



Big idea

This unit introduces you to several new concepts like: what is a function, how to use function notation, domain and range, as well as how to find an inverse of a function. All of these concepts are building the foundation for the grade 12 advanced functions course. The key characteristics you will study here will enable you to learn about more complex type functions in the next units of grade 11: quadratic, exponential, trigonometric, and discrete functions.



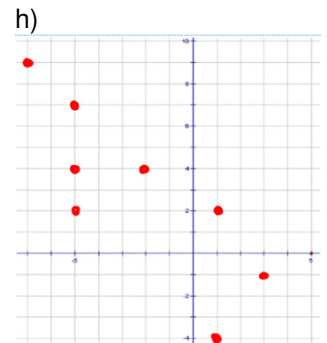
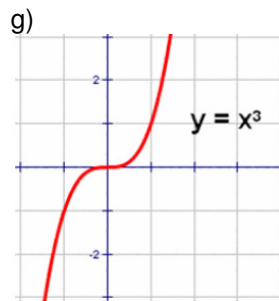
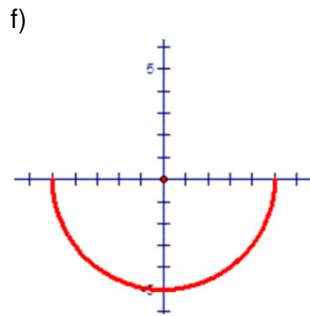
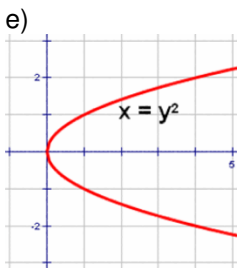
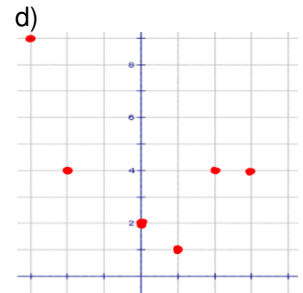
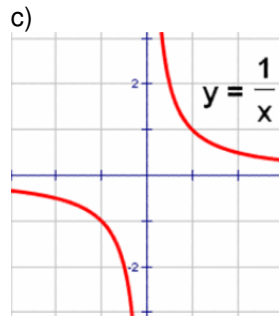
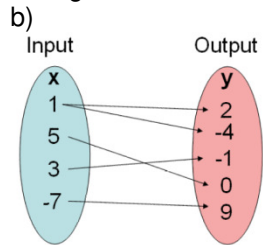
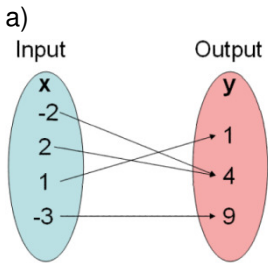
Feedback & Assessment of Your Success

			Finished assignment pages?	Summarized notes in a journal?	How many extra practice questions did you try in each topic?	Tentative TEST date:
Date	Pages	Topics	Made corrections?	Added your own explanations?		Questions to ask the teacher:
1.5days	2-5	Functions & Symbols (MCR) Journal #1				
	6-8	Function Notation (MCR) Journal #2				
1.5days	9-11	Domain Range (MCR) Journal #3				
	12-14	New Function Shapes (MCR) Journal #4				
1.5days	15-18	Transformations (MCR) Journal #5				
1.5days	19-22	Inverses (MCR) Journal #6				

ASSIGNMENT Relations and Functions (MCR)

1. Mathematics is a study of relationships. These relationships when written with variables are called **relations**. Sometimes a problem may relate variables that have interdependence. We usually call ____ variable as independent and the ____ variable the dependent. The independent variable can also be called the _____ and dependent variable the _____. This terminology is often used when the relations are **functions**.
 What is a function?

2. Determine if each of the following are functions or not.



i) $[(2, 4), (3, 7), (4, 10), (5, 13)]$

j) $[(-5, 3), (-3, 2), (-1, 1), (-3, 0)]$

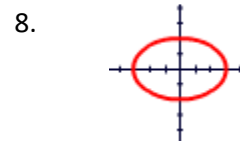
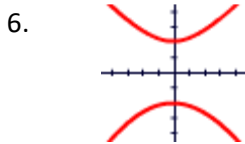
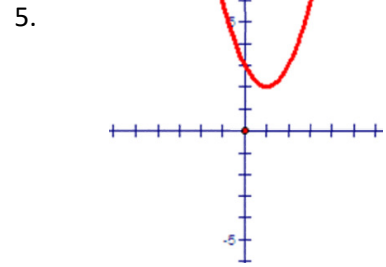
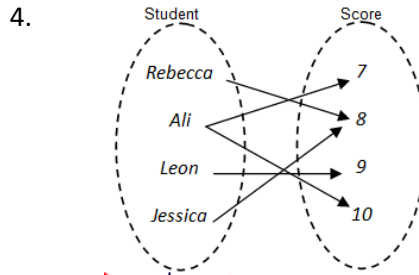
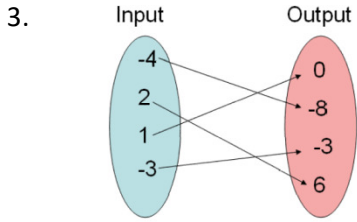
k) $[(-2, 5), (-1, 2), (0, 1), (1, 1), (2, 5)]$

l) $x - 6 = y^4 + x^2$

m) $x^2 + y = 5$

n) $4 + \sqrt{2y - 3x} = 10$

Determine if each of the following are functions or not.



9. $y = 3 - 6x$

10. $x^2 + 2xy + y^2 = 8$

11. $y = x^2 - 2x + 3$

12. $\{(-3,9), (-2,8), (-1,2), (-2,-4)\}$

13. $\{(2,5), (3,5), (4,10), (5,0)\}$

14. A vending machine produces pop, gum, chocolate bars, etc. depending on the button pressed.

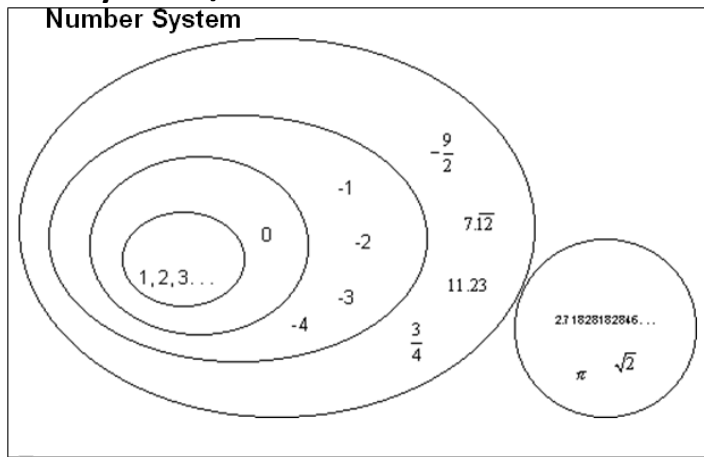
15. The input is the street address; the output is the postal code.

16. The input is the postal code, the output is the street address.

17. Sara asked each of her extended family members to measure his/her foot length. Then she graphed the relationship between foot length and age, using the age as an independent variable.

18. Come up with your own examples of real life relation that is a non-function and a relation that is a function.

New Symbols/Math shorthand



1. How to convert terminating decimals to fractions:
N= 34.765

2. How to convert repeating decimals to fractions:
N=45.1232323....

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Draw the following sets on a number line, if possible

3. $\{x \in \mathbb{Z} \mid -5 \leq x < 2\}$

4. $\{x \in \mathbb{N} \mid x < 4\}$

5. $\{x \in \mathbb{Q} \mid 0 < x < 1\}$

Draw the following sets on a number line, then convert to better notation.

6. $\{x \in \mathbb{R} \mid x > 3 \cap x \leq 4\}$

7. $\{x \in \mathbb{R} \mid x > 3 \cup x \leq 4\}$

8. $\{x \in \mathbb{Z} \mid -2 \leq x < 3 \cap -1 < x \leq 4\}$

9. Write the set of whole numbers using set notation.

10. Write the set of rational numbers using set notation.

11. Is $x > 3$ same or different to $3 < x$?

12. Is $\{3, 5, 10\}$ same or different to $\{5, 3, 10\}$?


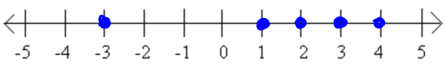
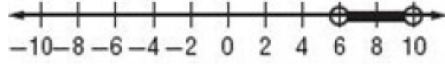

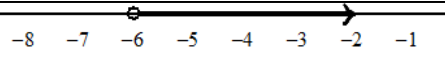
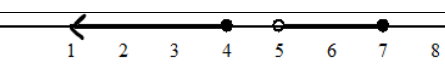
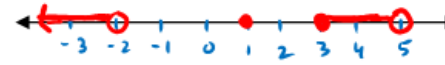
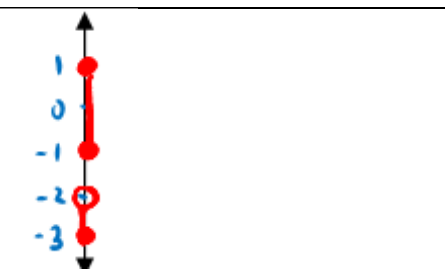
13. Is $(2, 3)$ same or different to $(3, 2)$?

14. Is $(2, 3)$ same or different to $x \in (2, 3)$?

15. What is wrong with $5 < x < -3$?

What is wrong with $x \in (6, 3)$?

Fill in the Table

	Picture	Set Notation	Interval Notation
16.			
17.			
18.			
19.			
20.			
21.			
22.			
23.			

ASSIGNMENT Function Notation (MCR)

1. What is **function notation**?
2. You have seen function notation for specific functions, but you just didn't know it. Indicate what is the name of the function, the input and the output of each of the following
 - a. $\cos(\theta)$.
 - b. $\sqrt[3]{(9)}$
3. $h(t) = |t + 2| + 3$; Find $h(-7)$
4. $g(a) = 3^{3a-2}$; Find $g(1)$
5. $h(n) = -2n^2 + 4$; Find $h(4)$
6. $h(t) = -2 \cdot 5^{-t-1}$; Find $h(-2)$
7. When can function notation be used? Explain by using this example: "Pythagorean theorem with one side as output and hypotenuse as input."
8. What could be confusing when using function notation? Explain by using this example:

Evaluate $m(3) - n$

 - a) if $m = 2$ and $n = 6$
 - b) if $m(x) = x^2$ and $n = 6$

For the functions $f(x) = 2x^4 - 3x^2$, $g(k) = \frac{6k}{k+1}$, $h(a) = \sqrt{a-5}$

9. $f(-3) + 4g(3)$

10. $2g\left(\frac{1}{2}\right)f(-1)$

11. $f(2)$

12. $2h(69) - 1$

13. $f(\sqrt{5}) - h(30)$

14. $g(3)^{h(6)}$

15. $g(n-1)$

16. $h(w^2 + 5)$

Isolate the output variable and record the answer in function notation.

17. $4x + \sqrt{y - 3x} = 1$

18. $3x = \frac{y+1}{y-2}$

19. $2x - 6\sin y = 18$

20. Write down the formula, in function notation for the perimeter of a rectangle, considering perimeter as input and length as output.

21. Write down the formula, in function notation for the volume of a rectangular prism considering height as input and length as output.

22. Write down the formula, in function notation for the area of the triangle, considering area as input and height as output.

23. Write down the formula, in function notation for the surface area of a cylinder, considering radius as input and height as output.

24. Write down the area of a circle as a function of its perimeter

ASSIGNMENT Domain and Range (MCR)

1. Domain and Range definitions:

2. Domain can be found without graphing. (Range is hard to find without graphing). What are two things that can make a relation undefined? How does that help you find domains of relationships without doing a graph?

3. Working with inequalities:

Find the domain of each function. Write in both set notation as well as interval notation

4. $y = 3x^2 - 8x + 1$

5. $y = \sqrt{2x+6} + 1$

6. $y = \sqrt{4-2x} - 9$

7. $y = \frac{1}{x-3}$

8. $y = \frac{1}{\sqrt{x-4}}$

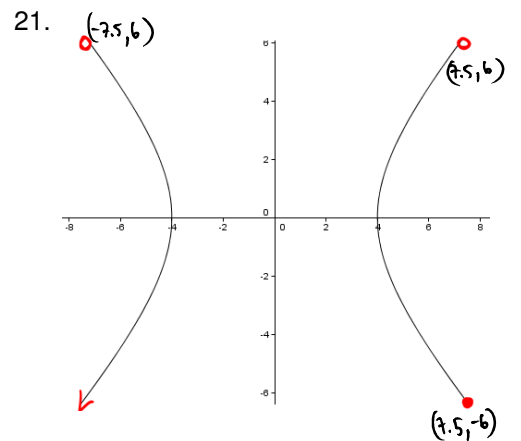
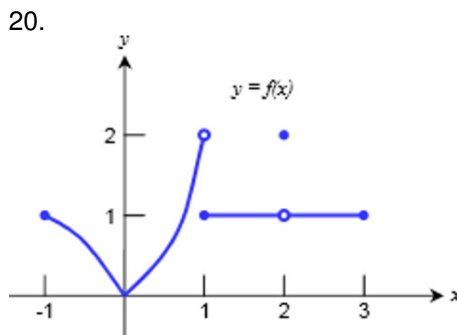
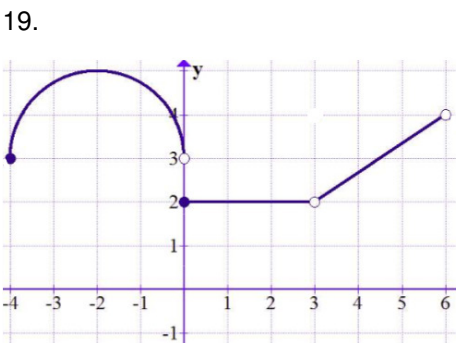
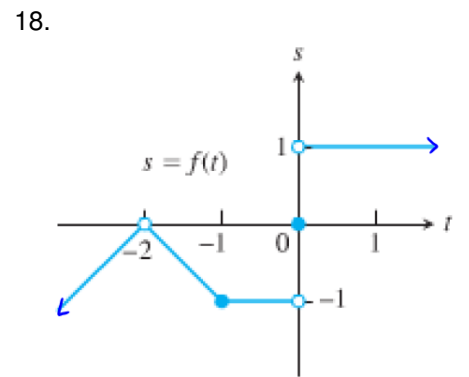
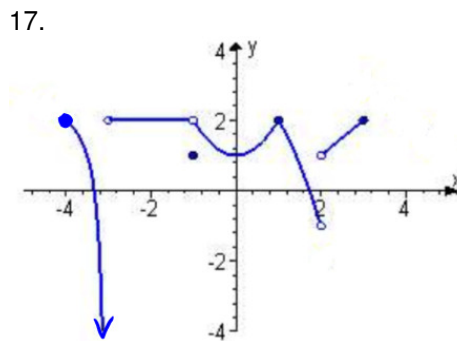
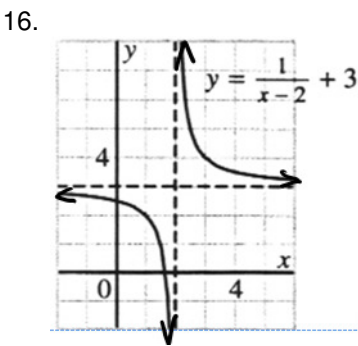
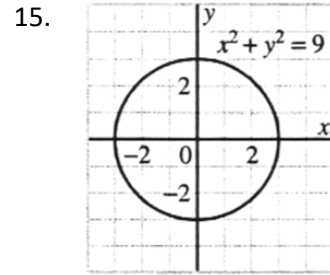
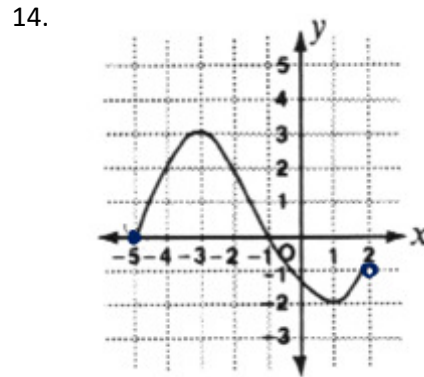
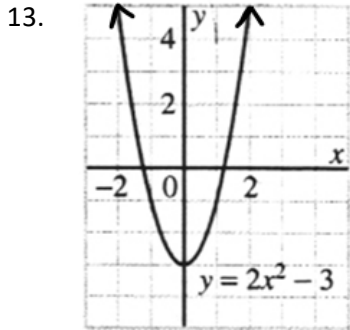
9. $y = \frac{2}{x^3 - 4x}$

Find the domain of each equation. Find both domain and range for graphs. Write #17-22 in set notation and #23-28 in interval notation

10. $f(x) = \frac{x+3}{x^3-49x}$

11. $y = \frac{x+5}{x^2+3x+2}$

12. $q = \sqrt{p-1} + 5$

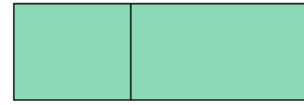


22. A golfer hits a ball into the air. The relationship between the height of the ball, h metres, and the time, t seconds, can be modelled by the equation

$$h = -4.9t^2 + 19.6t$$

- Determine the length of time that the ball is in the air.
- Determine the maximum height.
- What is the domain and range in the context of the real life problem.

23. A farmer has 540 m of fencing to enclose a rectangular area and divide it into two sections as shown.



- Write an equation to express the total area enclosed as a function of the width.
- Determine the domain and range of this area function.
- Determine the dimensions that give the maximum area.

ASSIGNMENT New Function Shapes (MCR)

Use the following website <https://www.desmos.com/calculator> Plot the following functions, sketch the shape, state domain and range.

LIST A (needed for this unit **MCR**)

$$f(x) = c \text{ any constant}$$

$$f(x) = x \text{ linear}$$

$$f(x) = x^2 \text{ quadratic}$$

$$f(x) = \sqrt{x} \text{ square root}$$

$$f(x) = |x| = \text{abs}(x)$$

absolute value

Note this is the same as

$$f(x) = |x| = \sqrt{x^2}$$

$$f(x) = \frac{1}{x} \text{ rational}$$

$$f(x) = x^3 \text{ cubic}$$

$$f(x) = \sqrt[3]{x} \text{ cube root}$$

LIST B (needed later in **MHF**)

even powers

$$f(x) = x^4$$

$$h(x) = x^6$$

odd powers

$$f(x) = x^5$$

$$h(x) = x^7$$

even roots

$$f(x) = \sqrt[4]{x}$$

$$h(x) = \sqrt[6]{x}$$

odd roots

$$f(x) = \sqrt[5]{x}$$

$$h(x) = \sqrt[7]{x}$$

Exponentials

$$f(x) = 2^x$$

$$g(x) = 3^x$$

$$h(x) = 0.5^x$$

$$j(x) = 0.1^x$$

Logarithms

$$f(x) = \log_5 x$$

$$g(x) = \log_{10} x$$

$$h(x) = \log_{0.8} x$$

$$j(x) = \log_{0.5} x$$

$$f(x) = e^x$$

natural exponential
where constant $e=2.718\dots$

$$f(x) = \log_e x = \ln x$$

natural logarithm

Continued LIST B (needed later in **MHF**) *SWITCH settings to DEGREES on x-axis!*

primary trigonometric

$$f(x) = \sin x$$

secondary (reciprocal) trig

$$f(x) = \frac{1}{\sin x} = \csc x$$

inverse trig

$$f(x) = \sin^{-1} x = \arcsin x$$

$$f(x) = \cos x$$

$$f(x) = \frac{1}{\cos x} = \sec x$$

$$f(x) = \cos^{-1} x = \arccos x$$

$$f(x) = \tan x$$

$$f(x) = \frac{1}{\tan x} = \cot x$$

$$f(x) = \tan^{-1} x = \arctan x$$

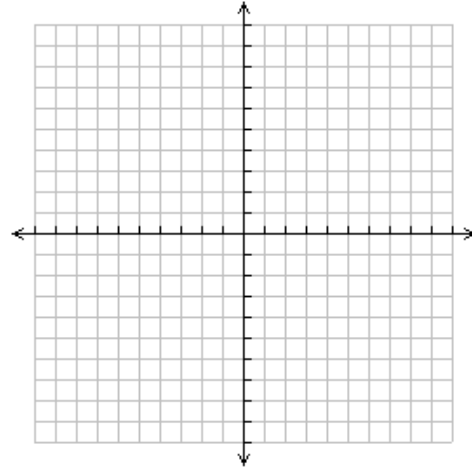
ASSIGNMENT Transformations (MCR)

Review what you know of quadratics $y = (6 - 2x)^2 - 5$

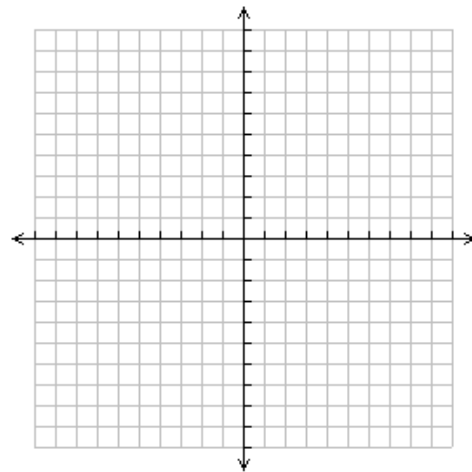
1. Factor out the coefficient on x and pull it out of the square function completely. Then state transformations, and sketch.
2. Not every function we will study will enable us to pull out the coefficient on x . Discuss what that coefficient could represent in terms of transformations. Use image points of both versions for comparison.
3. In general $y = af(k(x - d)) + c$. Describe what each letter means.

Identify a) the parent function, b) the transformations that were applied, c) sketch, include the parent sketch for comparison d) domain & range

4. $y = -3\sqrt{0.5x - 3} - 5$



5. $y = \frac{5}{0.25x + 1}$



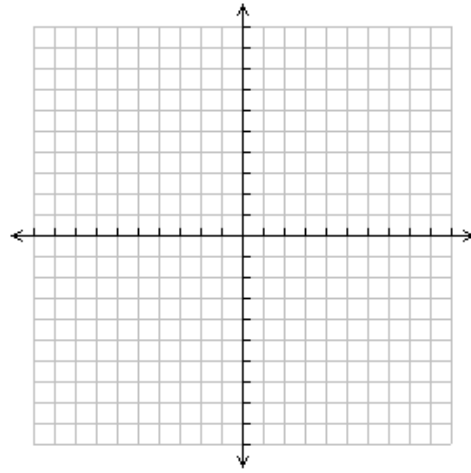
6. Write the equation given the parent function and the transformations. Sketch a small version of the resulting shape

a. $f(x) = |x|$, reflected in x-axis, horizontal compression by 2 (decide to use 2 or $\frac{1}{2}$), shift up by 5

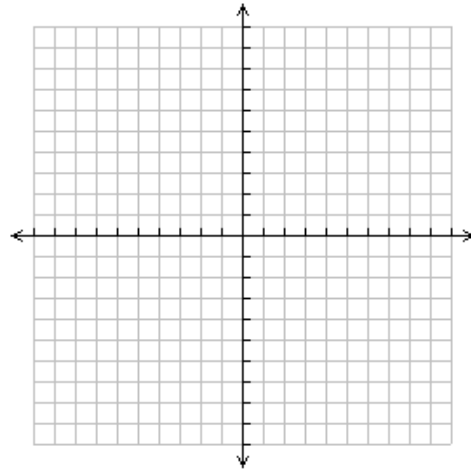
b. $g(x) = x^3$, reflected in y-axis, horizontal stretch by 3 (decide to use 3 or $\frac{1}{3}$), vertical compression by 2 (decide to use 2 or $\frac{1}{2}$), shift left 4.

Identify a) the parent function, b) the transformations that were applied, c) sketch, include the parent sketch for comparison d) domain & range

7. $i(x) = 2\sqrt[3]{4-x} - 1$



8. $f(x) = -|0.5x + 3| + 8$



9. Write the equation given the parent function and the transformations. Sketch a small version of the resulting shape

a. $f(x) = \frac{1}{x}$

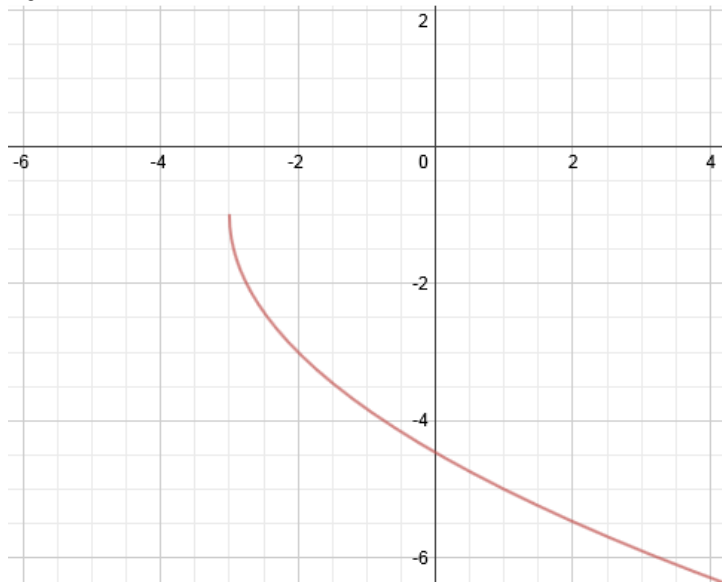
- vertical stretch by 2 (decide to use 2 or $\frac{1}{2}$)
- reflection in y-axis
- horizontal shift left by 8 units
- vertical shift up 6 units

b. $g(x) = \sqrt{x}$

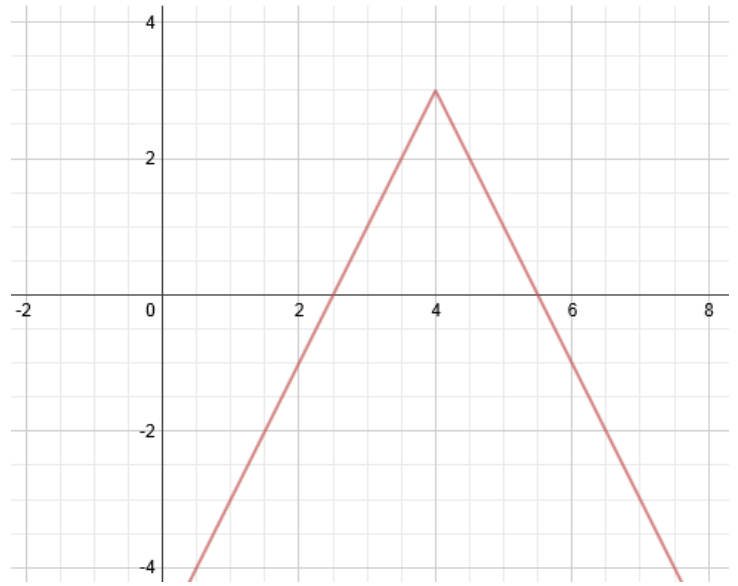
- horizontal compression by 2 (decide to use 2 or $\frac{1}{2}$)
- horizontal shift left 5 units
- vertical shift down 7 units

Identify the parent function, the transformations and write an equation for the graph shown.

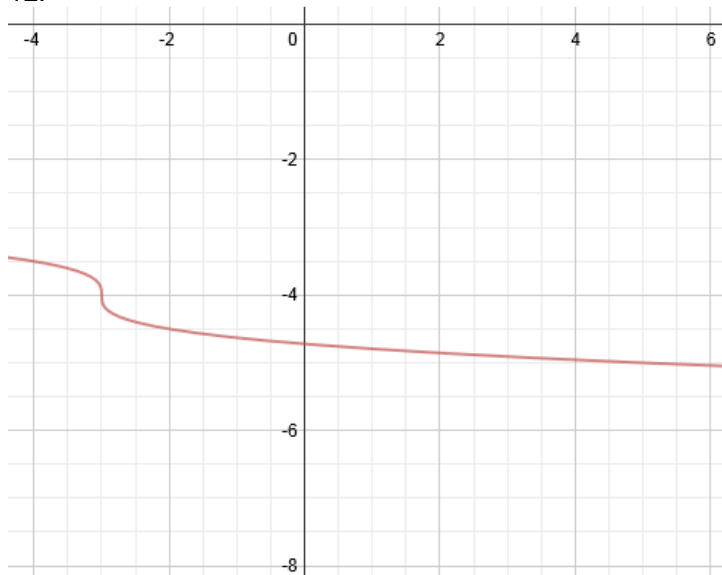
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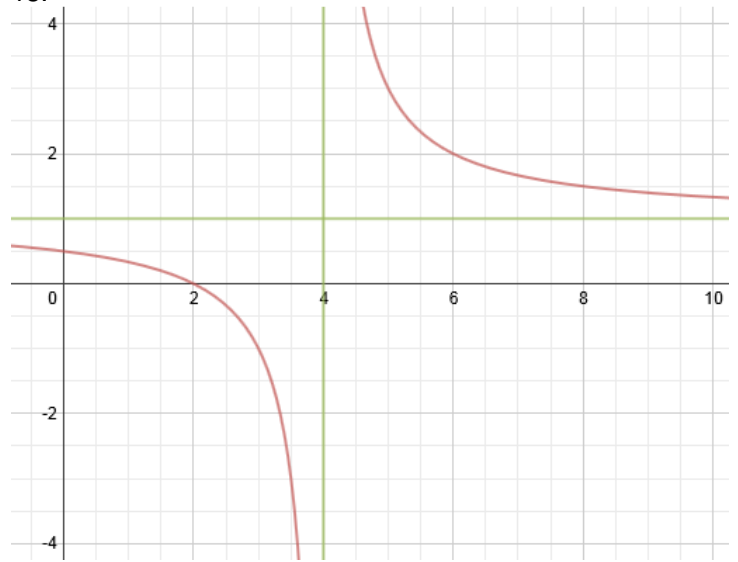
11.



12.



13.



ASSIGNMENT Inverses (MCR)

1. Clarify the differences between the words that sometimes get confused.

OPPOSITE

RECIPROCAL

INVERSE

You have already seen inverses in action any time you had to rearrange a formula

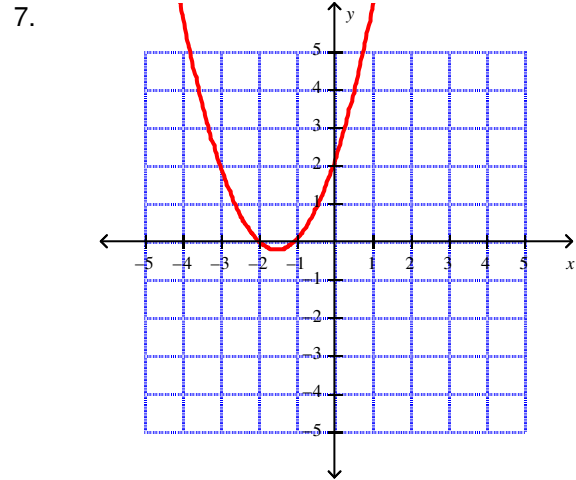
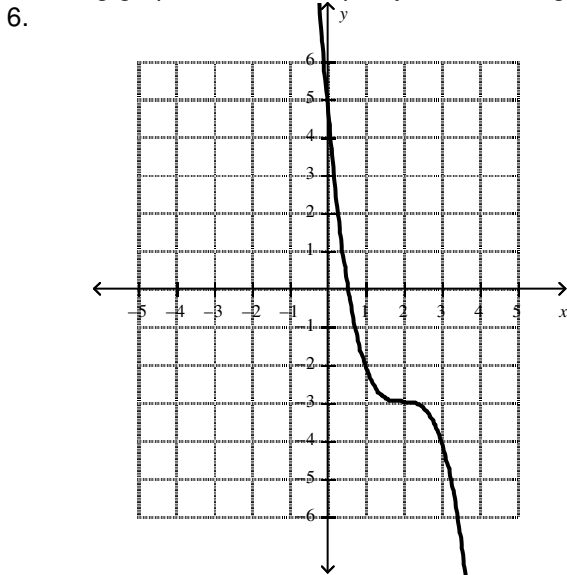
2. $V = \frac{D}{T}$, solve for T

3. $V = \pi r^2 h$, solve for r

4. $A = p + prt$, solve for p

5. $A = 2\pi r^2 + 2\pi rh$, solve for r

Notice in all examples of rearranging the formula the input became the output and vice versa. Sketch the inverses of the following graphs. Do the shapes you're drawing look familiar? Are all inverses functions?



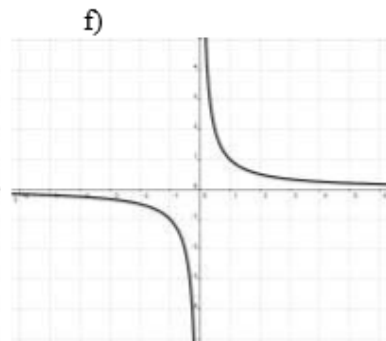
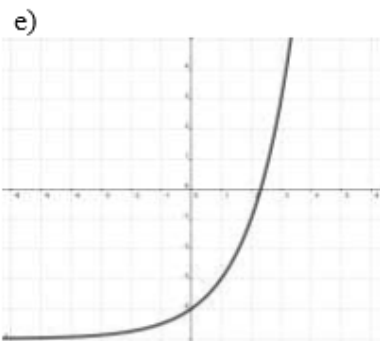
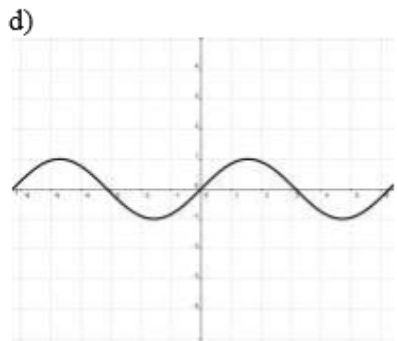
8. One to one functions:

9. Label each relation properly: “Not a function”; “Function (*not* one-to-one)”; “Function (one-to-one)”.

- a) (3, 2)
(-2, 7)
(-2, 5)
(1, 3)
(5, 9)

- b) (0, 3)
(-2, 5)
(1, 6)
(2, -6)
(3, 4)

- c) (-6, 2)
(-4, 6)
(1, 3)
(-1, 7)
(0, 2)



In the formulas you were rearranging on previous page notice it was easy if the variable you were solving for appeared once. It's all about getting the variable to appear once – common factoring or completing the square usually does the trick. After that its just doing BEDMAS backward. Find the inverses of the following – at the end, since these are generic variables x and y , switch them so input is always denoted with x and output with y . If variables are not generic, like formulas above, do not do the switch. The inverses can also be notated using $f^{-1}(x)$ for the new output. **NOT a POWER!**

10. $f(x) = -2(15 - 5x)^3 + 16$

11. $f(x) = \frac{x}{x-3}$

Find the inverses of the following and notate the answer in inverse function notation

12. $f(x) = \frac{x+1}{x-2}$

13. $4 + \sqrt{2-3x} = f(x)$

14. Find the inverse and state the domain of $g^{-1}(x)$ that would ensure the inverse is one-to-one

$$g(x) = \sqrt{\frac{1-x}{2}} - 6$$

15. Restrict the domain of $f(x)$ so that the function is one to one and find the inverse corresponding to your choice of domain

$$f(x) = 3(x+5)^2 + 2$$

16. Domain of $j(x)$ is $-4 \leq x \leq 6$ and the range is $0 \leq y \leq 10$. A new function $k(x)$ is formed when transformations are applied as follows: $k(x) = -2j(3x+6) - 4$, what is the range of $k^{-1}(x)$?

17. Restrict the domain so that the function is one to one and find the inverse corresponding to your choice of domain

$$f(x) = -x^2 + 2x - 9$$

18. Check if the following functions are inverses using the definition

f and f^{-1} are inverses if and only if

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

a.

$$g(x) = -\frac{2}{x} - 1$$

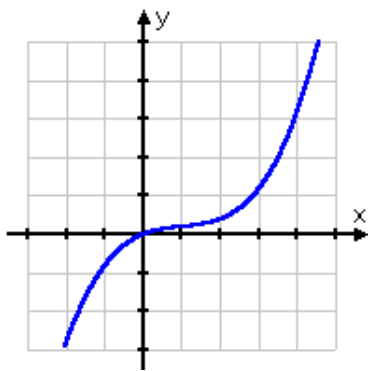
$$f(x) = -\frac{2}{x+1}$$

b.

$$f(n) = -(n+1)^3$$

$$g(n) = 3 + n^3$$

19. Sketch the inverse



20. Can you think of two different type of functions for which the inverse is itself ie. the graph will remain the same when input and output get switched?