

Vectors UNIT B

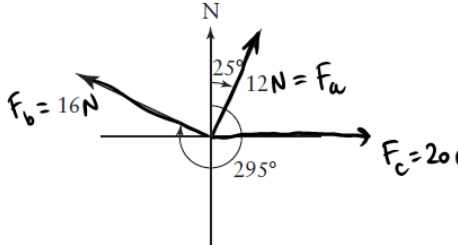
APPLICATION OF VECTORS – journal

NAME: _____

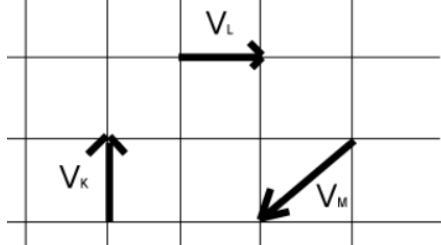
Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. FORCES

- A. State Newton’s 1st & 2nd & 3rd laws of motion. State the gravitational acceleration constant for earth, and discuss units that are valid for acceleration, mass and force.
- B. Define Applied force, Gravity force, Normal force, Friction force, Tension force, Buoyant force, Air resistance force, Resultant/Net force and Equilibrant force.

<p>C. Find resultant and equilibrant forces. State in algebraic and geometric (using true bearings form)</p>	<p>D. Ramp type:</p>	<p>E. Hanging weight type:</p>
	<p>Devon is holding his father’s wheelchair on a ramp inclined at an angle of 20° to the horizontal with a force of magnitude 400 N parallel to the surface of the ramp.</p> <ul style="list-style-type: none"> a) Draw FBD (define what it is) b) Find the weight of Devon’s father with wheelchair (distinguish between the mass) and also find the magnitude of the normal force. 	<p>A box weighing 450 N is hanging from two chains attached to an overhead beam at angles of 70° and 78° to the horizontal.</p> <ul style="list-style-type: none"> a) Draw FBD b) Draw vector diagram showing equilibrium or net force is zero c) Find the tensions in the chains

2. VELOCITY

<p>A. Find velocity of L with respect to K and with respect to M</p>	<p>B. Plane and wind type</p>	<p>C. Boat and current type</p>
	<p>An airplane is flying at an airspeed of 400 km/h on a heading of 220°. A 46-km/h wind is blowing from a bearing of 060°. Determine the ground velocity of the airplane.</p>	<p>A ship’s course is set at a heading of 143° at 18 knots. A 10-knot current flows at a bearing of 112°. What is the ground velocity of the ship?</p>

3. DOT PRODUCT

- A. Copy/Paste the following

Dot Product with Geometric Vectors

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

← angle between tails 0° ≤ θ ≤ 180°

Dot Product with Algebraic Vectors

$$\vec{u} = (a_1, b_1, c_1) \quad \vec{v} = (a_2, b_2, c_2)$$

$$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2 + c_1 c_2$$

Dot Product Properties

- ① $\vec{u} \cdot \vec{v} = 0$ if $\vec{u} \perp \vec{v}$
- ② $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$ commutative
- ③ $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
- ④ $\vec{u} \cdot \vec{0} = 0$ (scalar) distributive
- ⑤ $\vec{u} \cdot (\vec{a} + \vec{b}) = \vec{u} \cdot \vec{a} + \vec{u} \cdot \vec{b}$
- ⑥ $k(\vec{u} \cdot \vec{v}) = (k\vec{u}) \cdot \vec{v} = \vec{u} \cdot (k\vec{v})$

- B. Find the angle between the following vectors
- C. Prove

$$\vec{u} = (-3, 1, 2) \text{ and } \vec{v} = (5, -4, -1)$$

$$\vec{u} \cdot \vec{v} = \frac{1}{4} \|\vec{u} + \vec{v}\|^2 - \frac{1}{4} \|\vec{u} - \vec{v}\|^2$$

- D. Use the dot product to prove that the diagonals of a rectangle are perpendicular if and only if the rectangle is a square

4. PROJECTIONS & CROSS PRODUCT

A. Copy/Paste the following

Scalar Projection

of \vec{a} on $\vec{b} = \left| \text{proj } \vec{a} \text{ on } \vec{b} \right|$

$$= \left| \vec{a} \downarrow \vec{b} \right|$$

$$= \left| a \right| \cos \theta$$

$$= \frac{\vec{a} \cdot \vec{b}}{\left| \vec{b} \right|}$$

Vector Projection of \vec{a} on \vec{b}

$$\text{proj}(\vec{a} \text{ on } \vec{b}) = \vec{a} \downarrow \vec{b}$$

$$= \frac{\vec{a} \cdot \vec{b}}{\left| \vec{b} \right|^2} \vec{b}$$

mag. dir. using unit vector $\hat{b} = \frac{\vec{b}}{\left| \vec{b} \right|}$

Cross Product down \rightarrow minus \rightarrow up

algebraic

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$$

geometric

$$\vec{a} \times \vec{b} = \left| \vec{a} \right| \left| \vec{b} \right| \sin \theta \hat{n}$$

\hat{n} normal (perpendicular) to both \vec{a} and \vec{b}

between two tails.

Cross Product Properties

- $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ not commutative
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ distributive, order matters!
- $k(\vec{a} \times \vec{b}) = \vec{a} \times k\vec{b} = k\vec{a} \times \vec{b}$ distributive
- $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$ not associative $\rightarrow (\vec{c} \cdot \vec{a})\vec{b} - (\vec{c} \cdot \vec{b})\vec{a}$
- $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$
TRIPLE scalar product
- $\vec{a} \parallel \vec{b}$ then $\vec{a} \times \vec{b} = \vec{0}$ used to prove coplanar if equals zero
- $\vec{a} \perp \vec{b}$ then $\left| \vec{a} \times \vec{b} \right| = \left| \vec{a} \right| \left| \vec{b} \right|$ at max

The Cross Product (Vector Product)

Right Hand Rule

\vec{a} = side of hand
 \vec{b} = curled fingers
 $\vec{a} \times \vec{b}$ = thumb

B. Explain and show how to draw the scalar projection $\left| \vec{v} \downarrow \vec{u} \right|$ on the diagram. Find the magnitude

E. $\vec{a} = [5, 4, -1]$ and $\vec{b} = [1, -2, 3]$.

- Find $\left| \vec{a} \downarrow \vec{b} \right|$
- Find $\vec{b} \downarrow \vec{a}$
- Find $\vec{a} \times \vec{b}$

C. Explain and show how to draw the vector projection $\vec{v} \downarrow \vec{u}$. Find the vector.

F. Under what circumstances is

- $\text{Proj}(\vec{u} \text{ onto } \vec{v}) = \text{Proj}(\vec{v} \text{ onto } \vec{u})$?
- $\left| \text{Proj}(\vec{u} \text{ onto } \vec{v}) \right| = \left| \text{Proj}(\vec{v} \text{ onto } \vec{u}) \right|$?

D. Find magnitude and discuss direction of $\vec{u} \times \vec{v}$

5. APPLICATIONS

- A. Define Work and Torque
- B. Copy/Paste the following

Work

$$W = \vec{d} \cdot \vec{F}$$

$$W = |\vec{d}| |\vec{F}| \cos \theta$$

N·m or Joules

Area of ||gm

$$\text{Area} = lw$$

$$= |\vec{b}| |\vec{a}| \sin \theta$$

$$= |\vec{a} \times \vec{b}|$$

Torque

$$\vec{T} = \vec{r} \times \vec{F}$$

$$|\vec{T}| = |\vec{r}| |\vec{F}| \sin \theta$$

Nm or J

C. Work example (Note that displacement must be in correct units)	D. Torque example	E. Find volume of parallelepiped given points A(0,1,3), B(1, 0, 2) C(1,2,0) E(4,4,4)
<p>How much work does it take to slide a crate 200cm along a loading dock by pulling on it with a 200 N force at an angle of 30° from horizontal?</p>	<p>A 20 N force is applied at the end of a wrench that is 40 cm in length. The force is applied at an angle of 60° to the wrench. Calculate the magnitude of the torque about the point of rotation M and discuss direction.</p>	