Key Concepts

- Always look for a common factor first when factoring a trinomial.
- You can factor a difference of squares as $a^2 - b^2 = (a + b)(a - b)$.
- You can factor a perfect square trinomial as $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$.

Communicate Your Understanding

C1 Use words and diagrams to explain why $x^2 + 9$ cannot be factored over the integers.

C2 When her classmate showed Barbara the first step in Example 3b), $25k^2 - 60km + 36m^2 = (5k)^2 - 2(5k)(6m) + (6m)^2$, Barbara asked, "Where did the 2 come from?" Answer Barbara's question.

Practise

For help with questions 1 and 2, see Example 1.

1. Factor.

a) $x^2 - 16$ **b)** $y^2 - 100$
c) $9k^2 - 36$ **d)** $4a^2 - 121$
e) $36w^2 - 49$ **f)** $144p^2 - 1$
g) $16n^2 - 25$ **h)** $100g^2 - 81$

2. Factor.

a) $m^2 - 49n^2$ **b)** $h^2 - 25d^2$
c) $100 - 9c^2$ **d)** $169a^2 - 49b^2$
e) $25x^2 - 36y^2$ **f)** $16c^2 - 9d^2$
g) $162 - 8s^2$ **h)** $75h^2 - 27g^2$

For help with question 3, see Example 2.

3. Verify that each trinomial is a perfect square. Then, factor.

a) $x^2 + 12x + 36$ **b)** $k^2 + 18k + 81$
c) $y^2 - 6y + 9$ **d)** $m^2 - 14m + 49$
e) $x^2 + 20x + 100$ **f)** $64 - 16r + r^2$

For help with question 4, see Example 3.

4. Verify that each trinomial is a perfect square. Then, factor.

a) $4c^2 + 12c + 9$ **b)** $16k^2 - 8k + 1$
c) $25x^2 + 70x + 49$ **d)** $9y^2 - 30y + 25$
e) $100c^2 - 180c + 81$ **f)** $25 + 80y + 64y^2$

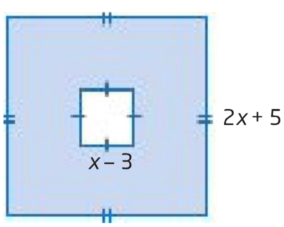
Connect and Apply

5. Each of the following is not factorable over the integers. Why not?
   a) $9x^2 - 16y$
   b) $36a^2 + 107a + 81$
   c) $10w^2 - 70wz + 49z^2$
   d) $25n^2 + 36m^2$

6. Factor fully, if possible.
   a) $4x^2 + 28xy + 49y^2$
   b) $9k^2 - 24km + 16m^2$
   c) $25p^2 + 60pq + 144q^2$
   d) $9y^2 - 7x^2$
   e) $2a^2 - 28ab + 98b^2$
   f) $196n^2 - 144m^2$
   g) $25x^2 + 70xy + 14y^2$
   h) $100f^2 - 120fg + 36g^2$
   i) $400p^3 - 900pq^2$

For help with question 7, see Example 4.

7. a) Find an algebraic expression for the area of the shaded region.
   b) Write the area expression in factored form.



8. Determine all values of b so that each trinomial is a perfect square.
   a) $y^2 + by + 121$
   b) $4x^2 + bx + 25$
   c) $9n^2 + bnp + 49p^2$
   d) $w^2 + 10w + b$
   e) $81m^2 - 90m + b$
   f) $16x^2 - 88xy + b^2y^2$

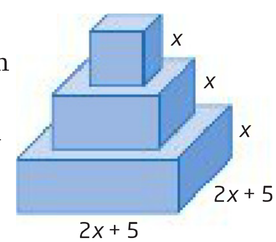
9. Determine two values of k so that each trinomial can be factored as a difference of squares.
   a) $m^2 - kn^2$
   b) $kx^2 - 9$
   c) $49c^2 - k$

10. Factor, if possible.
    a) $9a^2b^2 - 24abcd + 16c^2d^2$
    b) $225 - (x + 5)^2$
    c) $(3c + 2)^2 - (3c - 2)^2$
    d) $4x^2 + 26x + 9$

11. The area of an unknown shape is represented by $9x^2 + 30x + 25$. If x must be an integer, what shape(s) could this figure be?

12. A box is in the shape of a rectangular prism. Its volume is given as $x^3 - 2x^2 + x$.
    a) Determine algebraic expressions for the dimensions.
    b) Describe the faces of the box.

13. **Chapter Problem** In Section 5.3, question 12, you found an algebraic expression for the total of the top surface areas of the three prisms used to make the pedestal.



    a) Write algebraic expressions for the exposed top surface areas of the middle and bottom layers of the pedestal.
    b) Factor each expression from part a).
    c) Compare the expressions for the exposed surface areas when x represents 5 cm.

14. The radius of a circle has been decreased by a certain amount. Its area is now given as $\pi r^2 - 14\pi r + 49\pi$, where r was the original radius, in centimetres.
    a) What was the decrease in radius?
    b) What was the decrease in area?

15. Is $x^2 - 1$ the same as $(x - 1)^2$? Explain using words and/or diagrams.

16. A parabola has equation $y = x^2 - 4x + 4$. Rewrite the equation in factored form to find the coordinates of the vertex.

17. Factor to evaluate each difference.

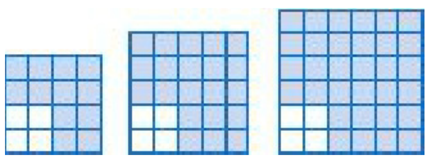
a) $15^2 - 11^2$

b) $37^2 - 27^2$

c) $98^2 - 97^2$

d) $28^2 - 22^2$

18. The first three diagrams in a pattern are shown.



a) Use a table to develop a formula to represent the number of shaded small squares in the nth diagram.

b) Write your formula in factored form.

c) Calculate the number of shaded small squares in the 10th diagram using both versions of your formula.

d) Which version is easier to use? Why?

Extend

19. A three-dimensional figure has volume given as $4\pi x^3 + 20\pi x^2 + 25\pi x$. What shape(s) could this figure be, and what are its dimensions?

20. Factor.

a) $(x - 4)^2 - 16$

b) $(x + 1)^2 + 2(x + 1) + 1$

c) $25x^4 - 9y^4$

d) $k^4 - 8k^2 + 16$

e) $a^6 + 20a^3 + 100$

f) $\frac{y^4}{81} - \frac{x^4}{625}$

21. Find all values of k so that each trinomial can be factored as a perfect square over the integers.

a) $81x^4 + kx^2 + 16$

b) $4y^4 + ky^2z^2 + 25z^4$

22. Use Technology

a) Use a CAS to factor each expression:
$x^2 - 1$
$x^3 - 1$
$x^4 - 1$
$x^5 - 1$

b) Look for a pattern in the factors. Which factored form does not appear to follow the pattern? Use a CAS to expand the last two factors of this factored form. Note what happens.

c) Use your pattern to predict the result of factoring $x^6 - 1$ into two factors. Check your prediction using a CAS. If necessary, expand factors.

23. Math Contest

a) Show that $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$.

b) Factor $m^3 - 64$.

c) Factor $27y^3 - 125z^6$.

24. Math Contest

a) Show that
$a^3 + 1000 = (a + 10)(a^2 - 10a + 100)$.

b) Factor $k^6 + 216e^3$.

c) Factor $343q^{12} + 729r^{24}$.

25. Math Contest

a) Expand $(a + b)^4$.

b) Factor.
$81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$

26. Math Contest If $a^2 + b^2 = 15$ and $ab = 3$, then the value of $(a - b)^2$ is

A 21

B 18

C 12

D 9

E 3

5.6 Factor a Perfect Square Trinomial and a Difference of Squares, pages 248–255

1. a) $(x + 4)(x - 4)$ **b)** $(y + 10)(y - 10)$
c) $9(k + 2)(k - 2)$ **d)** $(2a + 11)(2a - 11)$
e) $(6w + 7)(6w - 7)$ **f)** $(12p + 1)(12p - 1)$
g) $(4n + 5)(4n - 5)$ **h)** $(10g + 9)(10g - 9)$

2. a) $(m + 7n)(m - 7n)$ **b)** $(h + 5d)(h - 5d)$
c) $(10 + 3c)(10 - 3c)$ **d)** $(13a + 7b)(13a - 7b)$
e) $(5x + 6y)(5x - 6y)$ **f)** $(4c + 3d)(4c - 3d)$
g) $2(9 + 2s)(9 - 2s)$ **h)** $3(5h + 3g)(5h - 3g)$

3. a) $x^2 + 12x + 36 = (x)^2 + 2(x)(6) + (6)^2; (x + 6)^2$
b) $k^2 + 18k + 81 = (k)^2 + 2(k)(9) + (9)^2; (k + 9)^2$
c) $y^2 - 6y + 9 = (y)^2 - 2(y)(3) + (3)^2; (y - 3)^2$
d) $m^2 - 14m + 49 = (m)^2 - 2(m)(7) + (7)^2; (m - 7)^2$
e) $x^2 + 20x + 100 = (x)^2 + 2(x)(10) + (10)^2; (x + 10)^2$
f) $64 - 16r + r^2 = (8)^2 - 2(8)(r) + (r)^2; (8 - r)^2$

4. a) $4c^2 + 12c + 9 = (2c)^2 + 2(2c)(3) + (3)^2; (2c + 3)^2$
b) $16k^2 - 8k + 1 = (4k)^2 - 2(4k)(1) + (1)^2; (4k - 1)^2$
c) $25x^2 + 70x + 49 = (5x)^2 + 2(5x)(7) + (7)^2; (5x + 7)^2$
d) $9y^2 - 30y + 25 = (3y)^2 - 2(3y)(5) + (5)^2; (3y - 5)^2$
e) $100c^2 - 180c + 81 = (10c)^2 - 2(10c)(9) + (9)^2;$ $(10c - 9)^2$
f) $25 + 80y + 64y^2 = (5)^2 + 2(5)(8y) + (8y)^2; (5 + 8y)^2$

5. Answers may vary. For example:
a) y is not squared.
b) 107 is not equal to 2(6)(9).
c) 10 is not a perfect square.
d) The expression is a sum of squares, not a difference of squares.

6. a) $(2x + 7y)^2$ **b)** $(3k - 4m)^2$
c) not possible **d)** not possible
e) $2(a - 7b)^2$ **f)** $4(7n + 6m)(7n - 6m)$
g) not possible **h)** $4(5f - 3g)^2$
i) $100p(2p + 3q)(2p - 3q)$

7. a) area: $(2x + 5)^2 - (x - 3)^2$ **b)** $(3x + 2)(x + 8)$

8. a) 22, −22 **b)** 20, −20
c) 42, −42 **d)** 25
e) 25 **f)** 11, −11

9. Answers may vary. For example:
a) 4, 9 **b)** 1, 25 **c)** 16, 25

10. a) $(3ab - 4cd)^2$ **b)** $(20 + x)(10 - x)$
c) $(6c)(4)$ **d)** not possible

11. The figure could be a square or a parallelogram with base equal to height.

12. a) $x(x - 1)^2$; height x, length $x - 1$, width $x - 1$
b) The box is a square-based rectangular prism, so the top and bottom are squares with area $(x - 1)^2$ and the four sides are rectangles with area $x(x - 1)$.

13. a) middle: $(2x + 2)^2 - (2x - 1)^2$; bottom: $(2x + 5)^2 - (2x + 2)^2$
b) middle: $(4x + 1)(3)$; bottom: $(4x + 7)(3)$
c) middle: 63 cm²; bottom: 81 cm²

14. a) 7 cm **b)** $(14\pi r - 49\pi)$ cm²

15. No. $(x - 1)^2 = (x - 1)(x - 1)$, while $x^2 - 1 = (x - 1)(x + 1)$. The two factored expressions are not equivalent. Alternatively, $(x - 1)^2 = x^2 - 2x + 1$, which is not equivalent to $x^2 - 1$.

16. $y = (x - 2)^2$; the vertex is (2, 0).

17. a) $(15 + 11)(15 - 11) = 104$
b) $(37 + 27)(37 - 27) = 640$
c) $(98 + 97)(98 - 97) = 195$
d) $(28 + 22)(28 - 22) = 300$

18. a) $s = (n + 3)^2 - 4$ **b)** $s = (n + 5)(n + 1)$
c) 165 **d)** Answers will vary.

19. The figure could be a square-based prism with height πx and base side length $2x + 5$ or a cylinder with radius $2x + 5$ and height x.

20. a) $(x)(x - 8)$ **b)** $(x + 2)^2$
c) $(5x^2 + 3y^2)(5x^2 - 3y^2)$ **d)** $(k^2 + 4)(k + 2)(k - 2)$
e) $(a^3 + 10)^2$
f) $\left(\frac{y^2}{9} + \frac{x^2}{25}\right)\left(\frac{y}{3} + \frac{x}{5}\right)\left(\frac{y}{3} - \frac{x}{5}\right)$

21. a) 72, −72 **b)** 20, −20

22. a) $x^2 - 1 = (x - 1)(x + 1)$; $x^3 - 1 = (x - 1)(x^2 + x + 1)$;
$x^4 - 1 = (x - 1)(x + 1)(x^2 + 1)$;
$x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

b) Answers may vary. For example: $x - 1$ is one of the factors of each of the expressions and the number of terms in the other factor is equal to the degree of the original expression. The terms in the other factor form a sum where the coefficient of each of the terms is one and the terms are the sum of the descending degrees of the variable starting with 1 less than the original expression. The factored form of $x^4 - 1$ does not appear to follow the pattern. When expanded, the last two terms of this factored form result in the expression $x^3 + x^2 + x + 1$, which does follow the pattern.

c) $x^6 - 1 = (x - 1)(x^5 + x^4 + x^3 + x^2 + x + 1)$, which is also $(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)$.

23. a) By expanding, $(x - 2)(x^2 + 2x + 4) = x^3 - 8$.
b) $(m - 4)(m^2 + 4m + 16)$
c) $(3y - 5z^2)(9y^2 + 15yz^2 + 25z^4)$

24. a) By expanding, $(a + 10)(a^2 - 10a + 100) = a^3 + 1000$.
b) $(k^2 + 6e)(k^4 - 6k^2e + 36e^2)$
c) $(7q^4 + 9r^8)(49q^8 - 63q^4r^8 + 81r^{16})$

25. a) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
b) $(3x - 2y)^4$

26. D