Review

December 14, 2014 12:03 AM

Formulas

(the ones with * still need to be developed with methods other than just one step antiderivative)

COMMON INTEGRALS			
$\int k\,dx = kx + C$	$\int e^x\,dx = e^x + C$	$\int \sec^2 x\,dx = \tan x + C$	* $\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$	$\int a^u du = \frac{a^u}{\ln a} + C$	$\int \sec x \tan x\,dx = \sec x + C$	* $\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$
$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	* $\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + c$	$\int \csc x \cot x\,dx = -\csc x + C$	$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$
* $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$	$\int \cos x\,dx = \sin x + C$	$\int \csc^2 x\,dx = -\cot x + C$	$\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$
* $\int \ln(x)\,dx = x\ln(x) - x + C$	$\int \sin x\,dx = -\cos x + C$	* $\int \tan x\,dx = \ln\lvert\sec x\rvert + C$ * $\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}}dx = \sec^{-1} x + C$

Pythagorean Identities	Sum and Difference Formulas	Half Angle Formulas	Double Angle Formulas
$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$	$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$ $\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$ $\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$	$\sin(2\theta) = 2\sin\theta\cos\theta$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$ $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

1) Substitutions

a) $\int 4x^2 5^{x^3+7} \sec^2\left(5^{x^3+7}\right) dx$

b) $\int 22x^2 \sin\left(5x^3\right) e^{\cos\left(5x^3\right)} dx$

c) $\int \frac{5}{x\ln x} dx$

d) $\int \frac{\csc(\ln x)}{ex} dx$

2) Trigonometric

a) $\int \frac{dr}{r\sqrt{r^2 - 9}}$

b) $\int \frac{4\,dx}{1 + (2x + 1)^2}$

c) $\int \frac{6\,dx}{x\sqrt{25x^2 - 1}}$

3) Completing the Square

a) $\int_1^2 \frac{8\,dx}{x^2 - 2x + 2}$

b) $\int \frac{dt}{\sqrt{-t^2 + 4t - 3}}$

c) $\int \frac{dx}{(x + 1)\sqrt{x^2 + 2x}}$

4) Trigonometric Identities

a) $\int (\sec x + \cot x)^2\,dx$

b) $\int \csc x \sin 3x\,dx$

5) Improper Fractions

a) $\int_{\sqrt{2}}^{3} \frac{2x^3}{x^2 - 1} dx$

b) $\int \frac{4t^3 - t^2 + 16t}{t^2 + 4} dt$

6) Separating Fractions

a) $\int_0^{\pi/4} \frac{1 + \sin x}{\cos^2 x} dx$

b) $\int_0^{1/2} \frac{2 - 8x}{1 + 4x^2} dx$

a $\int_0^{\pi/4} \frac{1+\sin x}{\cos^2 x}\,dx$		b $\int_0^{1/2} \frac{2-8x}{1+4x^2}\,dx$	

7 **Multiplying by a Form of 1**

a $\int \frac{1}{1+\cos x}\,dx$		b $\int \frac{1}{1-\sec x}\,dx$	

8 **Eliminating Square Roots**

a $\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\,dx$		b $\int_{\pi/2}^{\pi} \sqrt{1+\cos 2t}\,dt$	
c $\int_{\pi/2}^{\pi} \sqrt{1-\sin^2\theta}\,d\theta$		d $\int_{-\pi/4}^{0} \sqrt{\sec^2 y - 1}\,dy$	

9 **Assorted**

a $\int_0^1 x\left(x^2+1\right)^3 dx$	b $\int \frac{2x}{(x+1)^2}\,dx$	c $\int \frac{dx}{x-\sqrt{x}}$	d $\int_0^1 \frac{16x\,dx}{8x^2+2}$

10 Using a vertical slicing method, find the area of the region in the first quadrant bounded by $y=\sqrt{x}$, $y=0$, and $y=x-2$
SHOW calculations for both vertical and horizontal slice methods.

11 Find the volume of the solid formed by rotating the region bounded by the x-axis, $y=\sqrt{x}$, and $x=1$ around the x-axis.
SHOW calculations for both vertical and horizontal slice methods.

12 Find the volume of the solid formed by rotating the region bounded by the $y=1$, $y=\sqrt{x}$, and $x=0$ around the line $y=1$.
SHOW calculations for both vertical and horizontal slice methods.

13 Find the volume of the solid formed by revolving the region bounded by the graphs of $y=x^2+1$, $y=0$, $x=0$, and $x=1$ about the y-axis.
SHOW calculations for both vertical and horizontal slice methods.

14 The region in the first quadrant of 1st revolution enclosed by the y-axis and the graphs of $y=\cos x$ and $y=\sin x$ is revolved about the x-axis to form a solid. Find its volume.

15 Find the volume of the solid formed when the R enclosed by the curves $y=x$ and $y=x^2$ is rotated about the following axes:

a) the line $y=2$ b) the line $y=-5$ c) the line $x=-1$

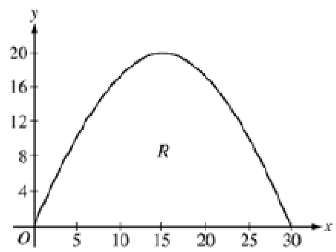
16 Find the volume of the solid whose base is bounded by the graphs of $y=x+1$ and $y=x^2-1$, with the following cross sections taken perpendicular to the x-axis:

Equilateral Triangles

following cross sections taken perpendicular to the x-axis.

Equilateral Triangles

17 (~~BC~~ 2009B-1) A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



AB (a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

AB (b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 grams of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

BC (c) Find the perimeter of the base of the cake.

18 (2003-~~BC~~ AB 1) Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

19 Find the length of the curve.

BC $x = \frac{2}{3}(y-5)^{\frac{3}{2}}$ from $(0,5)$ to $(\frac{2}{3}, 6)$

20 Find the area of the surface generated by

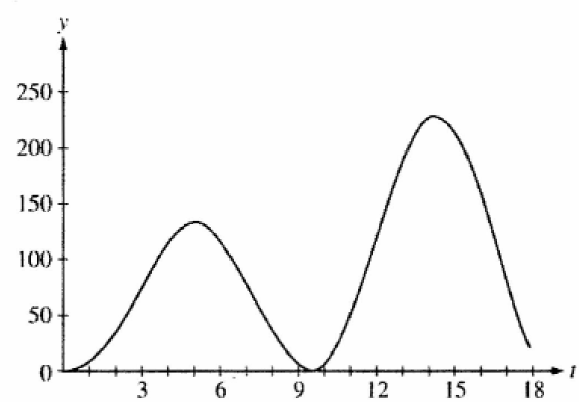
BC

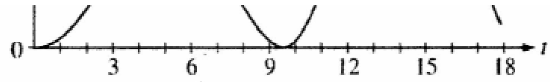
$y = \frac{1}{3}\left(x^2+2\right)^{3/2}, \quad 0 \le x \le \sqrt{2}; \quad y-axis$

BC 21 Find the area of the surface generated by $x = \sqrt{2y-1} \quad \frac{5}{8} \le y \le 1, \quad y-axis$

22.

2006 AP® CALCULUS AB FREE-RESPONSE QUESTIONS





At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

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2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justifv vour answers.

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2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.