Big idea

We live in a world that is always in flux. Sir Isaac Newton's name for calculus was "the method of fluxions." He recognized, as you probably recognize today, that understanding change is important. Newton was what we might call a "mathematical physicist." He developed calculus to gain a better understanding of the natural world, including motion and gravity. But change is not limited to the natural world, and, since Newton's time, the use of calculus has spread to include applications in the social sciences. Psychology, business, and economics are just a few of the areas in which calculus continues to be an effective problem-solving tool. As we shall see in this unit, anywhere functions can be used as models, the derivative is certain to be meaningful and useful.

Feedback & Assessment of Your Success

Tentative TEST date: ______________________

Date	**Pages**	**Topics**	**Finished assignment pages?** **Made corrections?**	**Summarized notes in a journal?** **Added your own explanations?**	**How many extra practice questions did you try in each topic?**	**Questions to ask the teacher:**
3.5days	2-12	Optimization (MCV) Journal #1				
3days	13-21	Related Rates (MCV) Journal #2				
1.5days	22-24	L'Hopital's Rule (AP - AB) Journal #3				
1.5days	25-27	Approximations (AP - AB) Journal #4, 5				
1.5days	28-30	OPTIONAL: Newton's Method + Taylor Series (AP – BC) Journal #6				

ASSIGNMENT Optimization (MCV)

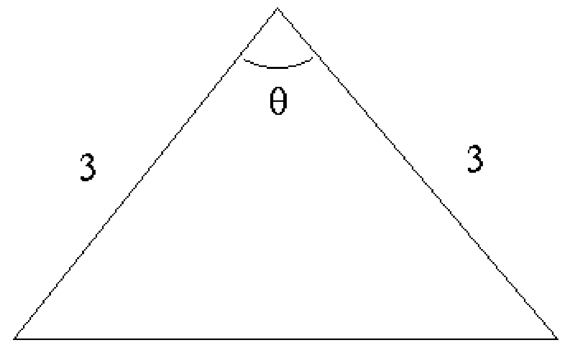
1. Find two numbers whose difference is 100 and whose product is a minimum.

2. **Medicine:** Blood pressure is the pressure exerted by circulating blood on walls of the arteries. Assume the pressure varies periodically according to the formula $p(t) = 90 + 15\sin(2.5\pi t)$, where t is the number of seconds since the beginning of a cardiac cycle. When is the blood pressure the highest for $0 \le t \le 0.8$? What is the maximum blood pressure? When is the pressure lowest and what is the corresponding blood pressure?

3. **Economics/Business:** A manufacturer produces lamps for $20 each and then sells them for $45 each. At this price they sell 200 per month. A market research survey indicates that by reducing their price by $2 they should gain 22 sales. What should the company charge to maximize profit?

4. **Economics/Business:** A grower has 100tonnes of potatoes that she can sell now for a profit of $500/tonne. For each week she delays shipment, she can produce an additional 10 tonnes of potatoes. Unfortunately, for each week she delays, the profit decreases by $25/tonne. When should she ship to maximize her profit?

5. What angle θ between two edges of length 3 will result in an isosceles triangle with the largest area?



6. **Economics:** A farmer wants to fence off a rectangular field with an area of $1000m^2$. What dimensions should it be so that fencing costs are minimized?

7. **Engineering/Design:** A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?

8. **Economics/Engineering:** A factory needs to run a pipeline across a 1 km wide river to a point that is also 2 km east of its factory. It costs $5 million/km of pipeline built under the river, and $3 million/km of pipeline built on land. In order to minimize costs, how far to the east of the company's factory should the pipeline be when it crosses out of the river?

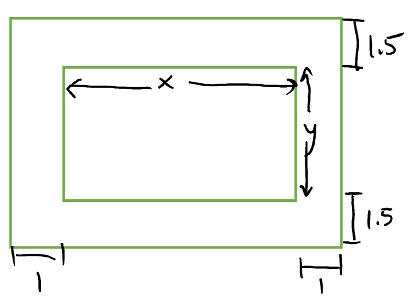
9. **Economics/Engineering/Design**: Consider a cylindrical tin can which is to be constructed by joining the ends of a rectangular piece of metal to form the cylindrical side, and then attaching circular pieces to form the top and bottom. There are seams around the perimeter of the top and bottom and there is one seam down the side surface (where the ends of the rectangle join together). Suppose the volume of the can is $1000cm^3$. Also suppose that the cost of the material is \$1.00 per m^2 and the cost of the seam is \$0.20 per meter. Find the dimensions of the can that will minimize the cost.

10. Find the largest possible area for a triangle inside a circle with radius of 6cm given that one vertex is at the centre of the circle and the other two vertices are on the circumference of the circle.

11. **Economics/Engineering/Design:** A rectangular box with an open top is constructed so that its volume is $10m^3$. The base of the box has length twice its width. Material for the base costs $\$4/m^2$, while material for the sides costs $\$2/m^2$. What dimensions of the box will minimize the cost.

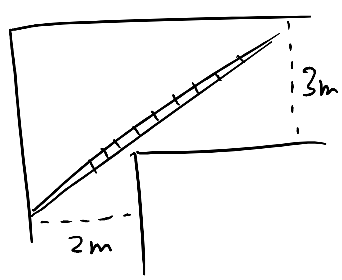
12. Find the dimensions of the rectangle of maximum area that can be inscribed in the region bounded by $y = 16 - 8x^2$ and the x-axis, provided that one side of the rectangle is on the x-axis.

13. **Economics:** A page is to contain 27cm^2 of print. The margins at the top and bottom are 1.5cm and at the sides are 1cm. Find the dimensions for the most economical area of the page.



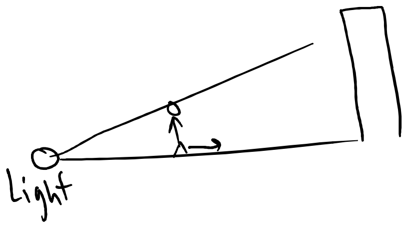
14. An athletic field with a 400m perimeter consists of a rectangle with a semi-circle at each end. Find the dimensions of the field so that the rectangular area is maximized.

15. A hallway of 2m turns into 3m after you turn a corner. Find the length of a ladder that can make it around this corner, so that the ladder's edges barely scrape by the walls.

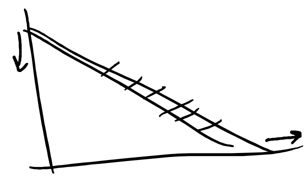


ASSIGNMENT Related Rates (MCV – usually do)

1. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a rate of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

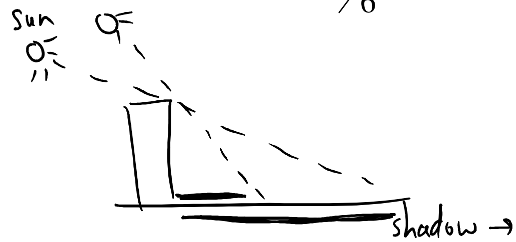


2. A 20 ft long ladder rests along a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.5 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 8 ft away from the wall?



3. A 5m long ladder is sliding down a wall at a rate of 10cm/min. How quickly is the angle of elevation of the ladder changing when the base is 3m from the wall?

4. The angle of elevation of the Sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400-ft tall building increasing when the angle of elevation of the sun is $\pi/6$?



5. **Medicine:** Blood flows faster the closer it is to the center of a blood vessel because of the reduced friction with cell walls. According to Poiseuille's laws, the velocity *V* of blood is given by $V = k(R^2 - r^2)$ where *R* is the radius of the blood vessel, *r* is the distance of a layer of blood flow from the centre of the vessel, and *k* is a constant (assume it to be 375). Suppose a skier's blood vessel has radius *R* = 0.08 mm and that cold weather is causing the vessel to contract at a rate of 0.01 mm/min. How fast is the velocity of the blood changing? Assume *r* is constant.

6. **Physics:** The combined electrical resistance *R* of R_1 and R_2, connected in parallel, is given by

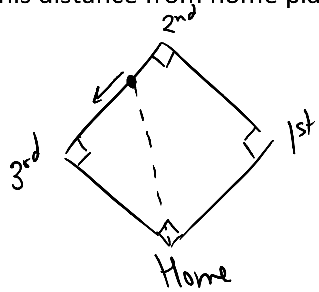
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of 1 and 1.5 ohms/s respectively, at what rate is *R* changing when $R_1 = 50$ ohms and $R_2 = 75$ ohms?

7. Suppose that liquid is poured into the conical filter, of diameter 8cm at the top and total depth of 16cm, at a constant rate of 2cm^3/min. At what rate is the depth of the liquid changing when it is 8cm deep?

8. Water flows at 8cm^3/min into a cylinder with a radius of 4cm. How fast is the water level rising when the water is 2cm deep?

9. Length of each side of a baseball diamond is 30m. A player runs from 2nd base to 3rd base at a speed of 10m/s. When he is 6m from 3rd base how quickly is his distance from home plate changing?

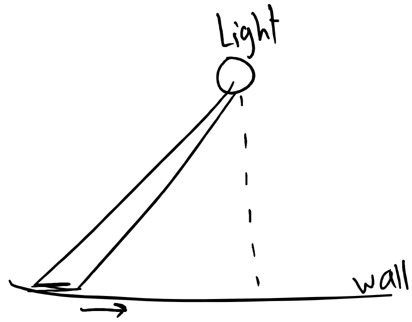


10. A sand storage tank used by the highway department for winter storms is leaking. As the sand leaks out, it forms a conical pile. The radius of the base of the pile increases at the rate of 1 cm/min. The height of the pile is always twice the radius of the base. Find the rate at which the volume of the pile is increasing at the instant the radius of the base is 5 cm.

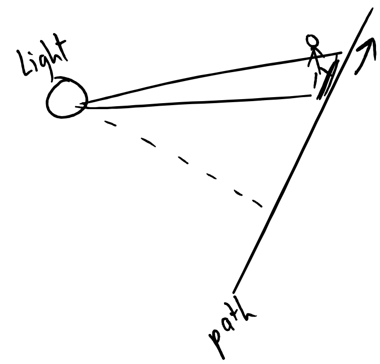
11. A triangle has two sides of constant length 10cm and 15cm. The angle between these two sides is increasing at 9^{o}/min.
    a) Find the rate at which the 3rd side is growing when the angle is 60^{o}.
    b) Find the rate at which the area of the triangle is changing when the angle is 60^{o}.

12. **Environmental Science**: Suppose that an oil slick has a circular area, and that its radius is expanding at 5 m/s. How fast is the area of the oil slick increasing when the radius is 30 m?

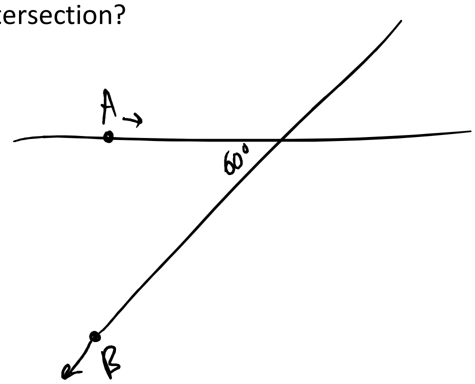
13. A light is rotating 3 times per minute. Determine how quickly it is moving along a straight wall given that the wall is 10m from the light and you wish to know the speed at a position 2m from a point that is directly across from the light.



14. A man walks along a straight path at a rate of 4ft/s. A spot light is located on the ground 20feet from the path is continuously focused on the man. At what rate is the light rotating when the man is 15feet from the point on the path closest to the light?

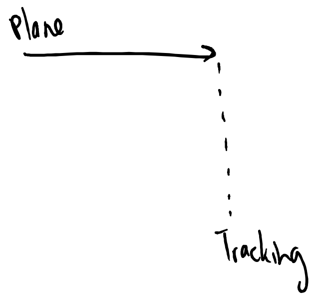


15. Car A is approaching an intersection at a speed of 70km/hr. Car B is leaving the intersection on another road at a speed of 60km/hr. Intersection makes a 60° angle. How quickly is the distance between the two cars changing when car A is 100m from the intersection and Car B is 200m from the intersection?

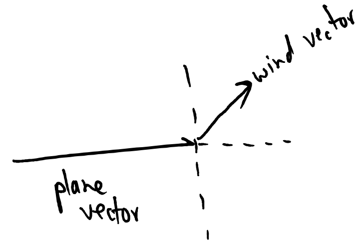


16. Two people are 50m apart standing on the east-west line. Person A stands still while person B walks north at some rate so that $\frac{d\theta}{dt} = 2^o\ /\ \text{min}$. At what rate is the distance between A and B changing when $\theta = 30^o$? The angle is measured by person A.

17. An airplane is flying on a horizontal flight path that will take it directly over a radar tracking station at an altitude of 6 miles. If the distance between the plane and the tracking station is decreasing at a rate of 400 mph when the straight-line distance between the plane and the tracking station is 10 miles, what is the speed of the plane?



18. A plane flies east at a speed of 250km/hr. There is a wind blowing 80km/h in a north-east direction. What is the speed of the plane, relative to the ground, 1 hour into the flight?



ASSIGNMENT L'Hopital's Rule For Indeterminate Limit Forms (AP – AB)

List indeterminate forms for LH: List other indeterminate forms: Not indeterminate:

Ones we did before with longer methods (factoring, rationalizing, squeeze law, etc) now can be done faster:

1. $\lim_{x \to 2} \frac{x^4 - 2x^2 - x - 6}{x^2 - 4}$

2. $\lim_{x \to 0} \frac{\sin x}{x}$

Ones we couldn't do before without sketching or plugging numbers:

3. $\lim_{x \to 0} \frac{2^x - 1}{x}$

4. $\lim_{x \to \infty} \frac{e^x}{x^2}$

5. $\lim_{x \to \infty} \frac{e^x + 3x}{\ln x}$

6. $\lim_{x \to 1} \frac{\ln x}{x - 1}$

7. $\lim_{x \to 0} \frac{4x}{\tan x} - \cos x$

8. $\lim_{x \to \infty} \frac{\ln x}{\sqrt[3]{x}}$

9. Consider an electrical circuit consisting of an electromotive force that produces a voltage *V*, a resistor with resistance *R*, and an inductor with inductance *L*. It is shown in electrical circuit theory that if the voltage is first applied at time $t = 0$, then the current *I* flowing through the circuit at time *t* is given by $I = \frac{V}{R}(1 - e^{-Rt/L})$.

What is the effect on the current at a fixed time *t* if the resistance approaches 0 (i.e. $R \to 0^+$)?

Other indeterminate forms where the rule can't be applied right away:

10. $\lim_{x \to -\infty} x^2 e^x$

11. $\lim_{x \to 0^+} \sin x \cdot \ln x$

12. $\lim_{x \to 6} \csc(\pi x) \ln(x - 5)$

13. $\lim_{x \to \infty} x \sin\left(\frac{7}{x}\right)$

14. $\lim_{x\to 0} \frac{1}{6x} - \frac{1}{e^{6x}-1}$

15. $\lim_{x\to \infty} x \ln\left(1+\frac{3}{x}\right)$

16. $\lim_{x\to \infty} x^{1/x}$

17. $\lim_{x\to 0^+} (e^x + x)^{1/x}$

18. $\lim_{x\to 0^+} [\cos(2x)]^{1/x^2}$

19. $\lim_{x\to \infty} (e^{-2x} + 3x)^{1/x}$

ASSIGNMENT Linear Approximations & Differentials (AP – AB)

1. Find a linear approximation to $f(x)=\ln x$ around $a=1$.

2. Find the linearization of $f(x)=\sin x$ at $a=0$.

3. Without using a calculator, find an approximate value for $\sqrt[3]{8.01}$.

   Without using a calculator is your answer too big or too small?

4. Using a linear approximation, find an approximate value for $\sqrt{26}$.

   Without using a calculator is your answer too big or too small?

5. If you know $f(8)=20$ and $f'(x)=\sqrt{x^3+1}$ find the approximate to $f(8.5)$ without using a calculator.

6. If you know $f(1)=5$ and $f'(x)=x^2-6x$ find the approximate to $f(1.01)$ without using a calculator.

7. For $\sqrt[3]{8.01}$ approximation that you did before, use differentials to find the change in output value

8. For $\sqrt{26}$ approximation that you did before, use differentials to find the change in output value

9. The radius, r, of a circle increases from 10cm to 10.1. Use differentials to estimate the change in Area of the circle.

10. An icicle is gradually increasing in length, while maintaining a cone shape with a length 15 times the radius. Find the approximate amount that the volume of the icicle increases when the length increases form 13 cm to 13.2 cm.

11. About how accurately should we measure the radius of a sphere to calculate its surface area to whithin 1% of its true value?

12. Flow through a pipe is given by $F = kr^4$. Suppose that the radius is increased by 10% what impact does this have on the flow through the pipe?

13. Suppose you want to paint a hemi-sphere (not the inside or base) of radius 5000cm. Determine the amount of paint needed if paint is to be 0.5cm thick.

14. Estimate the volume of material in a cylindrical shell with height of 30cm, radius of 6cm and shell thickness of 0.5cm.

OPTIONAL Newton's Method + Taylor Series (AP - BC)

Find a solution accurate to 5 decimal places

1. $2x^2 + 5 = e^x$

2. $2 - x^2 = \sin(x)$

3. Use Newton's method to estimate the real solution of $2x - x^2 + 1 = 0$. Start with $x = 0$ for the left-hand zero and with $x = 2$ for the zero on the right. Then, in each case, find x_5

4. Find the Maclaurin series for e^x at x=0

5. Find the Taylor series for e^x at x=1

6. Find first three non-zero terms of the Maclaurin series for
$f\left(x\right)=x^3e^{-x}$

7. Find the Maclaurin series for $\sin x$ at x=0

Distinguish between types of word problems

Classify which word problems are optimization type, related rates type and differentials type, then solve

1. In a manufacturing process, the radius of a circular disk is to be 17 cm with a maximum error of 0.3 cm. Approximate the maximum error in the area using differentials. What is the relative error in the area?

2. A barrel is designed to hold 20Litres. What should its dimensions be in order to minimize cost?

3. Assure that oil spilled spreads out in a circular pattern whose radius increases at a constant rate of 0.8m/s. How fast is the area of the spill increasing when radius is 20m?