Calculus UNIT 6

APPLICATIONS of INTEGRATION (AB some BC)– journal questions

Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. U-SUBSTITUTION METHOD
   a. Copy/Paste the following

U-Substitution (Change of variable):

If $u = g(x)$ is a differentiable function whose range is an interval *I*, and *f* is continuous on *I*, then

$$\int f'(g(x))\,g'(x)dx = \int f'(u)du$$

STEPS:

1) Let u=(inside quantity) usually
2) Find du=______dx
3) Change variables; Balance constants
4) Integrate with respect to u, (don't forget + C)
5) Back substitute to original variable

STRATEGIES

- Recognize the antiderivative, divide out the "correction constants"
- Recognize the chain rule, ie. identify both the "inner" and "outer" functions and do U-sub
- Expand brackets by distribution
- Use Trig identities
- Factor or complete the square
- Long divide
- Distribute the denominator
- When doing U-sub, do implicit derivative of "let u=" statement
- Multiply by a factor equivalent to ONE

Other for BC topics (learn next unit):

- Partial Fractions – opposite of LCD
- By Parts
- Trig Sub

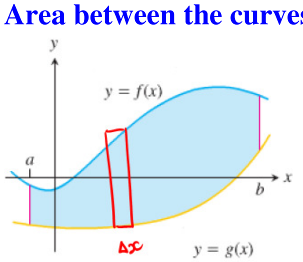
   b. Identify the strategy and find the antiderivative

i. $\int \frac{dx}{3\lvert x\rvert\sqrt{9x^2-4}}$	ii. $\int e^{-x}\sqrt{4+e^{-x}}dx$	iii. $\int \frac{dx}{x^2+x+1}$
iv. $\int \frac{x^3}{x^2+2}dx$	v. $\int x\sqrt{x+4}dx$	vi. $\int \frac{\tan x+\cos x}{\sin x}dx$

   c. Explain how to deal with a definite integral $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}dx$

2. AREAS BETWEEN CURVES
   a. Copy/Paste the following

Area between the curves



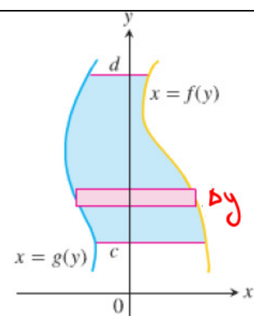
A = [Length][Thickness]

$A = [L][\Delta x]$

$$\int_{leftbound}^{rightbound} (top - bottom)dx$$

right and left bounds are x-coordinates of the intersection.

dx implies everything is in terms of x



$$\int_{lowerbound}^{upperbound} (right - left)dy$$

upper and lower bounds are y-coordinates of the intersection.

dy implies everything is in terms of y

STEPS:

- Draw a picture, clearly shade in the region
- Decide how to slice the region—vertically or horizontally, label the rectangular length and thickness (thickness is the variable of integration).
- Find POI to identify intervals of integration
- Set up the definite integral and integrate

This works as long as you can make your "cuts" from one function to another function. Your cuts should never run from one function to itself.

   b. Show how to find area between curves $y = \sqrt{x}$, $y = x - 2$ and $x = 0$ with vertical slices as well as with horizontal slices.

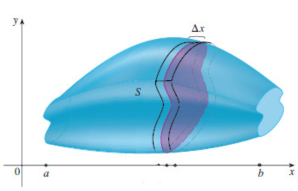
3. VOLUMES
   c. Copy/Paste the following

Not a solid that comes from a rotated shape? Slicing Method:

V = [Area][Thickness]

$V = [A][\Delta x]$

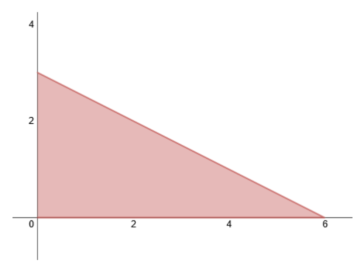
$V = \int_a^b A(x)dx$



d. Calculate the volume of the solid whose base is the shaded region shown and whose cross sections are

   i. Squares perpendicular to y-axis
   ii. Semicircles perpendicular to x-axis

include detailed diagrams



a. Copy/Paste the following

Cross-sections (slices/representative rectangles) must be PERPENDICULAR to the axis of rotation.

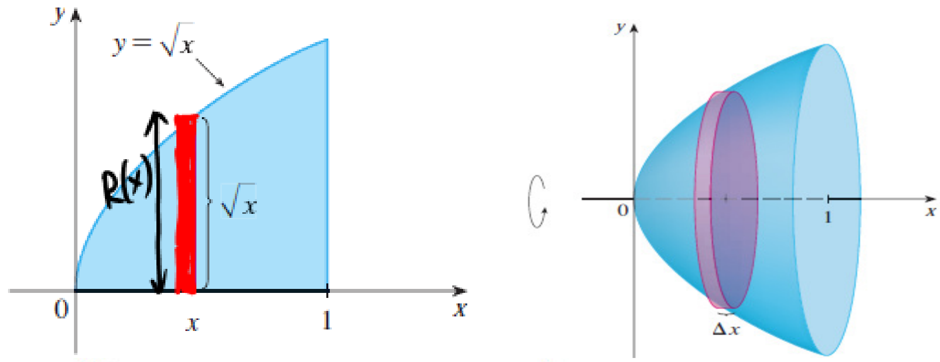
perpenDISCular

V = [Area][Thickness]

$V = [\pi R^2][\Delta x]$

$$V = \pi \int_a^b R(x)^2 \, dx$$

Disc/Washer Method for Volumes of Solids of Rotation



Cross-sections (slices/representative rectangles) must be PERPENDICULAR to the axis of rotation.

perpenWASHular

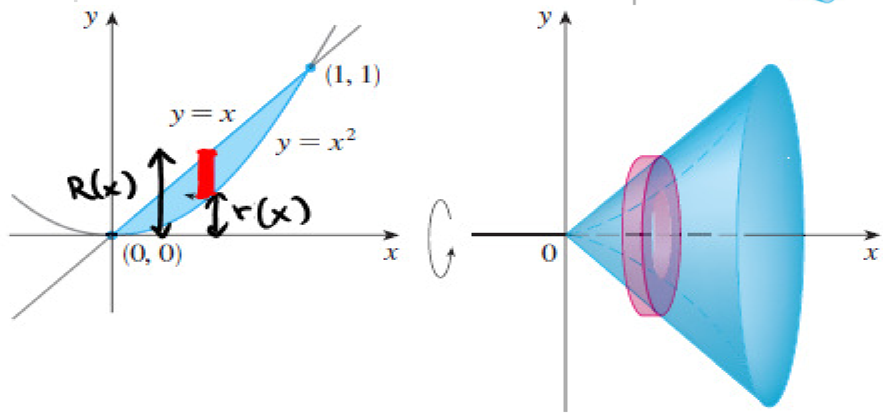
V = [Big circle − Small circle][Thickness]

$V = \left(\pi R^2 - \pi r^2\right)[\Delta x]$

$$V = \pi \int_a^b \left[(R(x))^2 - (r(x))^2 \right] dx$$

NOTE

radii are squared BEFORE subtracting



Cross-sections (slices) must be PARALLEL to the axis of rotation, ie. representative rectangle is taken for spin around the axis parallel to it.

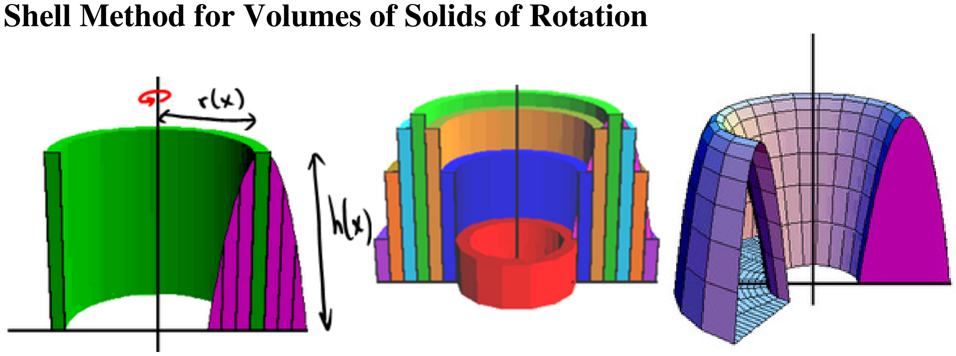
paraSHELL

V = [Circumference][Height][Thickness]

$V = [2\pi R][H][\Delta x]$

$$V = 2\pi \int_a^b r(x) \cdot h(x) \, dx$$

Shell Method for Volumes of Solids of Rotation



STEPS:

- Sketch the bounded area by shading it in, label any points of intersection and label the axis of revolution
  - If shaded region is always beside the axis of rotation then it is a DISC
  - If shaded region and axis of rotation are separated by a gap (even if a partial gap) then it is a WASHER
  - SHELL method doesn't change whether there is a gap or not, since the gap will be ensured by the limits on the integral
- Drawing the rectangle slice helps as well:
  - Pay attention to what function/value the edges of the rectangle touch. Use them for "top-bottom" or "right-left" Remember that for Length/Height/Radius you cannot subtract a function from itself so that can help you determine which way to slice. (Note: seeing if the function is possible to isolate for the other variable also helps to decide this)
  - The thickness of the rectangle determines the variable of integration, ensure that all variables are in terms of that one.
  - Imagine dragging that rectangle through the whole shaded region, will the Length/Height/Radius expression stay the same or will you need to split the integral over two intervals?
  - In the case where the rectangle represents RADIUS then you will always do "top-axis" or "right-axis" (the axis of rotation is always the edge of the radius)
  - Sometimes the expression for Radius in SHELL method will be just the variable x or just the variable y since the slice is allowed to move/vary it's position

b. Find the volume generated by rotating the region about the x-axis bounded by $y = x^2$ and $y = x$ show that you get the same result with Disk/Washer Method as with Shell Method. Include detailed diagrams.

4. ARC LENGTH & AREA of SURFACE of REVOLUTION (BC)

Use the following equation to answer the questions	
$x = 2\sqrt{4-y}$, $0 \le y \le \frac{15}{4}$	a. Summarize how to find arc length. To solve this one you may need TI-89 b. Summarize how to find area of surface of revolution around the y-axis, include detailed diagrams. This one try to simplify and integrate by hand.