MCV4U1

Unit #2 – Algebraic Vectors

Progress Check

1. Complete the following table:

Geometric Vector	Algebraic Vector Coordinate Form	Algebraic Vector Unit Vector Form
u = 35 cm [S40^{o}W]		
		v = $10\vec{i} + 7\vec{j}$
	w = (-3, 0, 5)	

2. Find the vector $\overrightarrow{UV}$ where U(3, -2, 9) and V(21, 25, -6).

3. Graph the following vectors:

a) $\vec{u} = (3, -5)$

b) $\vec{v} = (4, -3, 5)$

4. If $\vec{a} = 15\vec{i} - 20\vec{j} + 7\vec{k}$ and $\vec{b} = 5\vec{j} + 13\vec{k}$, find $|4\vec{a} - 3\vec{b}|$.

5. If the points A(x, 10, -2), B(25, 2, 15) and C(69, -14, z) are collinear, solve for x and z.

6. Determine whether or not $\vec{u}$ = (10, 20, 30) and $\vec{v}$ = (2, -1, 1) are perpendicular.

7. Calculate the dot product of the following:

a) $\vec{u}$ = (-10, 13, -4) and $\vec{v}$ = (2, 0, 15)

b) $|\vec{u}| = 80$, $|\vec{v}| = 25$, $\theta = 78^{o}$

8. Calculate the cross product of $\vec{a}$ = (100, 56, 243) and $\vec{b}$ = (-30, 28, 95).

9. Calculate $|4\vec{a} \times 3\vec{b}|$ where $|\vec{a}| = 15$, $|\vec{b}| = 38$ and $\theta = 11^{o}$.

10. Calculate the area of a parallelogram with sides $\vec{a}$ = (4, -20, 15) and $\vec{b}$ = (9, 12, -5).

11. Calculate the area of triangle ABC, where A(0, 2, 7), B(11, -14, 8), and C(-7, 7, 1).

12. Miss Pick pushes a box of textbooks 3 metres across the floor with her foot. If she has to work against a frictional force of 80 N, how much work must she do to move the box of books.

Solutions

MCV4U1

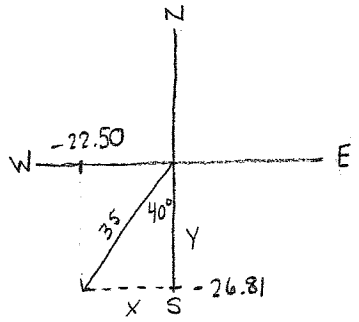
Unit #2 – Algebraic Vectors

Progress Check

1. Complete the following table:

	Geometric Vector	Algebraic Vector Coordinate Form	Algebraic Vector Unit Vector Form
①	$\vec{u} = 35$ cm [S40°W]	$\vec{u} = (-22.50, -26.81)$	$\vec{u} = -22.50\vec{i} - 26.81\vec{j}$
②	$\vec{v} = \sqrt{149}$ units [N55.01° E]	$\vec{v} = (10, 7)$	$\vec{v} = 10\vec{i} + 7\vec{j}$
③	$\vec{w} = \sqrt{34}$ units [120.96° to x] [90° to y] [30.96° to z]	w = (-3, 0, 5)	$\vec{w} = -3\vec{i} + 5\vec{k}$

① $\vec{u} = 35$ cm [S40°W]



$\sin\theta = \frac{o}{h}$

$\sin 40 = \frac{x}{35}$

$35\sin 40 = x$

$22.50 \doteq x$

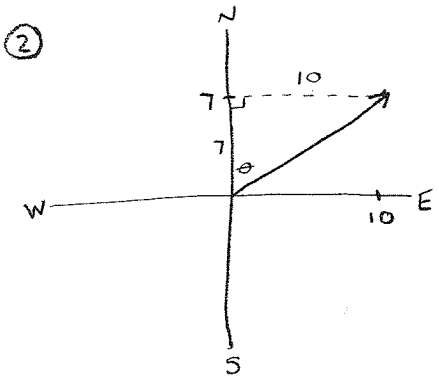
$\cos\theta = \frac{a}{h}$

$\cos 40 = \frac{y}{35}$

$35\cos 40 = y$

$26.81 \doteq y$

②



$a^2 + b^2 = c^2$

$7^2 + 10^2 = c^2$

$\sqrt{149} = c$

$\tan\theta = \frac{o}{a}$

$\theta = \tan^{-1}\left(\frac{10}{7}\right)$

$\theta = 55.01°$

③ $|\vec{w}| = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(-3)^2 + (0)^2 + (5)^2}$

$= \sqrt{34}$

$\cos A = \frac{x}{|\vec{w}|}$

$A = \cos^{-1}\left(\frac{-3}{\sqrt{34}}\right)$

$A = 120.96°$ to x-axis

$\cos B = \frac{y}{|\vec{w}|}$

$B = \cos^{-1}\left(\frac{0}{\sqrt{34}}\right)$

$B = 90°$ to y-axis

$\cos C = \frac{z}{|\vec{w}|}$

$C = \cos^{-1}\left(\frac{5}{\sqrt{34}}\right)$

$C = 30.96°$ to z-axis

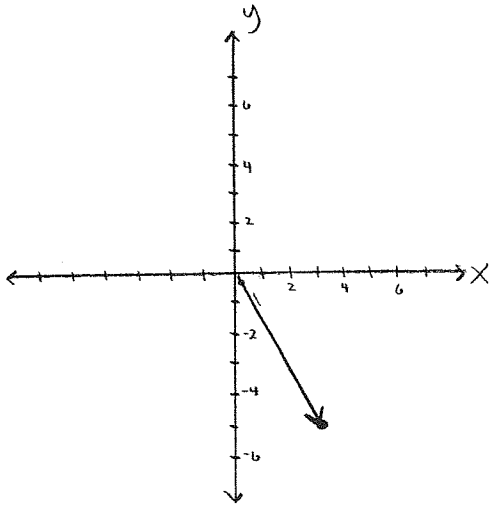
2. Find the vector $\overrightarrow{UV}$ where U(3, -2, 9) and V(21, 25, -6).

$\overrightarrow{UV} = (21-3, 25-(-2), -6-9)$

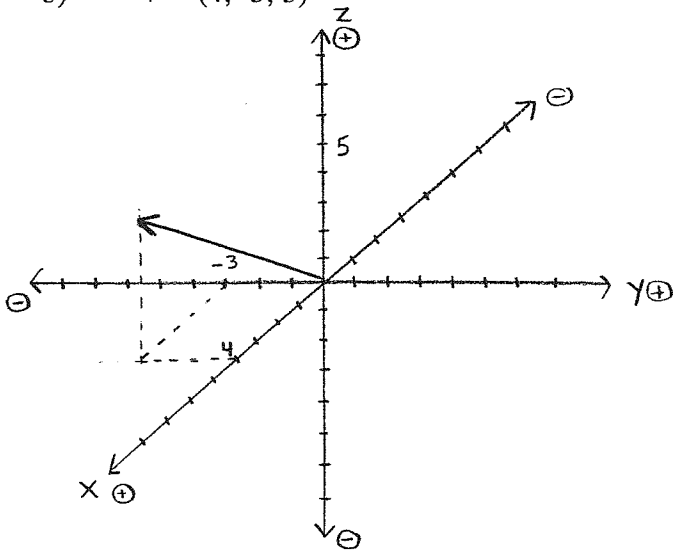
$\overrightarrow{UV} = (18, 27, -15)$

3. Graph the following vectors:

a) $\vec{u} = (3, -5)$



b) $\vec{v} = (4, -3, 5)$



4. If $\vec{a} = 15\vec{i} - 20\vec{j} + 7\vec{k}$ and $\vec{b} = 5\vec{j} + 13\vec{k}$, find $|4\vec{a} - 3\vec{b}|$.

$\vec{a} = (15, -20, 7)$ $\vec{b} = (0, 5, 13)$

$4\vec{a} - 3\vec{b}$

$= 4(15, -20, 7) - 3(0, 5, 13)$

$= (60, -80, 28) + (0, -15, -39)$

$= (60, -95, -11)$

$|4\vec{a} - 3\vec{b}| = \sqrt{x^2 + y^2 + z^2}$

$|4\vec{a} - 3\vec{b}| = \sqrt{(60)^2 + (-95)^2 + (-11)^2}$

$\boxed{|4\vec{a} - 3\vec{b}| = \sqrt{12746}}$

(*) 5. If the points A(x, 10, -2), B(25, 2, 15) and C(69, -14, z) are colinear, solve for x and z.

$\vec{AB} = (25 - x, 2 - 10, 15 - (-2))$

$\vec{AB} = (25 - x, -8, 17)$

$\vec{BC} = (69 - 25, -14 - 2, z - 15)$

$\vec{BC} = (44, -16, z - 15)$

$-8 \times k = -16$

$k = \frac{-16}{-8}$

$k = 2$

$(25 - x)(2) = 44$

$50 - 2x = 44$

$-2x = -6$

$\boxed{x = 3}$

$17(2) = z - 15$

$34 = z - 15$

$\boxed{49 = z}$

6. Determine whether or not $\vec{u} = (10, 20, 30)$ and $\vec{v} = (2, -1, 1)$ are perpendicular.

$\vec{u} \cdot \vec{v} = 10(2) + 20(-1) + 30(1)$

$\vec{u} \cdot \vec{v} = 20 - 20 + 30$

$\boxed{\vec{u} \cdot \vec{v} = 30}$

$\therefore$ No, they are not perpendicular

7. Calculate the dot product of the following:

a) $\vec{u} = (-10, 13, -4)$ and $\vec{v} = (2, 0, 15)$

$\vec{u} \cdot \vec{v} = -10(2) + 13(0) + (-4)(15)$

$\vec{u} \cdot \vec{v} = -20 - 60$

$\boxed{\vec{u} \cdot \vec{v} = -80}$

b) $|\vec{u}| = 80, |\vec{v}| = 25, \theta = 78^\circ$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

$\vec{u} \cdot \vec{v} = 80(25) \cos 78$

$\boxed{\vec{u} \cdot \vec{v} \doteq 415.82}$

8. Calculate the cross product of $\vec{a} = (100, 56, 243)$ and $\vec{b} = (-30, 28, 95)$.

$\vec{a} \times \vec{b} = (y_1 z_2 - y_2 z_1,\ z_1 x_2 - z_2 x_1,\ x_1 y_2 - x_2 y_1)$

$\vec{a} \times \vec{b} = (56(95) - 243(28),\ 243(-30) - 100(95),\ 100(28) - 56(-30))$

$\boxed{\vec{a} \times \vec{b} = (-1484, -16790, 4480)}$

56×28

243×95

100×-30

56×28

9. Calculate $|4\vec{a} \times 3\vec{b}|$ where $|\vec{a}| = 15$, $|\vec{b}| = 38$ and $\theta = 11^\circ$.

$|4\vec{a}| = 4(15) \quad |3\vec{b}| = 3(38)$

$= 60 \quad = 114$

$|4\vec{a} \times 3\vec{b}| = |4\vec{a}||3\vec{b}| \sin\theta$

$|4\vec{a} \times 3\vec{b}| = 60(114) \sin 11$

$\boxed{|4\vec{a} \times 3\vec{b}| \doteq 1305.13}$

10. Calculate the area of a parallelogram with sides $\vec{a}$ = (4, -20, 15) and $\vec{b}$ = (9, 12, -5).

-20×12
15×-5
4×9
-20×12

$A = |\vec{a} \times \vec{b}|$

$A = \sqrt{x^2+y^2+z^2}$

$A = \sqrt{(-80)^2+(155)^2+(228)^2}$

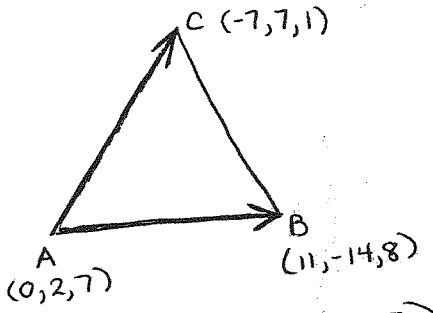
$\boxed{A = \sqrt{82409}}$

$\vec{a} \times \vec{b} = (100-180, 135+20, 48+180)$

$\vec{a} \times \vec{b} = (-80, 155, 228)$

$\therefore$ the area is $\sqrt{82409}$ units^2

11. Calculate the area of triangle ABC, where A(0, 2, 7), B(11, -14, 8), and C(-7, 7, 1).



$\vec{AB} = (11-0, -14-2, 8-7)$

$\boxed{\vec{AB} = (11, -16, 1)}$

$\vec{AC} = (-7-0, 7-2, 1-7)$

$\boxed{\vec{AC} = (-7, 5, -6)}$

$A = \frac{|\vec{a} \times \vec{b}|}{2}$

$A = \frac{|\vec{AB} \times \vec{AC}|}{2}$

$\boxed{A = \frac{\sqrt{15011}}{2}}$

$\vec{AB} \times \vec{AC} = (96-5, -7-(-66), 55-112)$

$= \boxed{(91, 59, -57)}$

$|\vec{AB} \times \vec{AC}| = \sqrt{x^2+y^2+z^2}$

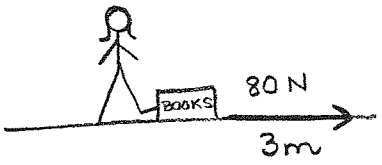
$= \sqrt{(91)^2+(59)^2+(-57)^2}$

$= \boxed{\sqrt{15011}}$

-16×5
1×-6
11×-7
-16×5

$\therefore$ The area is $\frac{\sqrt{15011}}{2}$ units^2.

12. Miss Pick pushes a box of textbooks 3 metres across the floor with her foot. If she has to work against a frictional force of 80 N, how much work must she do to move the box of books.



$W = \vec{F} \cdot \vec{d}$

$W = |\vec{F}||\vec{d}|\cos\theta$

$W = 80(3)\cos 0$

$W = 240$ J

$\therefore$ She must do at least 240 J of work.