac{16}{27}}$ (b) $\sqrt{\frac{75}{4}}$

65. (a) $\sqrt{72x^3}$ (b) $\sqrt{\dfrac{18^2}{z^3}}$

66. (a) $\sqrt{54xy^4}$ (b) $\sqrt{\dfrac{32a^4}{b^2}}$

67. (a) $\sqrt[3]{16x^5}$ (b) $\sqrt{75x^2y^{-4}}$

68. (a) $\sqrt[4]{3x^4y^2}$ (b) $\sqrt[5]{160x^8z^4}$

69. (a) $2\sqrt{50} + 12\sqrt{8}$ (b) $10\sqrt{32} - 6\sqrt{18}$

70. (a) $4\sqrt{27} - \sqrt{75}$ (b) $\sqrt[3]{16} + 3\sqrt[3]{54}$

71. (a) $5\sqrt{x} - 3\sqrt{x}$ (b) $-2\sqrt{9y} + 10\sqrt{y}$

72. (a) $8\sqrt{49x} - 14\sqrt{100x}$ (b) $-3\sqrt{48x^2} + 7\sqrt{75x^2}$

73. (a) $3\sqrt{x+1} + 10\sqrt{x+1}$ (b) $7\sqrt{80x} - 2\sqrt{125x}$

74. (a) $-\sqrt{x^3 - 7} + 5\sqrt{x^3 - 7}$ (b) $11\sqrt{245x^3} - 9\sqrt{45x^3}$

In Exercises 75–78, complete the statement with <, =, or >.

75. $\sqrt{5} + \sqrt{3}$ ▢ $\sqrt{5+3}$ **76.** $\sqrt{\dfrac{3}{11}}$ ▢ $\dfrac{\sqrt{3}}{\sqrt{11}}$

77. 5 ▢ $\sqrt{3^2 + 2^2}$ **78.** 5 ▢ $\sqrt{3^2 + 4^2}$

In Exercises 79–82, rationalize the denominator of the expression. Then simplify your answer.

79. $\dfrac{1}{\sqrt{3}}$ **80.** $\dfrac{5}{\sqrt{10}}$

81. $\dfrac{2}{5 - \sqrt{3}}$

82. $\dfrac{3}{\sqrt{5} + \sqrt{6}}$

∫ **In Exercises 83–86, rationalize the numerator of the expression. Then simplify your answer.**

83. $\dfrac{\sqrt{8}}{2}$

84. $\dfrac{\sqrt{2}}{3}$

85. $\dfrac{\sqrt{5} + \sqrt{3}}{3}$

86. $\dfrac{\sqrt{7} - 3}{4}$

In Exercises 87–94, fill in the missing form of the expression.

Radical Form	*Rational Exponent Form*
87. $\sqrt{9}$	
88. $\sqrt[3]{64}$	
89.	$32^{1/5}$
90.	$-(144^{1/2})$
91. $\sqrt[3]{-216}$	
92.	$(-243)^{1/5}$
93. $\left(\sqrt[4]{81}\right)^3$	
94.	$16^{5/4}$

In Exercises 95–98, perform the operations and simplify.

95. $\dfrac{(2x^2)^{3/2}}{2^{1/2}x^4}$

96. $\dfrac{x^{4/3}y^{2/3}}{(xy)^{1/3}}$

97. $\dfrac{x^{-3} \cdot x^{1/2}}{x^{3/2} \cdot x^{-1}}$

98. $\dfrac{5^{-1/2} \cdot 5x^{5/2}}{(5x)^{3/2}}$

In Exercises 99 and 100, reduce the index of each radical.

99. (a) $\sqrt[4]{3^2}$ (b) $\sqrt[6]{(x+1)^4}$

100. (a) $\sqrt[6]{x^3}$ (b) $\sqrt[4]{(3x^2)^4}$

In Exercises 101 and 102, write each expression as a single radical. Then simplify your answer.

101. (a) $\sqrt{\sqrt{32}}$ (b) $\sqrt{\sqrt[4]{2x}}$

102. (a) $\sqrt{\sqrt{243(x+1)}}$ (b) $\sqrt{\sqrt[3]{10a^7b}}$

103. *Period of a Pendulum*   The period T (in seconds) of a pendulum is

$$T = 2\pi\sqrt{\dfrac{L}{32}}$$

where L is the length of the pendulum (in feet). Find the period of a pendulum whose length is 2 feet.

The symbol ∫ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

The symbol ⬚ indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

104. *Erosion*   A stream of water moving at the rate of v feet per second can carry particles of size $0.03\sqrt{v}$ inches. Find the size of the largest particle that can be carried by a stream flowing at the rate of $\frac{3}{4}$ foot per second.

105. *Mathematical Modeling*   A funnel is filled with water to a height of h centimeters. The formula

$$t = 0.03[12^{5/2} - (12 - h)^{5/2}], \quad 0 \le h \le 12$$

represents the amount of time t (in seconds) that it will take for the funnel to empty.

⬚ (a) Use the table feature of a graphing utility to find the times required for the funnel to empty for water heights of $h = 0$, $h = 1$, $h = 2$, ... $h = 12$ centimeters.

(b) What value does t appear to be approaching as the height of the water becomes closer and closer to 12 centimeters?

106. *Speed of Light*   The speed of light is approximately 11,180,000 miles per minute. The distance from the sun to Earth is approximately 93,000,000 miles. Find the time for light to travel from the sun to Earth.

Synthesis

True or False?   **In Exercises 107 and 108, determine whether the statement is true or false. Justify your answer.**

107. $\dfrac{x^{k+1}}{x} = x^k$

108. $(a^n)^k = a^{n^k}$

109. Verify that $a^0 = 1$, $a \ne 0$. (*Hint:* Use the property of exponents $a^m/a^n = a^{m-n}$.)

110. Explain why each of the following pairs is not equal.

(a) $(3x)^{-1} \ne \dfrac{3}{x}$ (b) $y^3 \cdot y^2 \ne y^6$

(c) $(a^2b^3)^4 \ne a^6b^7$ (d) $(a + b)^2 \ne a^2 + b^2$

(e) $\sqrt{4x^2} \ne 2x$ (f) $\sqrt{2} + \sqrt{3} \ne \sqrt{5}$

111. *Exploration*   List all possible digits that occur in the units place of the square of a positive integer. Use that list to determine whether $\sqrt{5233}$ is an integer.

112. *Think About It*   Square the real number $2/\sqrt{5}$ and note that the radical is eliminated from the denominator. Is this equivalent to rationalizing the denominator? Why or why not?