Calculus UNIT 3

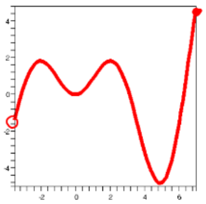
APPLICATION of DERIVATIVES Part 1 (MCV + AP)– journal questions

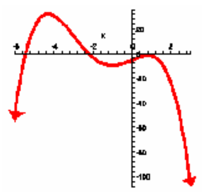
Summarize everything you need to know about these topics. Use examples and concise (not long – but with enough detail) explanations. Include definitions and diagrams if necessary

1. DEFINITIONS & THEOREMS
   a. Copy/Paste the following

A function *f* has an **absolute maximum** at *c* if $f(c) \geq f(x)$ for all *x* in the domain. We call $f(c)$ the **maximum value**. Similarly, *f* has an **absolute minimum** at *c* if $f(c) \leq f(x)$ for all x in the domain; $f(c)$ is then the **minimum value**.

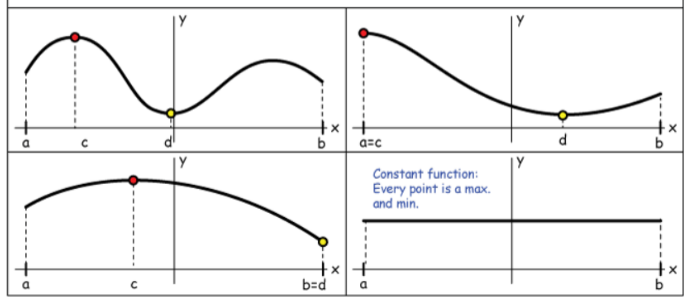
A function *f* has a **local maximum** at *c* if $f(c) \geq f(x)$ when *x* is near *c*. Similarly, *f* has a **local minimum** at *c* if $f(c) \leq f(x)$ when *x* is near *c*.





The Extreme Value Theorem

If **f** is a continuous function on the closed interval [a, b], then **f** has both an absolute maximum (●) value, f(c), and an absolute minimum (○) value, f(d), for some numbers c and d on [a, b].



Fermat's Theorem: If *f* has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

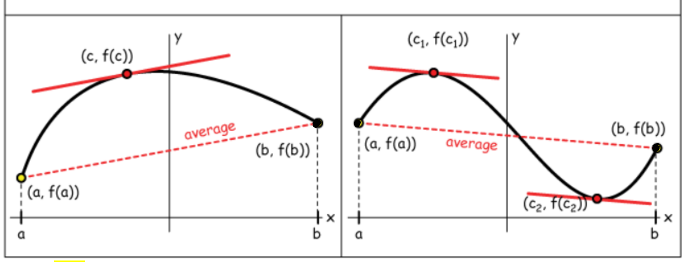
b. (MCV) Draw examples of functions that are
   i. continuous $\forall x \in \mathbb{R}$ yet have no absolute max and no absolute min
   ii. continuous $\forall x \in \mathbb{R}$, with abs max at (0,4) yet has no absolute min
   iii. continuous on $x \in (-1,1)$ with no absolute max and with an absolute minimum value of 3

c. (AP)
   i. Is the converse of Fermat's theorem true? Ie. if $f'(c) = 0$ then *c* is a local max/min? Explain or show a counterexample. Also, do local max/min values occur only at points where $f'(c) = 0$? Explain with examples.
   ii. Use the Closed Interval Method (or EVT) to find the absolute extreme values of $f(x) = \dfrac{\ln x}{x}$ on $x \in [1,3]$ Clarify the Max/Min value itself versus point versus where it occurs.

Mean Value Theorem

If **f** is a function that is continuous on [a, b] and differentiable on (a, b), then there is a number c ∈ [a, b] such that:

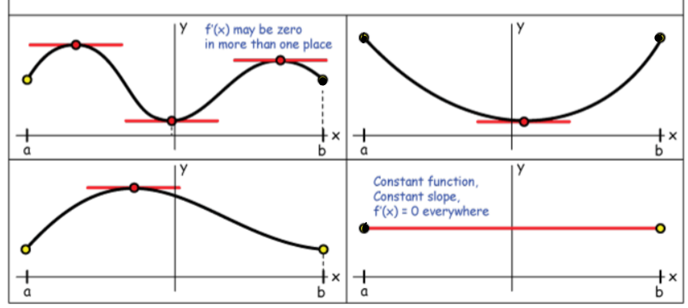
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$ which can be rearranged to $$f(b) - f(a) = f'(c)(b - a)$$



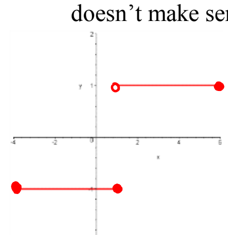
Rolle's Theorem

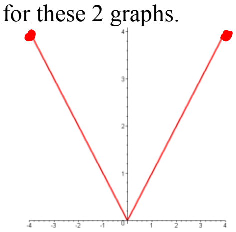
If **f** is a function that is continuous on [a, b] and differentiable on (a, b), and f(a) = f(b), then there is a number c ∈ [a, b] ∋ f'(c) = 0.

"is an element of" "such that"



d. (AP)
   i. If policewoman Betsy sees Paula enter the 407 at the QEW in Burlington (just outside Hamilton) at precisely 12:00 noon, and then her husband policeman Bobby sees Paula 107 km away exiting the 407 at HWY 7 in Pickering at precisely 12:45 pm, can they give her a speeding ticket? Why? Justify using theorem(s)
   ii. What conditions of Rolle's theorem are broken? Show how the conclusion doesn't make sense to be applied for these 2 graphs.





2. FUNCTION ANALYSIS (MCV)

a. Copy/Paste the following

If the graph of f lies above all of its tangents on an interval *I*, then it is called **concave upward** on *I*. If the graph of f lies below all of its tangents on *I*, it is called **concave downward** on *I*.	A **critical number** of a function *f* is a number *c* in the domain of *f* such that either $f'(c) = 0$ or DNE. A point *P* on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward (or vice versa) at *P* and $f''(c) = 0$ or DNE
Increasing/Decreasing Test: If $f'(x) > 0$ on an interval, then f is **increasing** on that interval. If $f'(x) < 0$ on an interval, then f is **decreasing** on that interval.	**Concavity Test** If $f''(x) > 0$ for all x in *I*, then the graph of f is concave upward on *I*. If $f''(x) < 0$ for all x in *I*, then the graph of f is concave downward on *I*.

b. Fill in the chart with conditions to check:.

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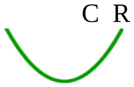
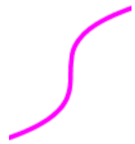
						
	HT & Inf Point	HT	HT	VT	VT	VT & Inf Point
1st deriv TEST						
2nd deriv TEST						

OTHER PTS		
	Corner or jump	VA or hole

c. Classify the critical points:

i) $y = x^{\frac{4}{5}}(2-x)^2$ apply the 2nd derivative test

ii) $y = xe^x$ apply the 1st derivative test

3. SKETCHING (MCV)

a. Read over PreCalculus Journal Unit 7 question #2 about the Algorithm for sketching. Then copy/paste the updated algorithm:

Note that * steps can be skipped since everything else may give enough info to determine all details of the graph.

- From f
  - Factor to see if any Holes exist then state VA's and Domain (watch out for restrictions in denominators, roots and log functions)
  - Find x and y intercepts
  - Decide on the end behavior (do the limit as $x \to \pm\infty$ to see if diverging or HA, for OA don't forget PreCalc Unit 7)
  - Look at multiplicities of zeros and VA's to determine function behavior near zeros and VA's
  - *Find positive/negative intervals ie. test intervals between zeros and VA's
- From f'
  - Find critical points
  - *Find increasing/decreasing intervals + classify the critical points using 1st derivative test
- From f''
  - Find possible inflection points
  - Find concave up/down intervals + decide if above pts are actual inflection points + classify critical points using 2nd derivative test
- At the end use f again to find the y values of critical and inflection points to order to sketch them. Don't forget intercepts.

b. Sketch and label all intercepts, asymptotes, critical points and inflection points. Show all justifying steps without use of desmos

i) $y = x^{\frac{1}{3}}(x-4)$ ii) $y = 2\ln|x| - x^2 + 1$ iii) $y = \frac{x^3}{x^2-1}$ **find derivatives using: http://goo.gl/lVbExS**

4. VELOCITY & ACCELERATION (MCV)

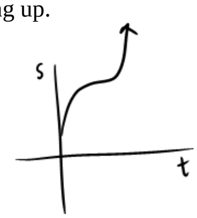
a. Copy/Paste the following

Displacement	$s(t)$	m
Velocity	$v(t) = \frac{ds}{dt}$	m/s
Acceleration	$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	m/s^2
Jerk/Turbulence	$j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$	m/s^3

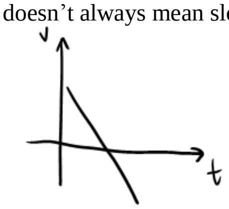
Condition	Event
s<0	Object to the left (below) of the origin
s=0	Object at the origin
s>0	Object to the right (above) of the origin
v<0	Moving to the left (downward)
v=0	At rest
v>0	Moving to the right (upward)
a<0	s-t graph concave down (slow down ; speed up) Acceleration directed to the left (downward)
a=0	Constant velocity
a>0	s-t graph concave up (slow down ; speed up) Acceleration directed to the right (upward)

Condition	Event
$s \cdot v < 0$	Object moving toward the origin
$s \cdot v > 0$	Object moving away from the origin
$s \cdot a < 0$	Acceleration is directed toward the origin
$s \cdot a > 0$	Acceleration is directed away from the origin
$v \cdot a < 0$	Object is slowing down
$v \cdot a > 0$	Object is speeding up

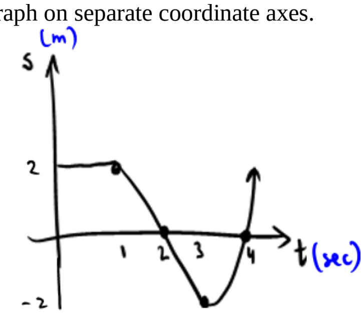
b. Explain what is happening in the following displacement graph. Then talk about why an increasing s-t graph may not mean necessarily speeding up.



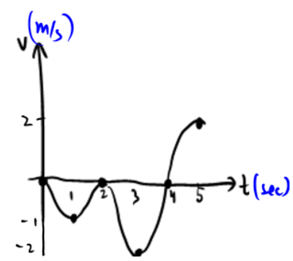
c. Explain what is happening in the following velocity graph. Then talk about why zero on a v-t graph does not mean object is at the origin. Also the graph shows negative acceleration, talk about that this doesn't always mean slowing down.



d. For the s-t graph below explain what's going on and sketch the v-t graph on separate coordinate axes.



e. For the v-t graph below explain what's going on and sketch the possible s-t graph and a-t graph on separate coordinate axes.



5. OTHER APPLICATIONS (MCV)

a.

A sperical balloon is being inflated. $V = \frac{4}{3}\pi r^3$

i. Find the rate of change in volume as radius changes from r = 1cm to r = 3cm

ii. Find the change in volume at r = 10cm

b.

An arrow is shot so that its height, in meters, above the ground is given by: $h(t) = 1.5 + 32t - 5t^2$ where t is time, in seconds

i. Determine the max height using the derivative

ii. What speed is it traveling at when it hits the ground?

iii. Determine the acceleration of the arrow and its relevance.