Big idea

In this unit you will learn how similar and right triangles apply to real life situations. You will learn how to use SINe, COSine, and TANgent buttons on the calculator to solve for sides or angles of right triangles. Make sure your calculator is in DEGREE mode so that the answers will always come up correctly.

Not all triangles you see in real life may be right angled. But _____ and _____ are only used on right triangles. How does one solve a non-right triangle? For that you will need two new laws: Sine and Cosine Laws. (There is no Tangent Law, since having the two that we develop enables us to solve ANY type of triangle). There are a lot of real life applications for trigonometry – these you will see in the word problems of this unit.

Feedback & Assessment of Your Success

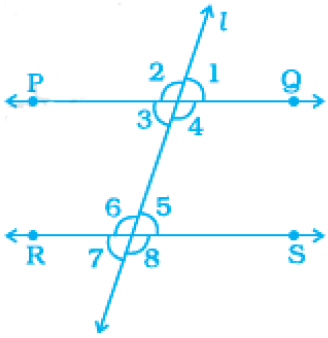
Date	Pages	Topics	Finished assignment pages? Made corrections?	Summarized notes in a journal? Added your own explanations?	How many extra practice questions did you try in each topic?	Tentative TEST date: _____ Questions to ask the teacher:
	2-4	Congruent Triangles (MPM) Journal #1 & 2				
	5-7	Similar Triangles (MPM) Journal #3				
2 days	8-13	SOH CAH TOA (MPM) Journal #4 & 5				
2 days	14-18	Sine & Cosine Laws (MPM) Journal #6				
2 days	19-22	Word Problems (MPM) Journal #7				

ASSIGNMENT Congruent Triangles (MPM)

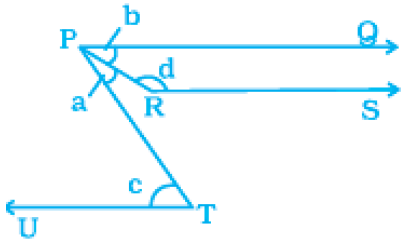
1. **Proving something is FALSE**
Conjectures can be proved **false** with a single _____
Example Conjecture: In a quadrilateral, if all angles are congruent, then all sides are congruent.

2. **Proving something is TRUE**
Conjectures can be proved **true** by using a logical argument, based on known facts. When a conjecture has been proved true, it is called a _____.

A proof is a logical argument. In math, something is considered true if it has been proved. It is not enough for something to *seem* true. In writing a proof, you can only use facts that have previously been proved, or facts that are assumed true without proof. In this class, we will assume journal #1a info is true.

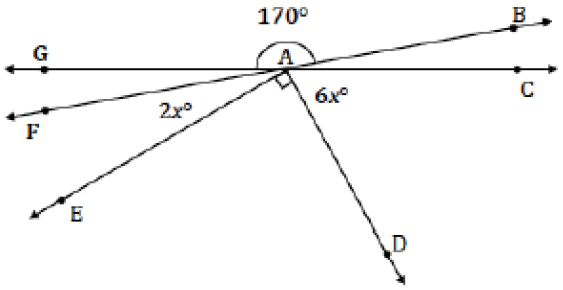
3. 

If PQ||RS, $\angle 1 = (2a+b)^o$ and $\angle 6 = (3a-b)^o$, then find $\angle 2$ in terms of b.

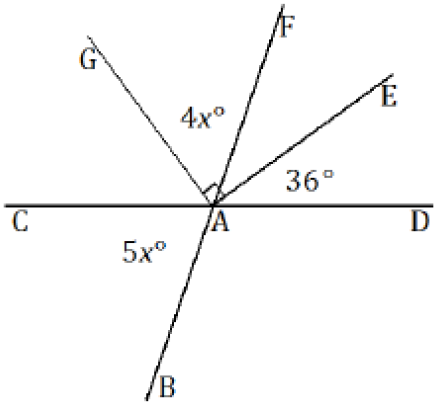
4. 

If PQ||RS||UT, $\angle c = 75^o$ and $a:c = 2:5$ then find $\angle b$.

5. Find the measurement of $\angle FAE$ and $\angle CAD$.

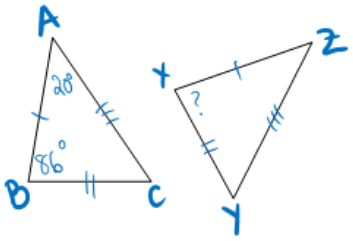


6. Find the measurement of $\angle GAF$ and of $\angle BAC$.

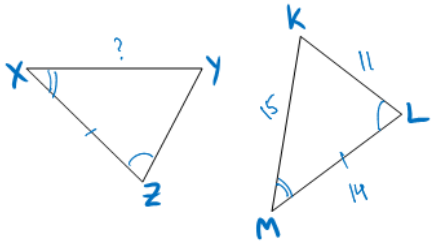


Prove Congruent then find the value of "?"

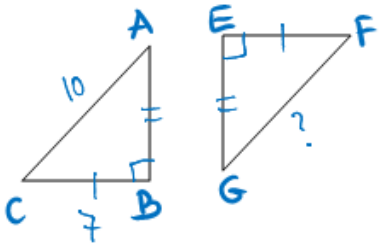
7. SSS



8. ASA or AAS

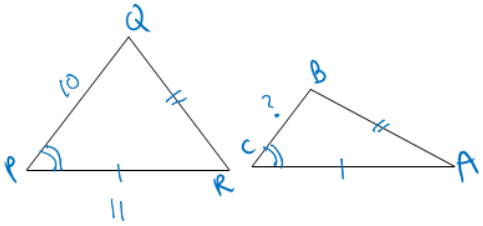


9. SAS

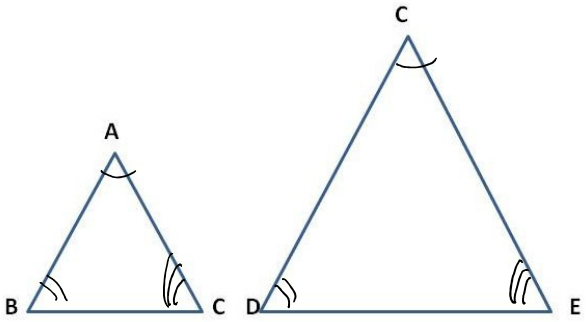


Talk about why SSA or AAA is not enough to prove congruency:

10. Not SSA



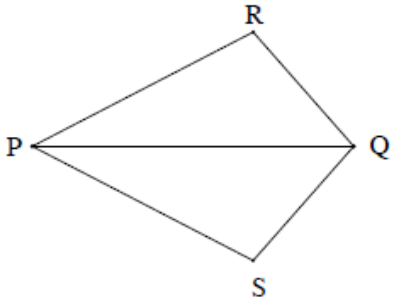
11. Not AAA



Two Formats for Proof

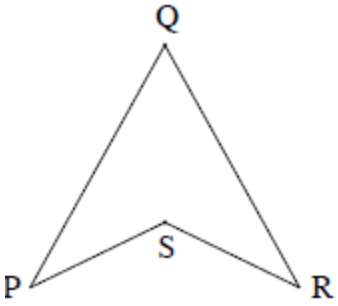
paragraph form – used by mathematicians

12.



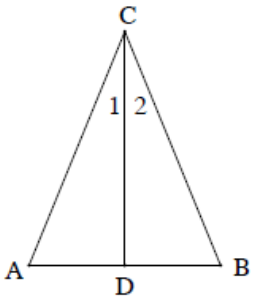
Given: RQ=SQ and RP=SP.
Prove: ∠ R = ∠ S.
The given information is: _____ and _____. PQ=PQ because _____. So ΔPRQ≅ ΔPSQ because of the _____ property for congruent triangles. Therefore, _____ because corresponding parts of congruent triangles are congruent.

14. Given: PQ = RQ and PS = RS.
Prove: ∠ P = ∠ R.



two-column form – students prefer this one

13.

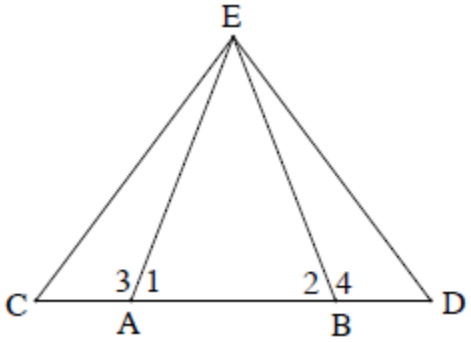


Given: AC=BC and ∠ 1 = ∠ 2.
Prove: ∠ A = ∠ B.

Proof:

Statements	Reasons
1. AC = BC	1.
2.	2. Given
3. CD = CD	3.
4.	4. by SAS property
5. ∠A = ∠ B	5.

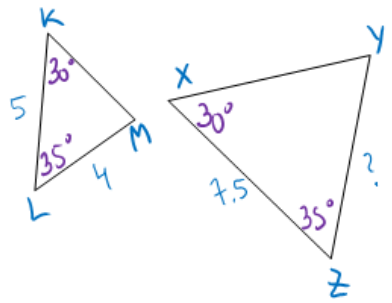
15. Given: AC = BD and ∠ 1 = ∠ 2.
Prove: ΔCAE≅ ΔDBE.



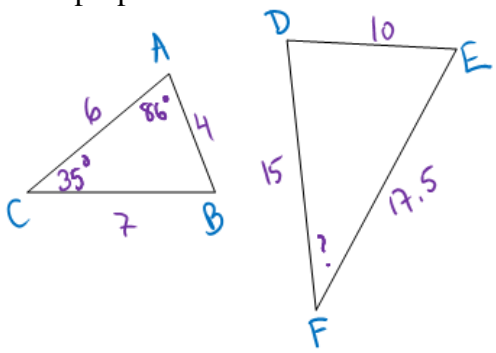
ASSIGNMENT Similar Triangles (MPM)

Prove Similar then find the value of "?"

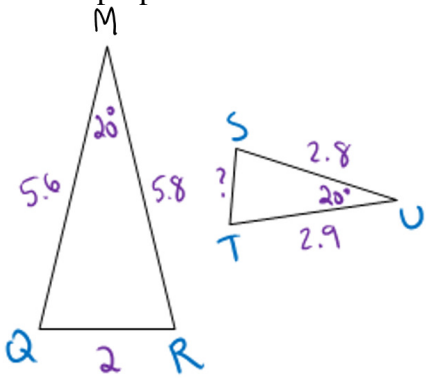
1. AAA or just AA



2. SSS in proportion

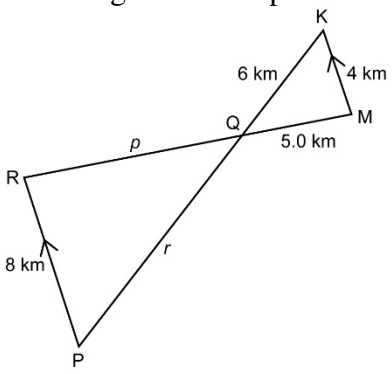


3. SAS in proportion

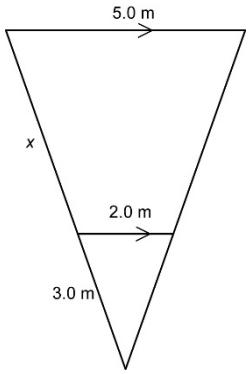


Prove the triangles in each pair are similar. Then find the unknown side lengths

4.



5.



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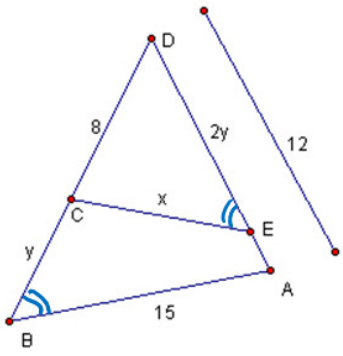
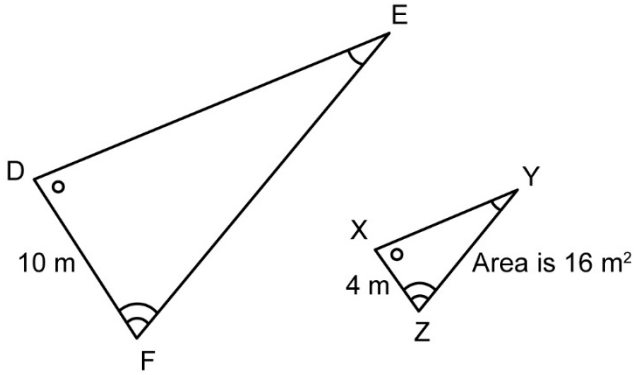


7. A right triangle has side lengths 5 cm, 12 cm, and 13 cm.
   a) A similar triangle has a hypotenuse 52 cm long. What is the scale factor?
   b) What are the lengths of the legs of the triangle in part a)?
   c) Find the area of each triangle.
   d) How are these areas related?

8. Bill placed a mirror on the ground 5 m from the base of a flagpole. He stepped back until he could see the top of the flagpole reflected in the mirror. Bill is 1.5 m tall and saw the reflection when he was 1.25 m from the mirror. How high is the flagpole?

9. A person 1.9 m tall casts a shadow 3.8 m long. At the same time a tree casts a shadow 18 m long. Find the height of the tree.

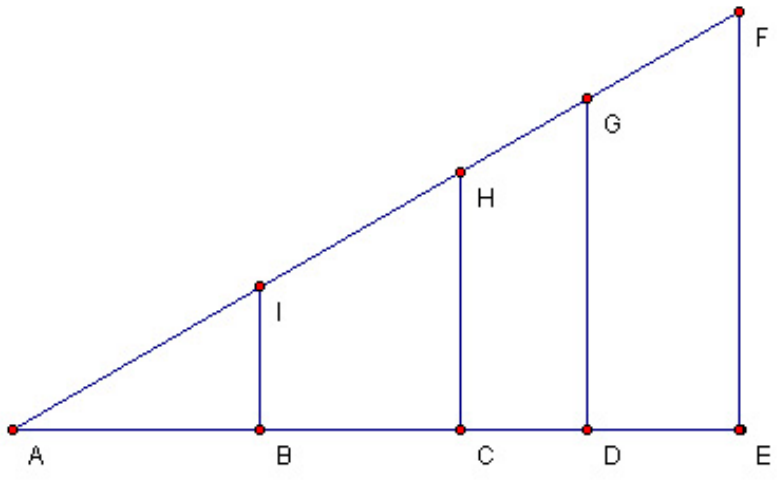
10. $\triangle DEF \sim \triangle XYZ$. Find the area of $\triangle DEF$.



ASSIGNMENT SOH CAH TOA (MPM)

In early times, similar triangles were used to solve problems about measurement.
One individual, Hipparchus, 140 B.C.E., found that right angle triangles had a special property.
Let's investigate what this special property is.

This diagram shows some similar **right** triangles with a common angle A. Measure the side lengths of each triangle and record your findings in the table below.



round to one decimal place

Triangles	Side Opposite to $\angle A$	Side Adjacent to (beside) $\angle A$	Hypotenuse	Calculate these Trigonometric Ratios		
				$\frac{opposite}{hypotenuse}$	$\frac{adjacent}{hypotenuse}$	$\frac{opposite}{adjacent}$
ΔABI						
ΔACH						
ΔADG						
ΔAEF						

1. Explain why the ratios of the triangles are the same.

2. Measure the angle A using the protractor: $< A =$ _____

3. MAKE SURE your calculator is in DEGREE mode!
   Use the calculator to calculate the following trigonometric functions

$\sin A =$ $\cos A =$ $\tan A =$

$\frac{opposite}{hypotenuse}$ is called _____ or _____ for short

$\frac{adjacent}{hypotenuse}$ is called _____ or _____ for short

$\frac{opposite}{adjacent}$ is called _____ or _____ for short

A way to remember Trig Ratios

— — — — — — — — —

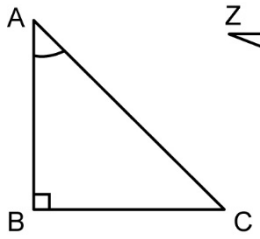
4. Try $\sin 90° =$ _____ $\cos 90° =$ _____ $\tan 90° =$ _____

5. What can you conclude about which angles you can use (highlight) when labeling opposite/adjacent/hypotenuse for SOH CAH TOA on the right triangle?

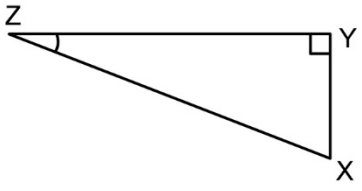
6. How and when to round decimals:

7. Practice labelling triangles: Label the hypotenuse, the opposite, and the adjacent sides relative to each marked angle.

a)



b)



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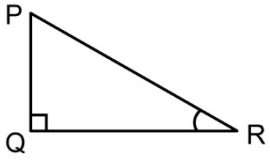


d)

8. Practice using the calculator: Make sure your calculator is in **DEGREE** (DEG) mode

Given the angle find the ratio

a) sin 45°

b) cos 98°

c) tan 4°

d) $\cos 76^o = \frac{3}{x}$

Given the angle fnd the angle (use the SHIFT or 2ND buttons)

e) sin A = 0.557

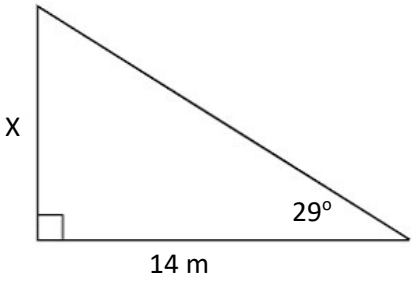
f) cos C = 0.705

g) tan B = 2.984

h) $\sin x = \frac{5}{6}$

9. Practice solving triangles: Determine the measure of the missing angles or sides

a.



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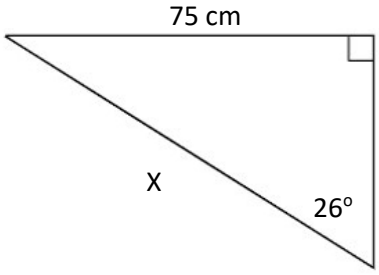

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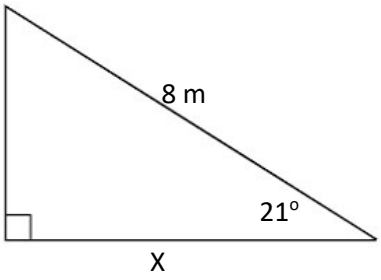
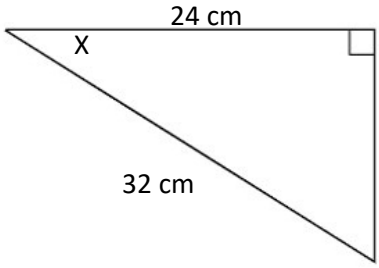
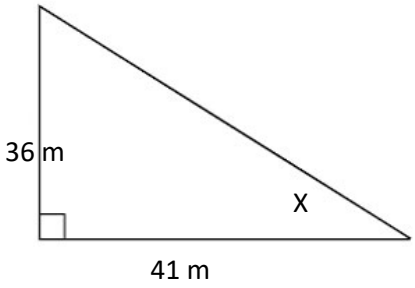


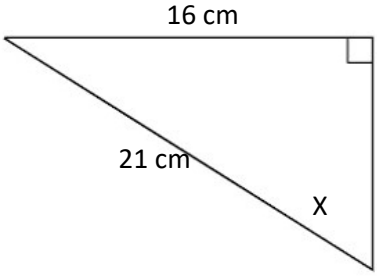
d.



e.

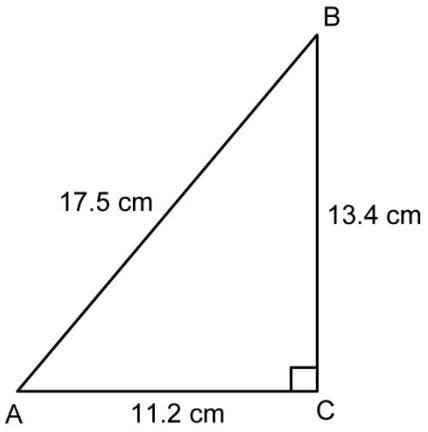


f.



10. Find the three primary trigonometric ratios for ∠A, to four decimal places.

a)



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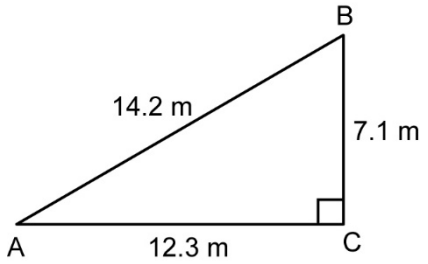


11. Evaluate with a calculator. Round your answers to four decimal places.

a) sin 72°

b) cos 36°

c) tan 57.4°

12. Find the measure of each angle, to the nearest degree.

a) $\sin \theta = 0.5189$

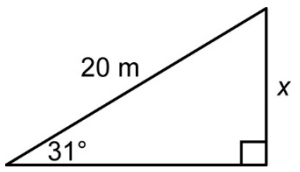
b) $\cos B = \frac{9}{10}$

c) $\tan \theta = \frac{9}{14}$

13. Find the measures of both angles A and B in part a) above. Discuss several methods of doing so.

14. Find the value of x, to the nearest tenth.

a)



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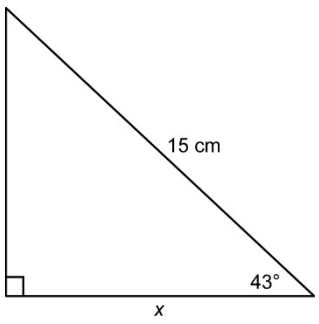
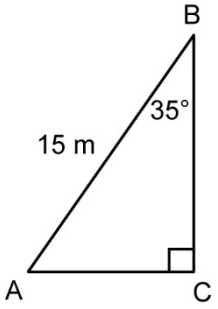


15. Solve each triangle. Round side lengths to the nearest tenth of a metre.

a)



b) In $\triangle PQR$, $\angle Q = 90°$, $p = 14$ m and $q = 20$ m.

16. In order to measure the height of a tree, Dan calculated that its shadow is 12 m long and that the line joining the top of the tree to the tip of the shadow forms an angle of 52° with the flat ground.
a) Draw a diagram to illustrate this problem.
b) Find the height of the tree, to the nearest tenth of a metre.

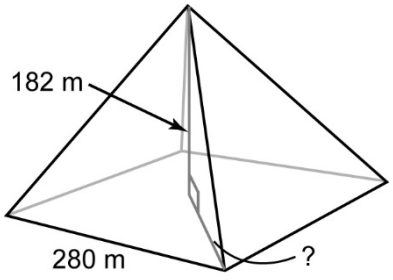
17. A monument casts a shadow 13 m long. The sun's rays form an angle of 63° with the ground. Calculate the height of the monument to on decimal place.

18. A ladder leans against a wall forming a 25° angle with the wall. If the ladder reaches 2.8 m up the wall, how long is the ladder?

19.
Brook is flying a kite while standing 24 m from the base of a tree at the park. Her kite is directly above the 3-m tree and the 25-m string is fully extended. Approximately how far above the tree is her kite flying?

20. A carpenter leans a 4 m ladder against a wall. It reaches 3.5 m up the wall. What is the angle the ladder makes with the wall?

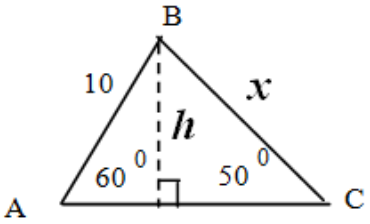
21.
A square-based pyramid has a height of 182 m and a base length of 280 m. Find the angle, to the nearest degree, that one of the edges of the pyramid makes with the base. Round your answer to the nearest degree.



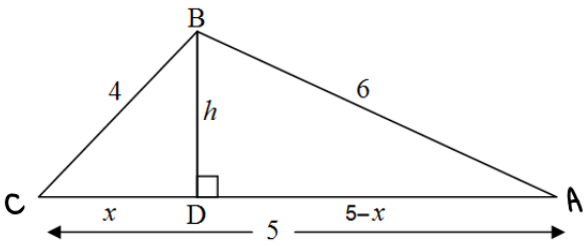
ASSIGNMENT Sine & Cosine Laws (MPM)

PROOFS:

Sine Law



Cosine Law



1. Draw a diagram and label the given information. Then, find the measure of the indicated side in each triangle, to the nearest tenth of a unit.

a) In acute $\triangle ABC$, $\angle A = 72°$, $\angle B = 68°$, and $a = 12$ cm. Find side b.

b) In acute $\triangle DEF$, $\angle D = 52°$, $\angle F = 71°$, and $e = 8.0$ m. Find side d.

2. Draw a diagram and label the given information. Then, find the measure of the indicated angle in each triangle, to the nearest degree.

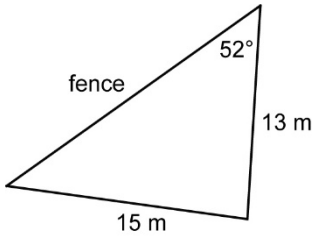
a) In acute $\triangle PQR$, $\angle P = 64°$, $p = 5.7$ cm, and $r = 4.1$ cm. Find $\angle R$.

b) In acute $\triangle STU$, $\angle S = 57°$, $s = 12$ m, and $u = 9$ m. Find $\angle U$.

3. Draw a diagram and label the given information. Then, solve the triangle.

In $\triangle DMC$, $\angle D = 55°$, $d = 21$ cm, and $m = 23$ cm.

4. Angela is building a garden in the shape of a triangle, as shown. She would like to put a fence on one side of the garden.

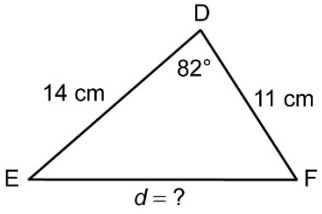


a) Find the angle formed by the fence and the side that is 15 m in length.

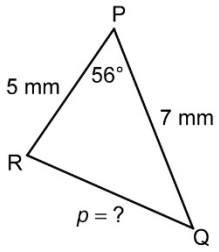
b) Find the length of the fence

5. Find the length of the indicated side in each triangle, to the nearest tenth of a unit.

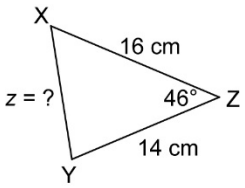
a)



b)



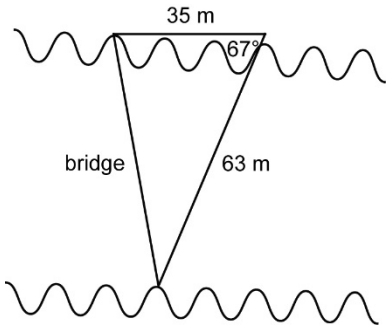
c)



6. Sketch the triangle and label the given information. Then, solve the triangle.

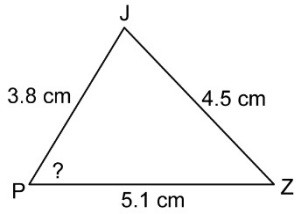
In $\triangle MCB$, $\angle M = 61°$, $c = 18$ cm, and $b = 21$ cm.

7. Find the length of the bridge, to the nearest metre.

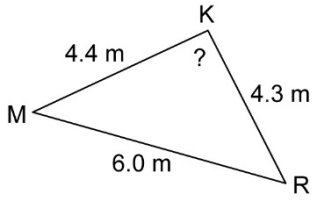


8. Solve for the indicated angle, to the nearest degree.

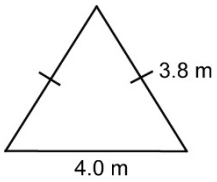
a)



b)



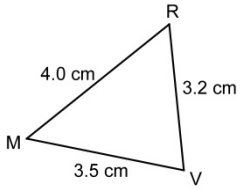
9. Laurissa is designing a reflecting pool, in the shape of a triangle, for her backyard.



a) Find the interior angles of the reflecting pool

b) Find the surface area of the water in the reflecting pool

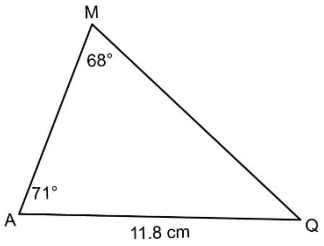
10. Solve the triangle.



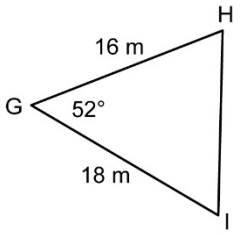
ASSIGNMENT Word Problems (MPM)

Determine whether the primary trigonometric ratios, the sine law, or the cosine law should be used to solve each triangle.

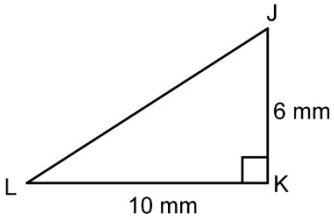
1.



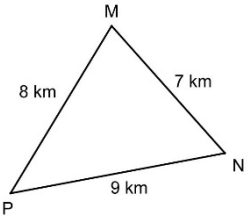
2.



3.



4.



5. A shed is 8 ft wide. One rafter makes an angle of 30° with the horizontal on one side of the roof. A rafter on the other side makes an angle of 70° with the horizontal. Calculate the length of the shorter rafter to the nearest foot.

6. A 10 m ladder leans against a wall. The top of the ladder is 9 m above the ground. Safety standards call for the angle between the base of the ladder and the ground to be between 70° and 80°. Is the ladder safe to climb?

7. An intersection between two country roads makes an angle of 68°. Along one road, 5 km from the intersection, is a dairy farm. Along the other road, 7 km from the intersection, is a poultry farm. How far apart are the two farms? Round the answer to the nearest tenth of a kilometre.

8. A triangle is built using three poles with lengths 17 m, 15 m and 9 m. What is the measure of the largest angle in the triangle?

9. Three islands - Fogo, Twillingate and Moreton's Harbour - form a triangular pattern in the ocean. Fogo and Twillingate are 15 nautical miles apart. The angle between Twillingate and Moreton's Harbour from Fogo is 45°. The angle between Moreton's Harbour and Fogo from Twillingate is 65°. How far is Moreton's Harbour from Fogo, to the nearest nautical mile?

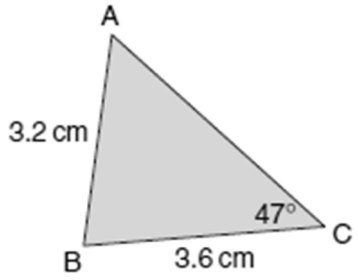
10. A golfer is faced with a shot that has to pass over some trees. The trees are 33 ft tall. The golfer finds himself 21 ft behind these trees, which obstruct him from the green. he decides to go for the green by using a 60° lob wedge. This club will allow the ball to travel at an angle of elevation of 60°. Did he make the right choice? Explain.

11. Two tracking stations, 5 km apart, track a weather balloon floating between them. The tracking station to the west tracks the balloon at an angle of elevation of 52°, and the station to the east tracks the balloon at an angle of elevation of 60°. How far is the balloon from the closest tracking station?

12. Three cell phone towers form a triangle. The distance between the first tower and the second tower is 16 km. The distance between the second tower and the third tower is 19 km. The distance between the first tower and the third tower is 19 km. Calculate the angles between the cell phone towers.

13. From the top of a cliff that is 70 m in height, the angle of depression of a sailboat on a lake is 41°. What is the distance from the base of the cliff to the sailboat?

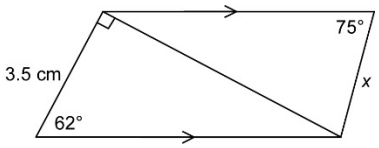
14. What are the measures of the other two angles in this triangle, to the nearest degree?



15. The foot of a ladder is 1.2 m from a fence that is 1.8 m high. The ladder touches the fence and rests against a building that is 1.8 m behind the fence. Draw a diagram, and determine the height on the building reached by the top of the ladder.

16. A tower casts a shadow 7 m long. A vertical stick casts a shadow 0.6 m long. If the stick is 1.2 m high, how high is the tower?

17. Find x, to the nearest tenth of a centimetre.

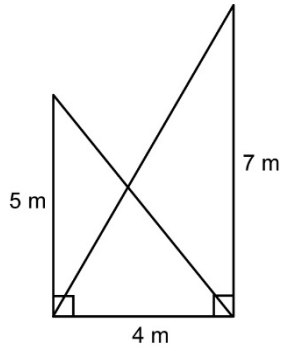


18. Savio is on a hiking trip. On the first section of the hike, he walks 5 km from the Loon Campsite to the Owl Campsite. Then, he turns and hikes 7 km to the Eagle Campsite, making a 68° angle from the first section of the hike. He then returns on the trail to the Loon Campsite.

a) What is the distance from the Eagle campsite to the Loon campsite, to the nearest kilometre?

b) At what angles are the three campsites situated with respect to each other?

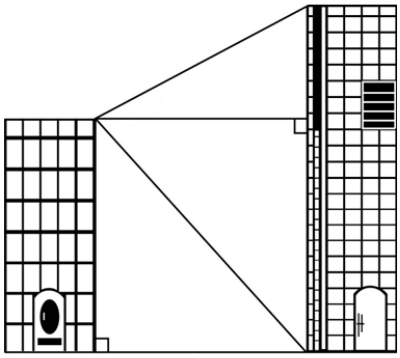
19. At the top of a hiking trail, there are two vertical posts. One is 5 m tall, and the other is 7 m tall. The ground between the posts is level, and the bases of the posts are 4 m apart. The posts are connected by two straight wires.



a) What angle does each wire make with the ground?

b) What is the length of each wire? Solve using different methods for each.

20. From the top of a building, the angle of elevation of the top of a nearby building is 28° and the angle of depression of the bottom of the nearby building is 48°. The distance between the two buildings is 50 m. What is the height of the taller building?



21. Aimee and Russell are facing each other on opposite sides of an 8-m telephone pole. From Aimee's point of view, the top of the telephone pole is at an angle of elevation of 52°. From Russell's point of view, the top of the telephone pole is at an angle of elevation of 38°. How far apart are Aimee and Russell?