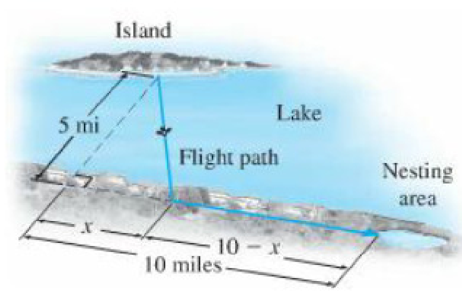
Review

October 31, 2014 10:58 AM

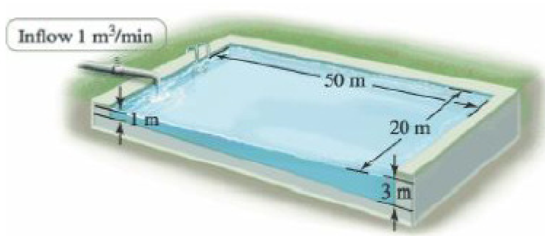
1. A cone-shaped icicle is dripping from the roof. The radius of the icicle is decreasing at a rate of 0.2 cm per hour, while the length is increasing at a rate of 0.8 cm per hour. If the icicle is currently 4 cm in radius and 20 cm long, is the volume of the icicle increasing or decreasing and at what rate?

2. Some birds tend to avoid flights over large bodies of water during daylight hours. Suppose that an adult bird with this tendency is taken from its nesting area on the edge of a large lake to an island 5 miles offshore and is then released

If it takes only 1.4 times as much energy to fly over water as land, how far up the shore (x, in miles) should the bird head to minimize the total energy expended in returning to the nesting area?

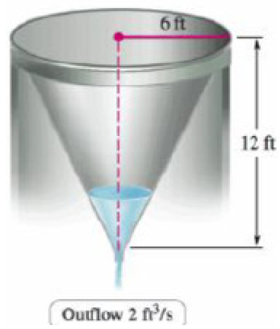


3. A swimming pool is 50 *m* long and 20 *m* wide. Its ~~length~~ depth decreases linearly along the length from 3 *m* to 1 *m*. It is initially empty and is filed at a rate of 1 m^3 / min .
   a) How fast is the water level rising 250 min after the filling begins?
   b) How long will it take to fill the pool?



4. The cost per hour for fuel to run a train is $\frac{v^2}{4}$ dollars, where v is the speed of the train in miles per hour. (Note that the cost goes up as the square of the speed.) Other costs, including labor are \$300 per hour. How fast should the train travel on a 360-mile trip to minimize the total cost for the trip?

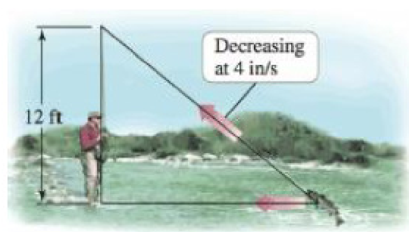
5. An inverted conical water tank with a height of 12 ft and a radius of 6 ft is drained through a hole in the vertex at a rate of 2 ft^3 / sec . What is the rate of change of the water depth when the water depth is 3 ft? (*Hint*: Use similar triangles.)



6. A commercial pear grower must decide on the optimum time to have fruit picked and sold. If the pears are picked now, they will bring 30¢ per pound, with each tree yielding an average of 60 pounds of salable pears. If the average yield per tree increases 6 pounds per tree per week for the next 4 weeks, but the price drops 2 ¢ per pound per week, when should the pears be picked to realize the maximum return per tree? What is the maximum return?

7. A company wants to manufacture cylinder aluminum can with a volume $1000cm^3$. What should the radius and height of the can be to minimize the amount of aluminum used?

8. A fisherman hooks a trout and reels in his line at 4 in/sec. Assume the trip of the fishing rod is 12 ft above the water directly above the fisherman and the fish is pulled horizontally directly towards the fisherman. Find the horizontal peed of the fish when it is 20 ft from the fisherman.



9. A spherical balloon is inflated with helium at the rate of $100\pi\ ft^3$ / min . How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast the surface area increasing?

10. A rectangular field is to be enclosed on four sides with a fence. Fencing costs $3 per foot for two opposite sides, and $4 per foot for the other two sides. Find the dimensions of the field of area 730 square feet that would be the cheapest to enclose.

11. Show that the linearization of

$f(x) = (1+x)^k$ at $x = 0$ is $L(x) = 1 + kx$.

(12.) Use the linearization $(1+x)^k \approx 1+kx$ to approximate the following. State how accurate your approximation is.

a. $(1.002)^{100}$

b. $\sqrt[3]{1.009}$

(13.) Find the linear approximation for the following functions at the given point.

a) $f(x)=x\sqrt{x^2+1}$ at $x=0$

b) $f(x)=\ln(1+x^2)$ at $x=2$

(14.) Use an appropriate local linear approximation to estimate the value of the given quantity.

a) $\sqrt[4]{15.5}$

b) $\cos 5°$

(15.) Find the differential dy.

a) $y=x^4-\sqrt[3]{x}+1$

b) $y=\sqrt{\dfrac{x+2}{x-2}}$

c) $y=\arctan(2x^2)$

d) $y=\tan\sqrt{x}$

e) $y=e^{\arcsin x}$

f) $y=\ln\csc x$

(16.) Use dy to approximate Δy.

a) $y=\sqrt{x^2+7}$ from $x=2$ to $x=2.03$

b) $y=\dfrac{x}{x^2+1}$ from $x=2$ to $x=1.96$

(17.) Use an differentials to estimate the value of the given quantity.

a) $\sqrt[3]{127}$

b) $(2.98)^4$

(18.)

a) The side of a cube is measured to be 25 cm, with a possible error of ± 1 cm. Use differentials to estimate the error in the calculated volume

b) The hypotenuse of a right triangle is known to be 10 cm exactly, and one of the acute angles is measured to be 30° with a possible error of $\pm 1°$.

Use differentials to estimate the errors in the sides opposite and adjacent to the measured angle.

19. Suppose that we don't have formula for g(x) but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x.

a) Use a linear approximation to estimate g(1.95) and g(2.05)

b) Are your estimates too large or too small? Explain.

20. Norris finds that it sells N units of a product after spending x thousands of dollars on advertising where: $N(x) = -x^2 + 300x + 6$. If Norris wants to increase the number of units sold by 15% how much will they need to increase their advertising budget? [*They are currently spending $100 thousand on advertising*.]

BC

1. Use Newton's method to estimate the real solution of $x^3 + 3x + 1 = 0$. accurate to 6 decimals.

2. Use Newton's method to estimate the 2 real solutions of $x^4 + x - 3 = 0$. Start with $x_0 = -1$ for the left-hand zero and with $x_0 = 1$ for the zero on the right. Then, in each case, find x_2

3. Apply l'Hôpital Rule to evaluate

a) $\displaystyle\lim_{x\to 0} \frac{x^2}{\ln(\sec x)}$

b) $\displaystyle\lim_{x\to 0} \frac{\left(e^x - 1\right)^2}{x \sin x}$

c) $\displaystyle\lim_{\theta\to 0} \frac{3^{\sin\theta} - 1}{\theta}$

d) $\displaystyle\lim_{x\to 0^+} (\ln x - \ln \sin x)$

e) $\displaystyle\lim_{x\to 1^+} x^{1/(x-1)}$

f) $\displaystyle\lim_{x\to \infty} (1+2x)^{1/(2\ln x)}$

4) Approximate f by a Taylor polynomial with degree n at the number a. Use this approximation to estimate the given number, and estimate the accuracy of that approximation.

a) $f(x)=\sqrt{1+x}$ $\quad n=1 \quad a=0 \quad \sqrt{1.1}$

b) $f(x)=\dfrac{1}{\sqrt[3]{x+2}}$ $\quad n=3 \quad a=6 \quad \frac{1}{\sqrt[3]{7.9}}$

c) $f(x)=\cos^3 x$ $\quad n=2 \quad a=\frac{\pi}{6} \quad \cos^3 31°$

5) Find the Taylor series for $f(x)$ at $x=a$

a) $f(x)=\sin x \quad a=\frac{\pi}{2}$

b) $f(x)=\dfrac{e^x+e^{-x}}{2} \quad a=0$

*Taylor series at $a=0$ are called Maclaurin Series