Review

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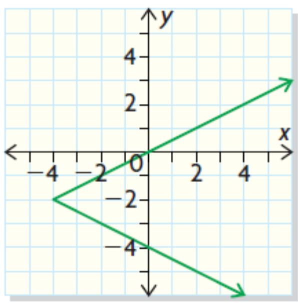
1.

For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.

a) $\{(-3, 0), (-1, 1), (0, 1), (4, 5), (0, 6)\}$

b) $y = 4 - x$

c)



d) $x^2 + y^2 = 16$

2.

What rule can you use to determine, from the graph of a relation, whether the relation is a function? Graph each relation and determine which are functions.

a) $\{(-2. 1), (1, 1), (0, 0), (1, -1), (1, -2), (2, -2)\}$

b) $y = 4 - 3x$

c) $y = (x - 2)^2 + 4$

d) $x^2 + y^2 = 1$

e) $y = \dfrac{1}{x}$

f) $y = \sqrt{x}$

3.

Sketch the graph of a function whose domain is the set of real numbers and whose range is the set of real numbers less than or equal to 3.

4.

If $f(x) = x^2 + 3x - 5$ and $g(x) = 2x - 3$, determine each.

a) $f(-1)$

b) $f(0)$

c) $g\left(\dfrac{1}{2}\right)$

d) $f(2b)$

e) $g(1 - 4a)$

f) x when $f(x) = g(x)$

5.

a) Graph the function $f(x) = -2(x - 3)^2 + 4$, and state its domain and range.

b) What does $f(1)$ represent on the graph? Indicate, on the graph, how you would find $f(1)$.

c) Use the equation to determine each of the following.

i) $f(3) - f(2)$

ii) $2f(5) + 7$

iii) $f(1 - x)$

6.

If $f(x) = x^2 - 4x + 3$, determine the input(s) for x whose output is $f(x) = 8$.

7.

A ball is thrown upward from the roof of a building 60 m tall. The ball reaches a height of 80 m above the ground after 2 s and hits the ground 6 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

c) Determine an equation for the function.

8.

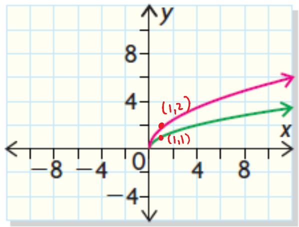
State the domain and range of each function.

a) $f(x) = 2(x - 1)^2 + 3$

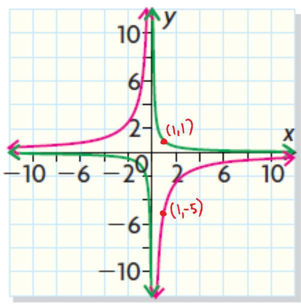
b) $f(x) = \sqrt{2x + 4}$

9. In each graph, a parent function has undergone a transformation of the form $f(kx)$. Determine the equations of the transformed functions graphed in red. Explain your reasoning.

a)



b)



10. For each set of functions, transform the graph of $f(x)$ to sketch $g(x)$ and $h(x)$, and state the domain and range of each function.

a) $f(x) = x^2, g(x) = \left(\frac{1}{2}x\right)^2, h(x) = -(2x)^2$

b) $f(x) = |x|, g(x) = |-4x|, h(x) = \left|\frac{1}{4}x\right|$

11. The point $(1, 4)$ is on the graph of $y = f(x)$. Determine the coordinates of the image of this point on the graph of $y = 3f[-4(x + 1)] - 2$.

12. In each case, write the equation for the transformed function, sketch its graph, and state its domain and range.

a) The graph of $f(x) = \sqrt{x}$ is compressed horizontally by the factor $\frac{1}{2}$, reflected in the x-axis, and translated 3 units right and 2 units down.

b) The graph of $y = \frac{1}{x}$ is stretched vertically by the factor 3, reflected in the x-axis, and translated 4 units left and 1 unit up.

13. If $f(x) = (x - 4)(x + 3)$, determine the x-intercepts of each function.

a) $y = f(x)$

b) $y = -2f(x)$

c) $y = f\left(-\frac{1}{2}x\right)$

d) $y = f(-(x + 1))$

14. A function $f(x)$ has domain $\{x \in \mathbf{R} \mid x \geq -4\}$ and range $\{y \in \mathbf{R} \mid y < -1\}$. Determine the domain and range of each function.

a) $y = 2f(x)$

b) $y = f(-x)$

c) $y = 3f(x + 1) + 4$

d) $y = -2f(-x + 5) + 1$

15. Determine the domain and range of each function. Show your steps.

a) $f(x) = \frac{1}{x - 2}$

b) $f(x) = \sqrt{3 - x} - 4$

c) $f(x) = -|x + 1| + 3$

16. find the inverse of the given function

a) $j(x) = (x+4)^2 + 9$, when $x \leq -4$

b) $f(x) = 3\sqrt{4 - x}$

c) $h(x) = \frac{4x}{x - 6}$

d) $g(x) = x^2 - 10x + 25$, when $x \geq 5$

ReviewSol

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1. a) domain $= \{-3, -1, 0, 4\}$, range $= \{0, 1, 5, 6\}$; not a function, because two y-values are assigned to $x = 0$

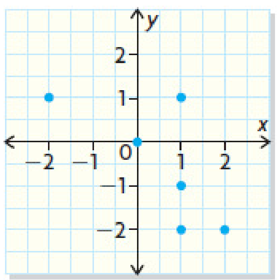
b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$; function, because each x-value has only one y-value assigned

c) domain $= \{x \in \mathbf{R} \mid x \geq -4\}$, range $= \{y \in \mathbf{R}\}$; not a function, because each $x > -4$ has two y-values assigned

d) domain $= \{x \in \mathbf{R} \mid -4 \leq x \leq 4\}$, range $= \{y \in \mathbf{R} \mid -4 \leq y \leq 4\}$; not a function, because each x except ± 4 has two y-values assigned

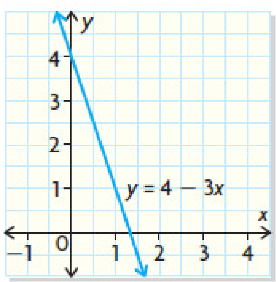
2. Vertical-line test

a)



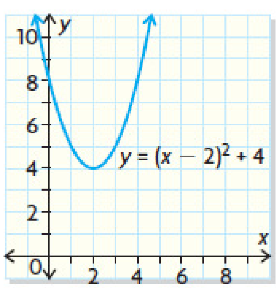
Not a function

b)



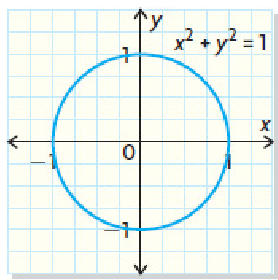
Function

c)



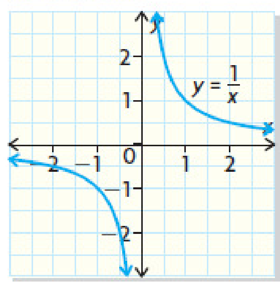
Function

d)



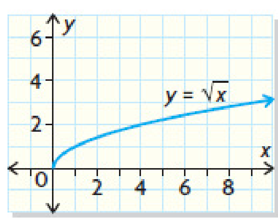
Not a function

e)



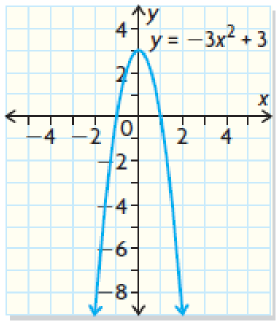
Function

f)



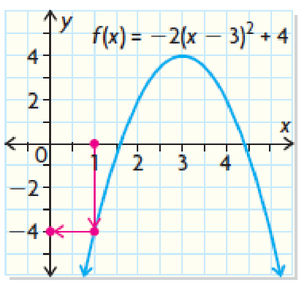
Function

3. Answers may vary; for example:



4. **a)** -7 **d)** $4b^2 + 6b - 5$
b) -5 **e)** $-8a - 1$
c) -2 **f)** 1 or -2

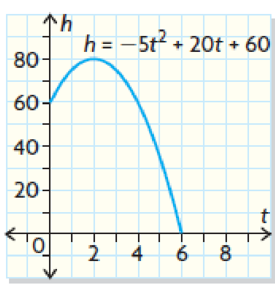
5. **a), b)**



a) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 4\}$
b) $f(1)$ represents the y-coordinate corresponding to $x = 1$.
c) **i)** 2 **ii)** -1 **iii)** $-2(-x - 2)^2 + 4$

6. $5, -1$

7. **a)**

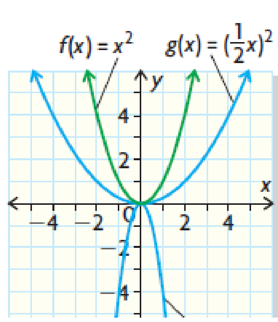


b) domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 6\}$,
range $= \{h \in \mathbf{R} \mid 0 \leq h \leq 80\}$
c) $h = -5t^2 + 20t + 60$

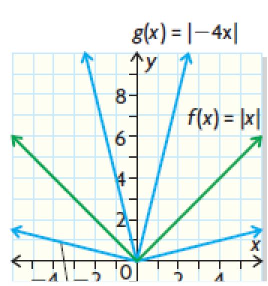
8. **a)** domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 3\}$
b) domain $= \{x \in \mathbf{R} \mid x \geq -2\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

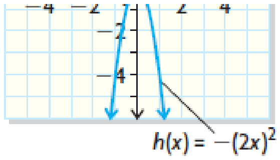
9. **a)** $y = \sqrt{4x}$
b) $y = \dfrac{1}{-\frac{1}{5}x}$

10. **a)**

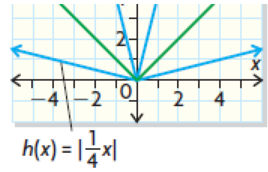


b)





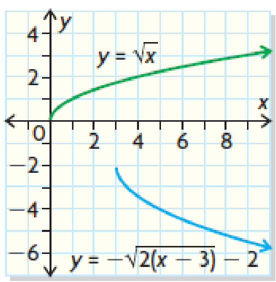
$f(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$g(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 0\}$



$f(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$g(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

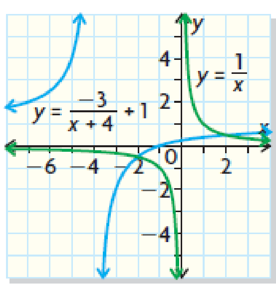
11. $\left(-\frac{5}{4}, 10\right)$

12. a) $y = -\sqrt{2(x-3)} - 2$



domain $= \{x \in \mathbf{R} \mid x \geq 3\}$, range $= \{y \in \mathbf{R} \mid y \leq -2\}$

b) $y = \dfrac{-3}{x+4} + 1$



domain $= \{x \in \mathbf{R} \mid x \neq -4\}$, range $= \{y \in \mathbf{R} \mid y \neq 1\}$

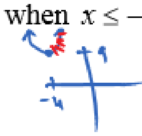
13. a) $4, -3$ b) $4, -3$ c) $-8, 6$ d) $-5, 2$

14. a) domain $= \{x \in \mathbf{R} \mid x \geqslant -4\}$, range $= \{y \in \mathbf{R} \mid y < -2\}$
b) domain $= \{x \in \mathbf{R} \mid x \leqslant 4\}$, range $= \{y \in \mathbf{R} \mid y < -1\}$
c) domain $= \{x \in \mathbf{R} \mid x \geqslant -5\}$, range $= \{y \in \mathbf{R} \mid y < 1\}$
d) domain $= \{x \in \mathbf{R} \mid x \leqslant 9\}$, range $= \{y \in \mathbf{R} \mid y > 3\}$

15. a) domain $= \{x \in \mathbf{R} \mid x \neq 2\}$, range $= \{y \in \mathbf{R} \mid y \neq 0\}$
b) domain $= \{x \in \mathbf{R} \mid x \leq 3\}$, range $= \{y \in \mathbf{R} \mid y \geq -4\}$
c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 3\}$

16.

a). $j(x) = (x+4)^2 + 9$, when $x \le -4$



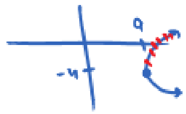
orig: $y = (x+4)^2 + 9$

inverse: $x = (y+4)^2 + 9$

$x - 9 = (y+4)^2$

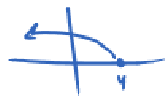
$\pm\sqrt{x-9} = y + 4$

$-4 \pm \sqrt{x-9} = y$



$\therefore j^{-1}(x) = -\sqrt{x-9} - 4, \; x \geq 9$

b) $f(x) = 3\sqrt{4-x}$



$x \le 4$

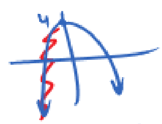
orig: $y = 3\sqrt{4-x}$

inverse: $x = 3\sqrt{4-y}$

$\frac{x}{3} = \sqrt{4-y}$

$\left(\frac{x}{3}\right)^2 = 4 - y$

$y = 4 - \left(\frac{x}{3}\right)^2$



$\therefore f^{-1}(x) = -\frac{x^2}{9} + 4, \; x \geq 0$

c). $h(x) = \frac{4x}{x-6}$

12

orig: $y = \frac{4x}{x-6}$

inverse: $x = \frac{4y}{y-6}$

$x(y-6) = 4y$

$xy - 6x = 4y$

$xy - 4y = 6x$

$y(x-4) = 6x$

$y = \frac{6x}{x-4}$

$\therefore h^{-1}(x) = \frac{6x}{x-4}$

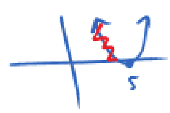
$x \neq 4$

14

d) $g(x) = x^2 - 10x + 25$, when $x \ge 5$

orig: $y = x^2 - 10x + 25$

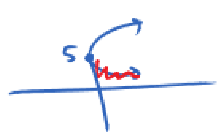
$y = (x-5)^2$ complete sq.



inverse: $x = (y-5)^2$

$\pm\sqrt{x} = y - 5$

$\pm\sqrt{x} + 5 = y$



$\therefore g^{-1}(x) = +\sqrt{x} + 5, \; x \geq 0$